Electromagnetic induction

Faraday's law: an emf is induced in a loop of wire when magnetic flux through the loop changes in time

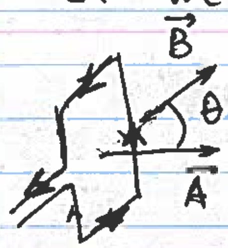
$$\mathcal{E}_{ind} = - \frac{d\Phi_B}{dt}$$

minus sign represents Lenz law

Lenz's law: the induced current is in the direction that produces magnetic field that opposes the change of the magnetic flux

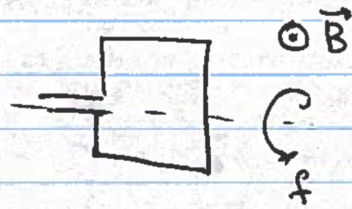
If we assume constant B throughout the loop

$\Phi_B = B \cdot A \cdot \cos\theta \; (\times N)$ (number of turns)



$\vec{A}$ (normal to the loop plane)

General idea of a power plant / AC generator

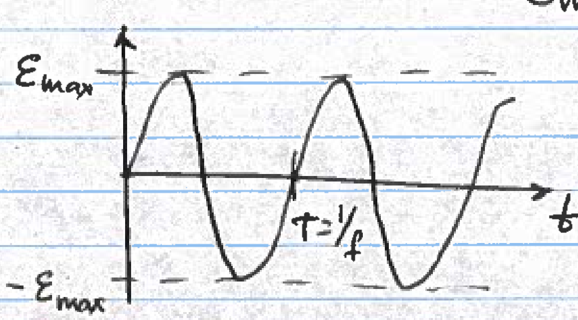


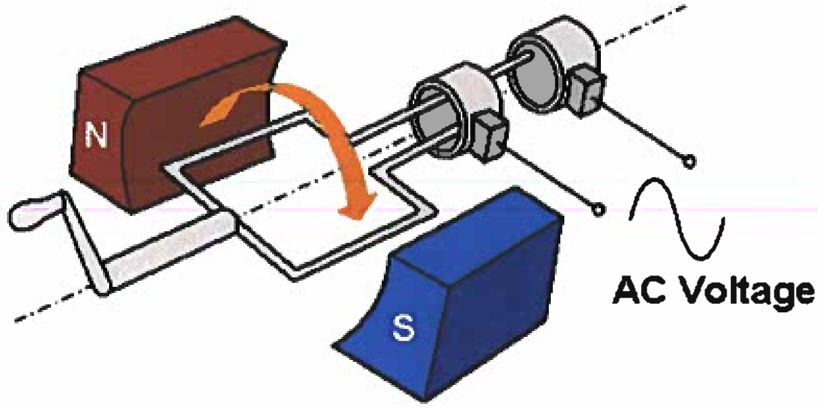
Rotating loop of wire

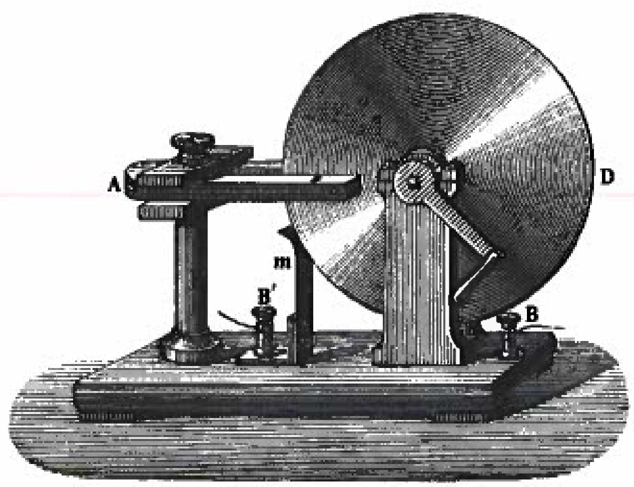
$\theta = 2\pi f \cdot t$

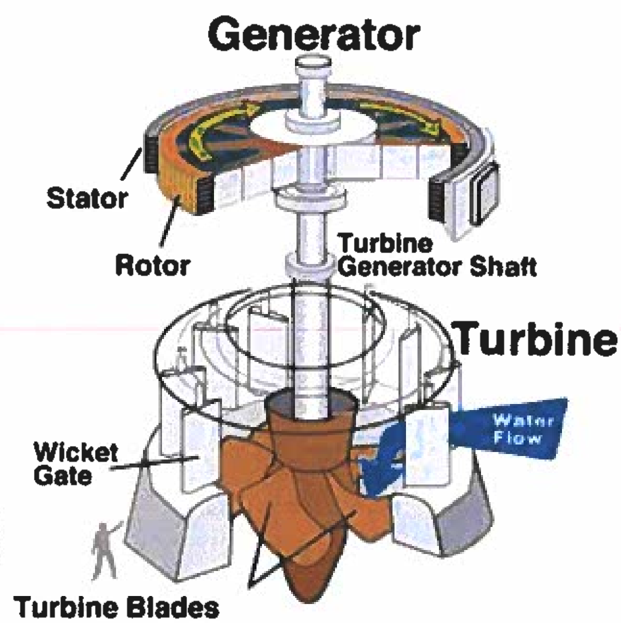
$$\Phi_B = BA\cos\theta = BA\cos 2\pi f t$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \underbrace{2\pi f \cdot BA}_{\mathcal{E}_{max}} \sin 2\pi f t$$

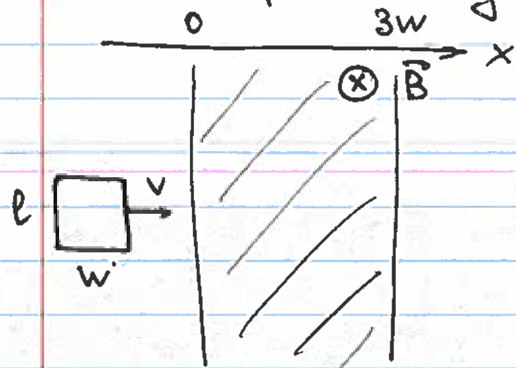



N
S
AC Voltage

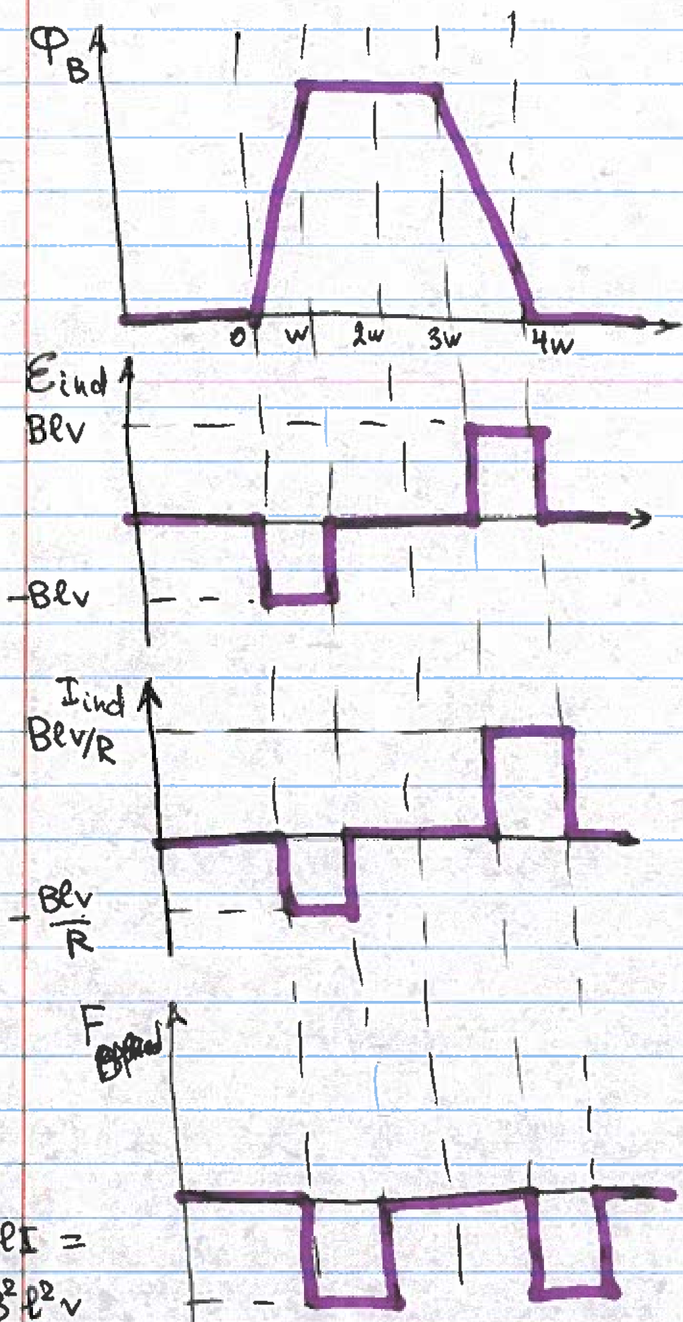

A
D
m
B'
B

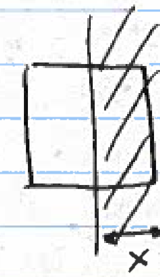

Generator
Stator
Rotor
Turbine
Generator Shaft
Turbine
Wicket
Gate
Water
Flow
Turbine Blades

A loop moving through a magnetic field



A ractangular loop $\ell \times w$ is moving with constant speed v along x through a region of constant magnetic field B as shown





$\Phi = B \cdot \ell x$
for $x < w$
$x = v \cdot t$

position of the front of the loop

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

for $0 < x < w$

$\mathcal{E}_{ind} = B\ell v \qquad 3w < x < 4w$

If R is a resistance of the loop

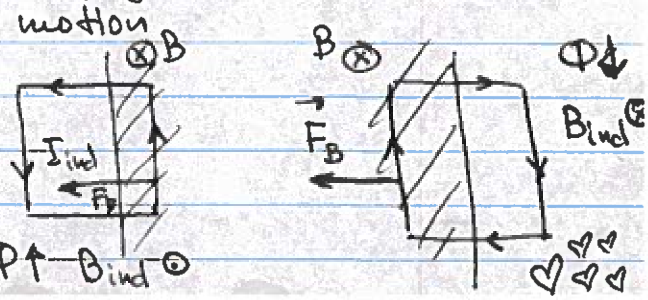
$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{B\ell v}{R}$$

Energy lost to heat

$P = I^2 \cdot R = (B\ell v)^2/R$

There will be a force trying to stop the loop motion

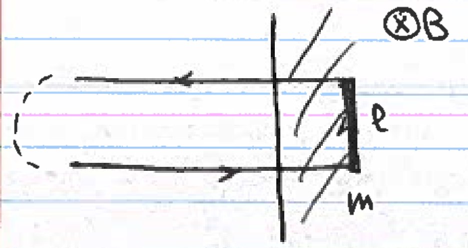
$-B\ell I = -\frac{B^2\ell^2 v}{R}$



One needs to push the loop to make it move w/constant v.

$F_{Bx} = -\ell B I_{ind}$ Φ↑ — B_{ind} ⊙

How a loop would move w/o pushing?
Lets first assume very long loop



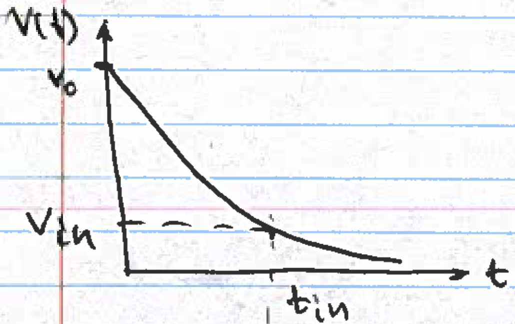
at $t=0$ the front enters magnetic field region w/ speed V_0

$$\mathcal{E}_{ind} = -B\ell v(t)$$

$$F_B = -\frac{\ell^2 B^2}{R} v(t) = -\beta \cdot v(t)$$

β

Newton's law: $ma = m\frac{dv(t)}{dt} = -\beta v(t)$



$$\int_{V_0}^{V(t)} \frac{dv(t)}{v} = -\frac{\beta}{m}\int_0^t dt$$

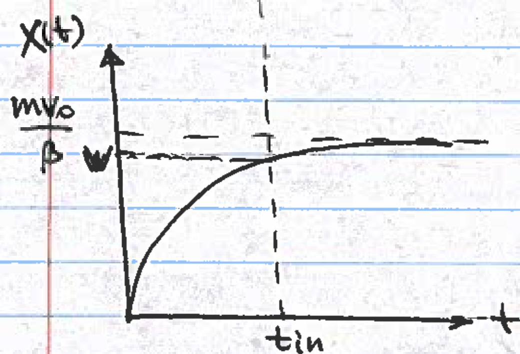
$$\ln\frac{V(t)}{V_0} = -\frac{\beta t}{m} \Rightarrow V(t) = V_0 e^{-\beta t/m}$$

$$V(t) = \frac{dx}{dt} = V_0 e^{-\beta t/m}$$

$$\int_0^{x(t)} dx = V_0 \int_0^t e^{-\beta t/m}\, dt$$

$$x(t) = -\frac{mV_0}{\beta}\left(e^{-\beta t/m} - 1\right)$$

$$x(t) = \frac{mV_0}{\beta}\left(1 - e^{-\beta t/m}\right)$$



If the loop has finite ~~length~~ width w, then once $x=w$, the loop will be completely inside B
$\mathcal{E}_{ind} = 0$, constant speed motion