Thin-film interference

Interference is visible when two fields of the same frequency overlap with a constant phase b/w them.

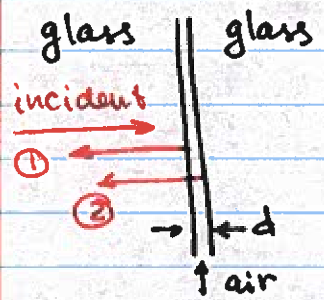
Phase $= 0, 2\pi, 4\pi \ldots = 2\pi m$ — constructive
$\pi, 3\pi, 5\pi \ldots = \pi + 2\pi m$ — destructive

If two waves travel different distance Δx aquired phase $k\Delta x = \frac{2\pi}{\lambda}\Delta x = 2\pi\,(\Delta x/\lambda)$

Constructive : $\Delta x = m\lambda$ destructive : $\Delta x = \lambda/2 + m\lambda$

Interference is sensitive to changes in pathlength of the order of wavelength (sub – micron)

Thus, we can see interference easily if ~~\#~~ light reflects from two very close surfaces



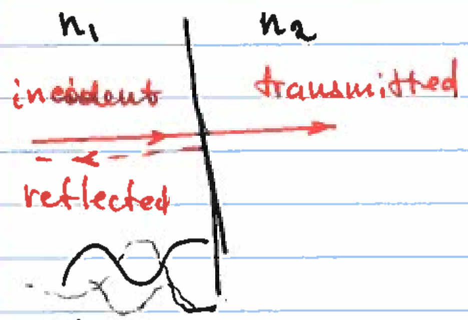
① reflected from the top

② reflected from the bottom
extra distance the second beam travels — $2 \cdot d$ (air gap thickness)
extra phase it aquires
$$\varphi = \frac{2\pi}{\lambda} \cdot 2d \, \underline{+\pi} = 4\pi\frac{d}{\lambda} + \pi$$

Note: every time an e-m wave reflects off a material with <u>higher</u> refractive index, it aquires extra phase shift of π.

Thin film interference: phase difference

Reflection



On the boundary

$E_r = r \cdot E_i$

For the normal incidence

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

Usually we measure light power or intensity

$$P = \frac{\text{energy}}{\text{time}} \qquad I = \frac{\text{energy}}{\text{time} \cdot \text{area}}$$

$$P \propto |E|^2 \qquad P_r = |r|^2 P_i = R \cdot P_i$$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

For the glass $n_2 = 1.5$, $n_1 = 1$

$$R = \left(\frac{0.5}{2.5} \right)^2 = \frac{1}{25} \approx 4\%$$

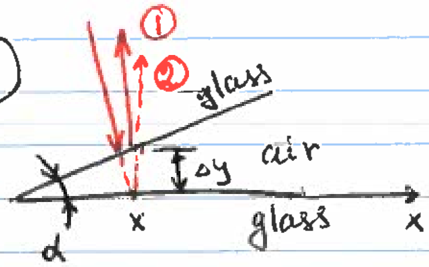
~~if we go from a~~

However $r = -\frac{1}{5}$ $\longrightarrow$ the phase of the reflected wave flips by 180°, if we go from air to glass (but does not if from glass $\to$ air)

This is a general rule $\longrightarrow$ the phase of the reflected wave at any angle flips by 180° when reflected of the material with higher refractive index, and does not if ~~the~~ travels into the material with a lower refractive index

Thin film interference

(1)



Beam 1: glass-air interface, no phase shift
Beam 2: ~~g~~ air-glass interface $+\pi$ phase shift
<u>and</u> travel in the gap

distance travelled $2\times\Delta y = \cancel{2x}\cdot \tan\alpha \approx 2x\cdot\alpha$

optical phase change $2\pi/\lambda \cdot 2\Delta y = 4\pi \frac{x\cdot\alpha}{\lambda}$

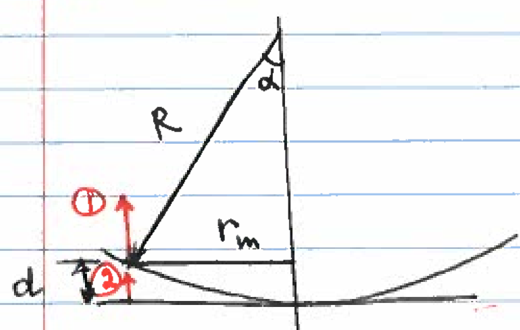
Total phase shift b/w the beams $= 4\pi\frac{x\cdot\alpha}{\lambda} + \pi$

$x=0 \rightarrow$ shift $=\pi \rightarrow$ dark fringe

distance b/w two bright or two dark fringes ~~Δx~~ $4\pi\frac{\Delta x\cdot\alpha}{\lambda} = 2\pi \Rightarrow \Delta x = \lambda/2\alpha$

smaller $\alpha \rightarrow$ larger Δx

(2) Newton rings



Like in the previous problem, the phase difference b/w two reflected beams is

$\frac{2\pi}{\lambda}\cdot 2d + \pi$

$d = R - R\cos\alpha = R(1-\cos\alpha)$

Constructive interference $\rightarrow$ phase difference $2\pi m$

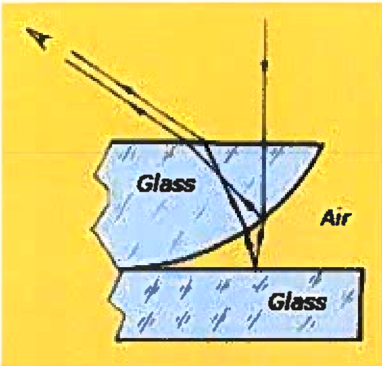
$$\frac{2\pi}{\lambda}\cdot 2R\underbrace{(1-\cos\alpha)}_{\approx \alpha^2/2 \text{ for } \alpha \ll 1} + \pi = 2\pi m (+2\pi) \qquad m=0,1,2\ldots$$

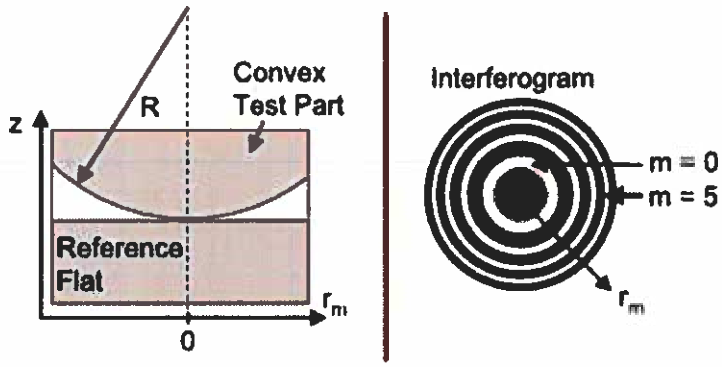
$$\frac{\cancel{2\pi}}{\lambda}\cdot R\alpha^2 = \cancel{2}\pi(m+1/2)$$

$R\alpha^2 = \lambda(m+1/2)$

$r_m^2 = R^2\alpha^2 = R\lambda(m+1/2)$

$$R = \frac{r_m^2}{\lambda(m+1/2)}$$





$$R = \frac{r_m^2}{\lambda\left(m + \frac{1}{2}\right)}$$