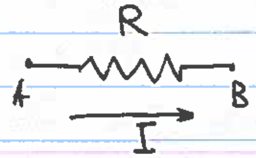
Basic circuit components

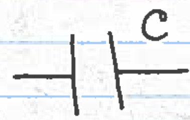
Resistor



$V_R = V_B - V_A = -IR$

voltage is proportional to current

Capacitor

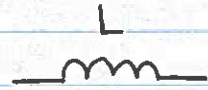


$V_C = Q/C = \frac{1}{C}\int_{-\infty}^{t} I(t')dt'$

* empty capacitor = wire

* full capacitor = no current circuit break

Inductance

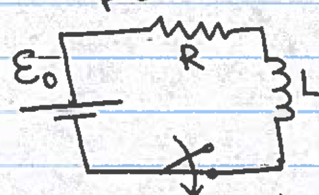


$V_L = -L\frac{dI}{dt}$

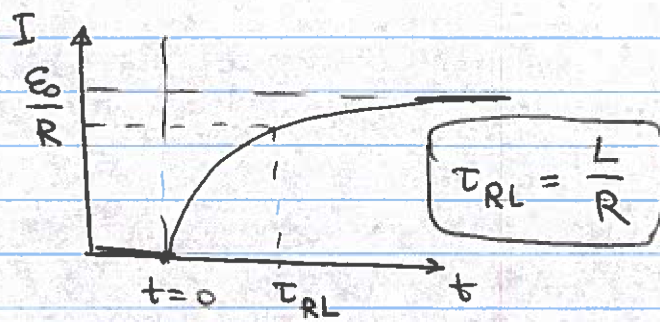
induced emf agains current change

* immediately after current change – same current (maintaindd by induced emf)
* in steady state = wire

Simple RL circuit

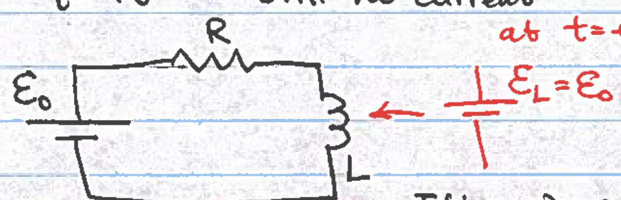


closed at $t=0$



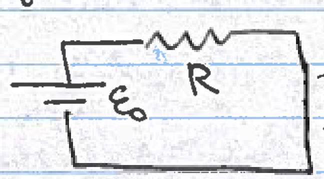
$t=-0$ → no current

$t=+0$ → still no current



$I(t=+0)=0$

$t \to \infty$



$I(t\to\infty) = \frac{\mathcal{E}_0}{R}$

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = \mathcal{E}_0 - IR$$

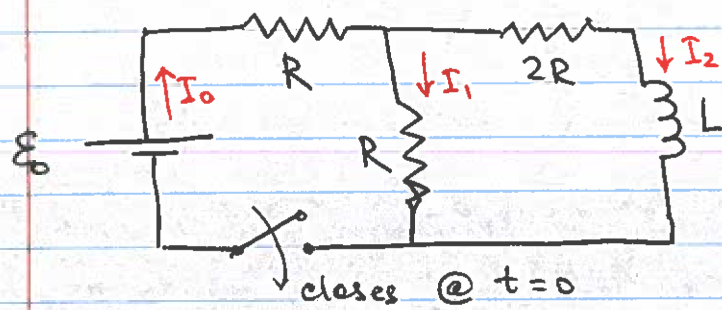
$$\frac{L}{R}\frac{dI}{dt} = \frac{\mathcal{E}_0}{R} - I$$

$$\int_0^{I(t)} \frac{dI}{\frac{\mathcal{E}_0}{R} - I} = \int_0^t \frac{R}{L}\,dt$$

$$\ln\frac{\mathcal{E}_0/R - I(t)}{\mathcal{E}_0/R} = -\frac{R}{L}t$$

$$I(t) = \frac{\mathcal{E}_0}{R} - \frac{\mathcal{E}_0}{R}e^{-\frac{R}{L}t}$$

More complicated circuit



$t<0$ $\quad I=0$

$I_1=0$

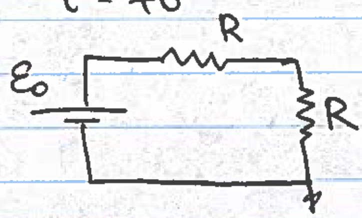
$I_2=0$

Immediately after the switch is closed

$I_2=0$ — no current in the second blanch

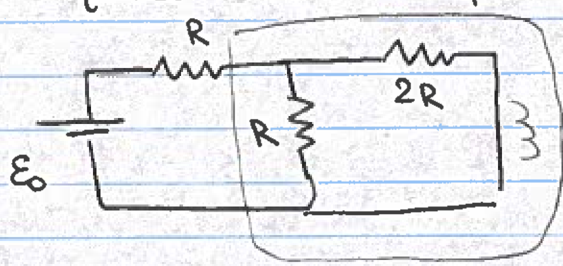
$t=+0$

$I=I_1 = \mathcal{E}_0/2R$

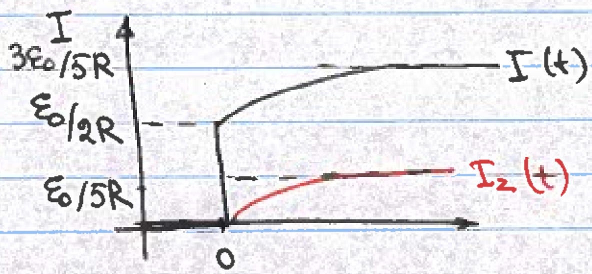


$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \qquad R_{eq} = \frac{2R}{3}$

$t \to \infty$



$I = \dfrac{\mathcal{E}_0}{R+\frac{2R}{3}} = \dfrac{3\mathcal{E}_0}{5R}$



Exact solution

① $I = I_1 + I_2$

② $\mathcal{E}_0 - IR - I_1 R = 0$

③ $\mathcal{E} - IR - I_2 \cdot 2R - L\dfrac{dI_2}{dt} = 0$

from ② $\quad I_1 = \mathcal{E}_0/R - I$

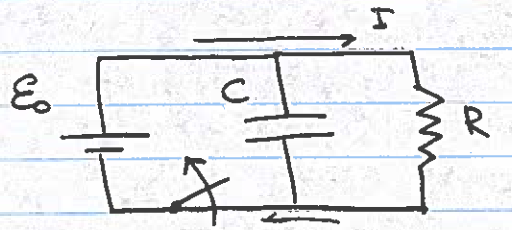
① $\quad I = \mathcal{E}_0/R - I + I_2 \Rightarrow I = \mathcal{E}_0/2R + I_2/2$

$$-L\frac{dI_2}{dt} - \frac{5}{2}RI_2 + \frac{\mathcal{E}_0}{2} = 0$$

$$I_2 = \frac{\mathcal{E}}{5R}\left(1 - e^{-\frac{5R}{2L}\cdot t}\right) \qquad \tau_{RL} = \frac{2L}{5R}$$

$$I = \frac{\mathcal{E}_0}{2R} + \frac{1}{2}I_2 = \frac{3\mathcal{E}_0}{5R} - \frac{\mathcal{E}_0}{10R}e^{-t/\tau_{RL}}$$

Both a capacitor and an inductor serve as a storage of electric or magnetic energy that ~~they~~ ~~release~~ allows them ~~the~~ to delay the responce to voltage or current change

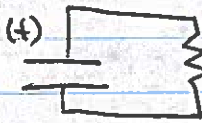


$t<0 \quad I_R = \mathcal{E}_0/R$

$V_C = \mathcal{E}_0$

no current through C

Once the battery is disconnected



$V_C(t) - IR = 0$

$-IR + Q/C = 0$

$I = -dQ/dt$

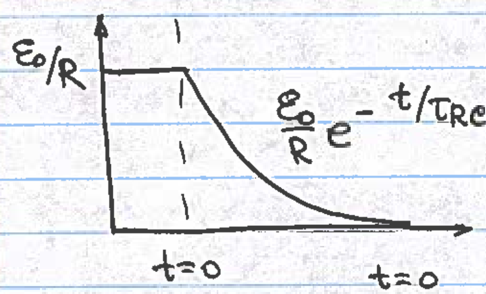
$$Q(t) = \mathcal{E}_0 C\, e^{-t/\tau_{RC}}$$

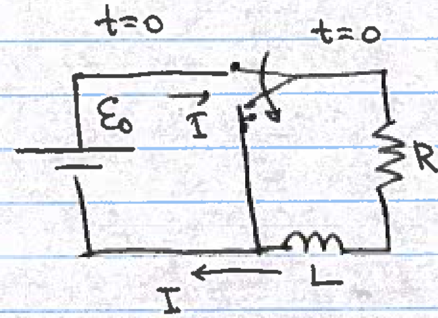
$$I(t) = \frac{\mathcal{E}_0}{R} e^{-t/\tau_{RC}}$$

$\tau_{RC} = RC$

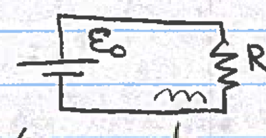
$$\frac{dQ}{dt} = -\frac{1}{RC} Q$$

$1/\tau_{RC}$



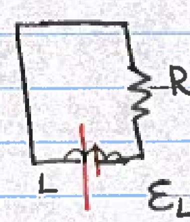


$t<0$



$I = \mathcal{E}_0/R$

$t = +0 \quad I = \mathcal{E}_0/R$ still!



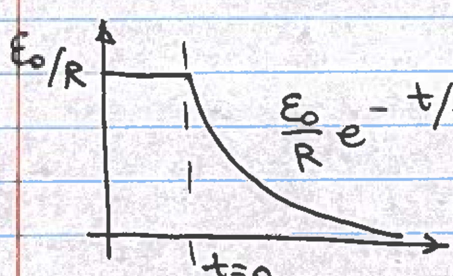
$\mathcal{E}_L - IR = 0$

$$-L\frac{dI}{dt} - IR = 0$$

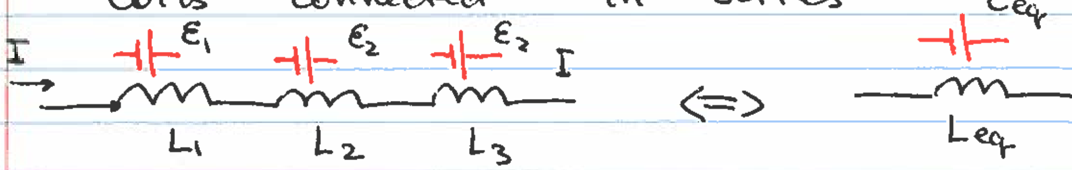
$$I(t) = \frac{\mathcal{E}_0}{R} e^{-t/\tau_{RL}}$$

$$\frac{dI}{dt} = -\frac{R}{L} I$$

$1/\tau_{RL}$



Coils connected in series



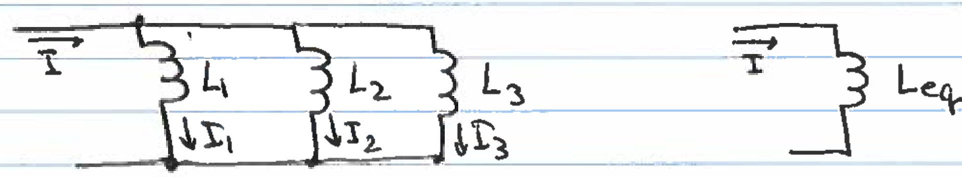
$$\varepsilon_1 = L_1 \frac{dI}{dt}, \quad \varepsilon_2 = L_2 \frac{dI}{dt}$$

$$\varepsilon_3 = L_3 \frac{dI}{dt}$$

$$\varepsilon_{tot} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \underbrace{(L_1 + L_2 + L_3)}_{L_{eq}} \frac{dI}{dt}$$

$L_{eq} = L_1 + L_2 + L_3$ for series connection

Parallel connection



$$\varepsilon_{ind} = L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_3 \frac{dI_3}{dt}$$

$$\frac{dI_1}{dt} = \frac{\varepsilon}{L_1} \qquad \frac{dI_2}{dt} = \frac{\varepsilon}{L_2} \qquad \frac{dI_3}{dt} = \frac{\varepsilon}{L_3}$$

$$\frac{dI}{dt} = \frac{\varepsilon_{eq}}{L_{eq}} = \frac{d(I_1 + I_2 + I_3)}{dt}$$

$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$ for parallel connection