Thin-film interference

Interference is visible when two fields of the same frequency overlap with a constant phase b/w them.

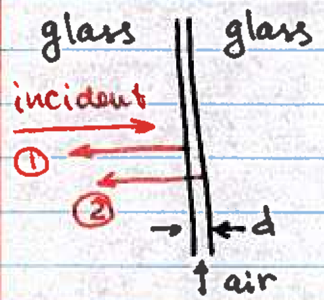
Phase $= 0, 2\pi, 4\pi \ldots = 2\pi m$ – constructive
$\pi, 3\pi, 5\pi \ldots = \pi + 2\pi m$ – destructive

If two waves travel different distance Δx aquired phase $k\Delta x = \frac{2\pi}{\lambda}\Delta x = 2\pi\left(\Delta x/\lambda\right)$

Constructive : $\Delta x = m\lambda$ destructive : $\Delta x = \lambda/2 + m\lambda$

Interference is sensitive to changes in pathlength of the order of wavelength (sub – micron)

Thus, we can see interference easily if light reflects from two very close surfaces



① reflected from the top

② reflected from the bottom
extra distance the second beam travels – $2\cdot d$ (air gap thickness)
extra phase it aquires
$$\varphi = \frac{2\pi}{\lambda}\cdot 2d \underbrace{+\pi} = 4\pi\frac{d}{\lambda} + \pi$$

Note: every time an e-m wave reflects off a material with higher refractive index, it aquires extra phase shift of π.

What color will be reflected the most?

Constructive interference: $4\pi \frac{d}{\lambda} + \pi = 2\pi m$

if $d < \lambda$: $4\pi \frac{d}{\lambda} + \pi = 2\pi \Rightarrow 4\pi \frac{d}{\lambda} = \pi$

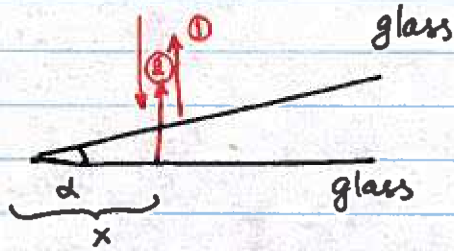
$d_{constr} = \frac{\lambda}{4} \quad \left(\frac{2m-1}{4} \lambda \right)$

Destructive interference (won't see this color)

$4\pi \frac{d}{\lambda} + \pi = \pi$ or $4\pi \frac{d}{\lambda} + \pi = 3\pi \Rightarrow 4\pi \frac{d}{\lambda} = 2\pi$

$d = 0$

$d_{destr} = \frac{\lambda}{2} \quad \left(\frac{m}{2} \lambda \right)$



$d \approx x \cdot \tan \alpha \approx x \cdot \alpha$

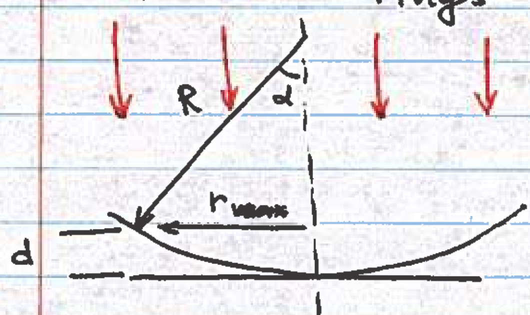
if illuminated by a monochromatic light

distance b/w bright or dark spots is

$\Delta x \cdot \alpha = \frac{\lambda}{2}$

$\Delta x = \lambda / 2\alpha$

Newton rings



extra distance

$d = R - R\cos\alpha = R(1-\cos\alpha)$

Constructive interference:

$\frac{2\pi}{\lambda} \cdot 2d_{max} + \pi = 2\pi m \qquad m = 1, 2, \ldots$

$4 \frac{d_{max}}{\lambda} = 2m-1$

$d_{max} = \frac{2m+1}{4} \lambda$; $\cancel{R(1-\cos\alpha)}$

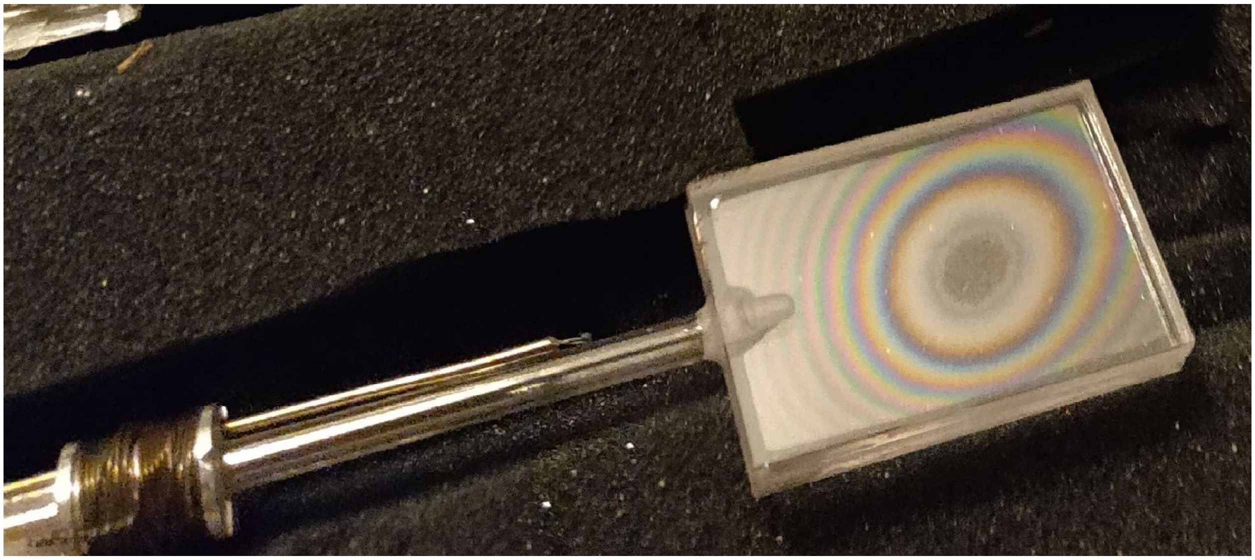
$r_{max} = R \sin\alpha \approx R \cdot \alpha$ for small α

$d = R(1-\cos\alpha) \approx \frac{1}{2} R\alpha^2$ for small α

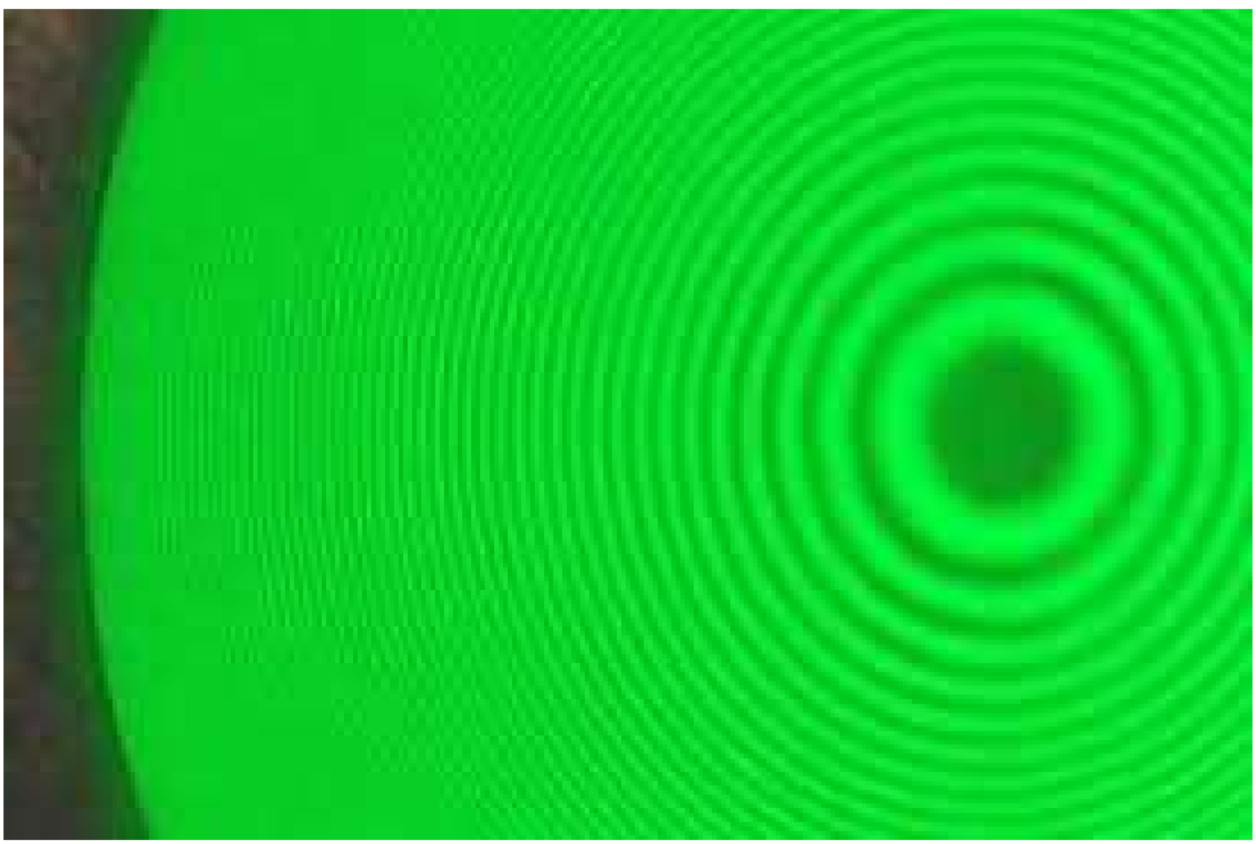
If we need to find the radius of the curvature

$R = \frac{r_{max}^2}{\lambda(m + 1/2)}$

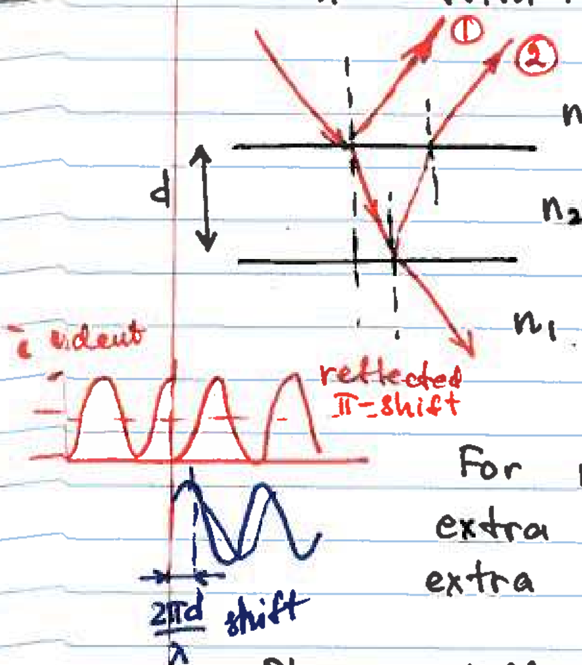
Interference in a small air gap between two flat glass windows



Newton rings (interference due to an air gap between flat and curved surfaces



Thin film : soab bubbles



Beam ① → phase flips by $180^\circ = \pi$ (since $n_1 < n_2$ @ reflectio

Beam ② → no phase flip (since $n_2 > n_1$ @ reflection) but has exta phase due to propagation in the film

For nearly - normal incidence

extra physical distance — $2 \times d = \Delta x$

extra optical phase — $\frac{2\pi n_2}{\lambda} \Delta x = \frac{4\pi \cdot d n_2}{\lambda}$

Phase difference b/w beams 1 and 2

$$\left(\frac{4\pi d n_2}{\lambda} - \pi \right) = 2\pi m \quad \text{for constructive interference} \quad m = 0, 1, 2 \ldots$$

$$= 2\pi m + \pi \quad \text{for destructive interference}$$

"Bright" reflection $\frac{4\pi d n_2}{\lambda} - \pi = 2\pi m \Rightarrow \frac{4\pi d n_2}{\lambda} = \pi(2m+1)$

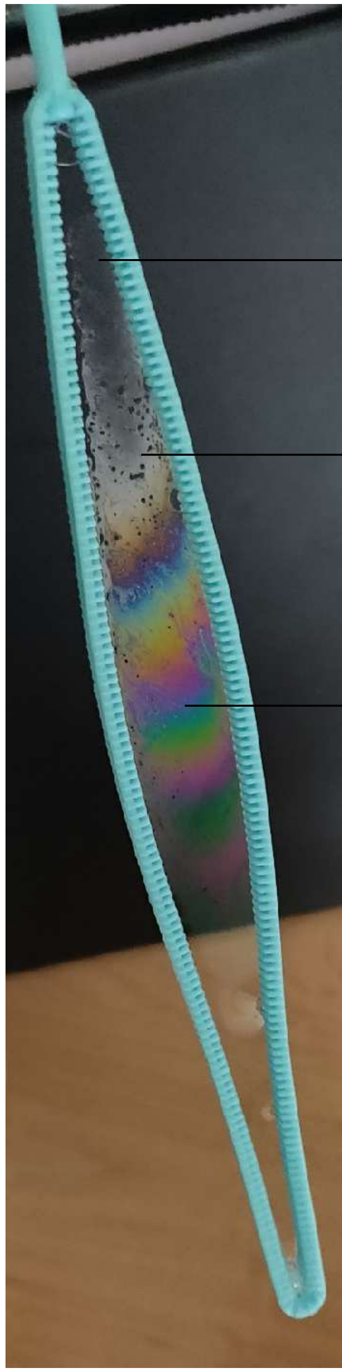
$$d_{max} = \frac{2m+1}{4} \cdot \lambda / n_2$$

"Dark" reflection $\frac{4\pi d n}{\lambda} \cancel{-\pi} = 2\pi m \cancel{-\pi} \Rightarrow \frac{4\pi d n_2}{\lambda} = 2\pi m$

$$d_{min} = \frac{m}{2} \cdot \lambda / n_2$$

Very thin film → min reflection for any color

First bright band occures for smaller wavelength — blue, then followes the spectrum to the red.



Constructive and destructive interference in reflection of white light from a vertical soap bubble. Due to gravity the thickness of the film changes from top to bottom,

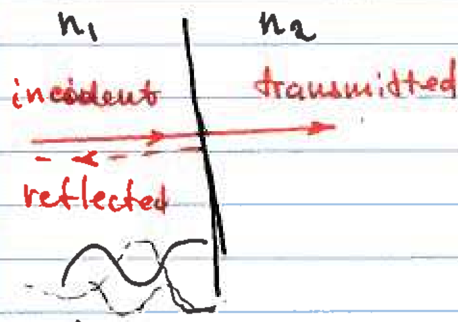
Very thin film, d<<λ
Almost perfect destructive interference

Thickness still significantly smaller than the light wavelength, all colors reflect a little, so overall reflection is white.

Thickness is comparable with visible light wavelength, conditions for constructive interference are distinctly different for each color.

Thin film interference: phase difference

Reflection



On the boundary

$E_r = r \cdot E_i$

For the normal incidence

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

Usually we measure light power or intensity

$$P = \frac{\text{energy}}{\text{time}} \qquad I = \frac{\text{energy}}{\text{time} \cdot \text{area}}$$

$$P \propto |E|^2 \qquad P_r = |r|^2 P_i = R \cdot P_i$$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

For the glass $n_2 = 1.5$, $n_1 = 1$

$$R = \left(\frac{0.5}{2.5}\right)^2 = \frac{1}{25} \approx 4\%$$

However $r = -\frac{1}{5}$ → ~~if we go from a~~ the phase of the reflected wave flips by 180°, if we go from air to glass (but does not if from glass → air)

This is a general rule → the phase of the reflected wave at any angle flips by 180° when reflected of the material with higher refractive index, and does not if ~~the~~ travels into the material with a lower refractive index