Black body radiation

When light hits any object, it can be transmitted, reflected/scattered, or absorbed.

Black body absorbs all radiation that falls on its surface.

In thermal equilibrium, it also emits the same amount of energy that it absorbs. However, the frequencies emitted are different from frequencies absorbed (in general), and reflect on the temperature of the black body.

Empirical laws:

Total emitted energy: $P_{\text{total}} = 6T^4 \cdot A$

Stefan-Boltzmann law

$6 = 5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$

Wein's displacement law

$\lambda_{\text{max}} \cdot T = 2.89 \cdot 10^{-3} \text{ m.K}$
Black body spectrum

Energy spectral density - how much energy $\Delta E$ is emitted in the narrow spectral region $\Delta \lambda$

$$\Delta E = u(\lambda, T) \Delta \lambda \cdot \nu$$

Plank found empirical (at first) formula

$$u(\lambda, T) = \frac{\text{const}}{\lambda^3 (e^{\frac{\text{const}}{\lambda k_B T}} - 1)}$$

Using existing understanding of thermodynamics this formula could not be reproduced.

Classical description

EM waves are emitted by oscillating charges (now known as atoms)

Simple model of a black body - all-absorbing box with radiation inside and small hall to let it escape
Inside such a box, EM waves must form standing waves:

\[ \lambda_i = L \quad \lambda_2 = L/2 \quad \lambda_n = L/n \quad \text{7 modes} \]

\[ f_1 = c/L \quad f_2 = 2c/L \quad f_n = nc/L \]

How many frequencies fall in the range \( \Delta f \) around some frequency \( f \)? Rayleigh showed that there are

\[ N(f) \, \Delta f = \frac{8\pi}{c^3} \, V \, f^2 \, \Delta f \]

Classical principle of statistical mechanics:

Equipartition of energy: each EM radiation mode receives some amount of energy \( \approx k_B T \)

\[ U(f, T) = \frac{\Delta E}{V \, \Delta f} \approx \frac{8\pi}{c^3} \, f^2 \, k_B T \quad f \to \infty \]

Ultraviolet catastrophe

Plank's desperate solution: quantization of energy:

Energy cannot be emitted continuously, but in finite "bunches" he called "quanta" (Latin "how much")

\[ E_f = h \cdot f \quad h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \]

Plank's constant
This resolves UV catastrophe:
For high frequencies if $hf > k_BT$, these high-energy quanta cannot be emitted — UV cut-off!

Planck didn't like his solution, and worked hard for 6 years (b/w 1900-1905) to prove it wrong and find "a better solution."

Einstein picked it up to explain a photoelectric effect (discovered by Hertz in 1887, characterized carefully by Lenard in 1900-1902)

The effect itself is nicely explained classically, but some things do not make sense:

Classical EM wave: Intensity $\propto$ amplitude $^2$
1. Energy of the electrons is proportional to frequency of light
2. Intensity of light changes only the number of emitted e

Einstein pointed out that quantum description of light resolves this paradox

$$hf = Ke + \Phi$$

work function,

binding energy of e inside the metal