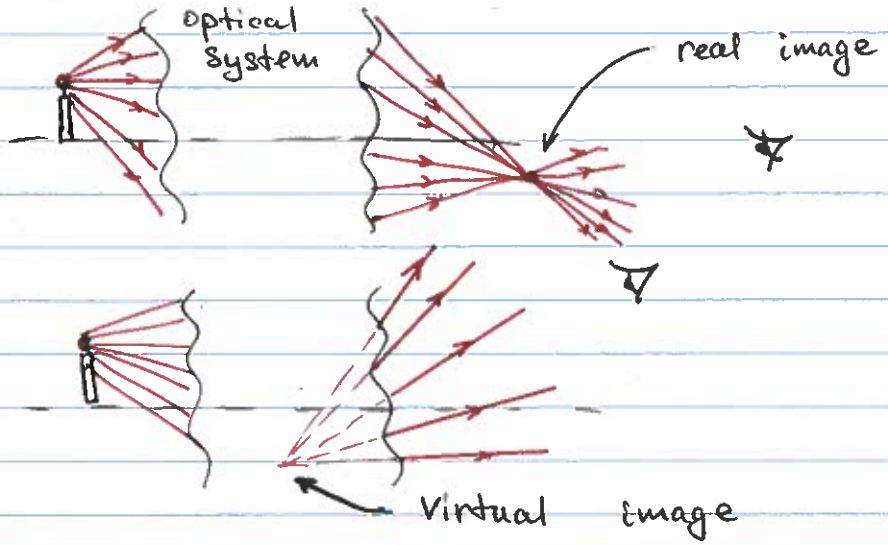
Image formation

Object — light source, emitting light rays in all directions

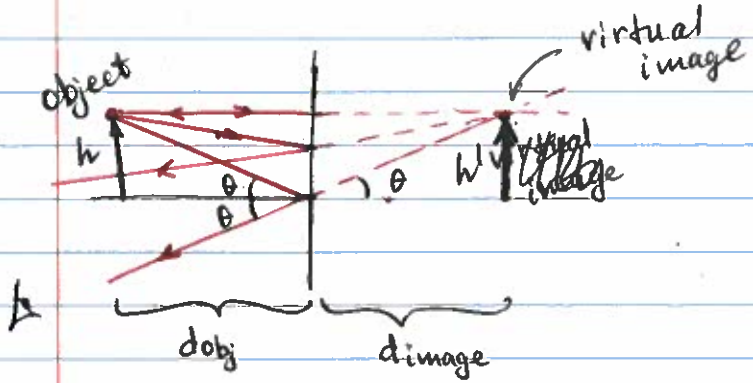
After some optical elements:

Real image — points when same of these beams cross again in space

Virtual image — (some) beams diverge as if from a different point



Flat mirror



Magnification

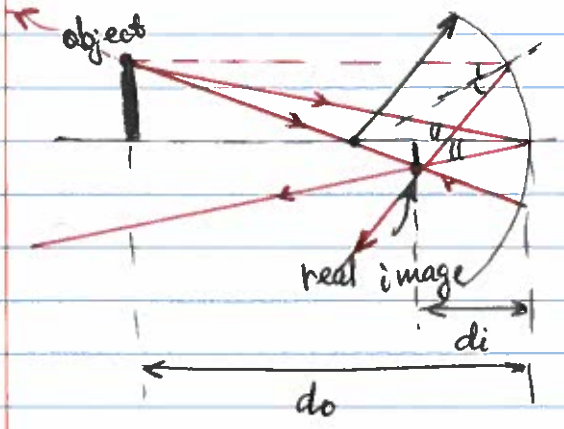
$$M = \frac{h'}{h} = 1$$

for a flat mirror

Since light rays are straight lines, we don't need to check all the direction, usually there are a few "convenient" beams. They intercept in the same point as all other rays.

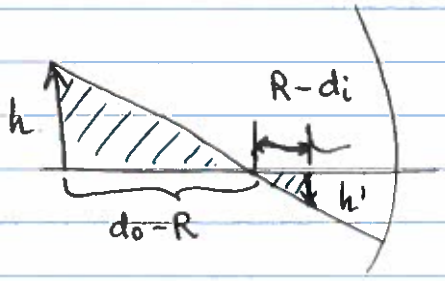
Curved mirrors

Spherical mirror of radius R



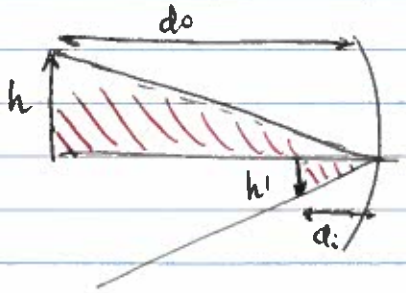
Two "easy" beams
- the ray that goes through the center of the sphere (reflects back on itself)
- the ray that goes to the center (on principle axis) (reflects symmetrically w/respect to the principle axis)

For a real object / image the distances to the mirror are positive



Similar triangles

$$\frac{d_o - R}{h} = \frac{R - d_i}{h'}$$



at the same time

$$\frac{d_o}{h} = \frac{d_i}{h'}$$

$$\frac{h'}{h} = \frac{d_i}{d_o} = \frac{R - d_i}{d_o - R}$$

Magnification

$$M = \frac{h'}{h} = -\frac{d_i}{d_o}$$

$$d_i d_o - d_i R = R d_o - d_o d_i$$

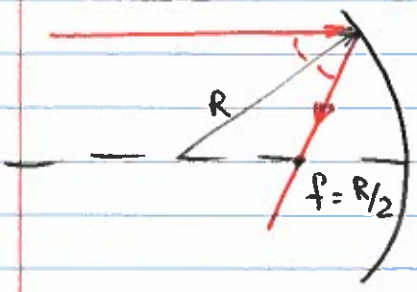
$$d_i R + d_o R = 2 d_i d_o \qquad \times \frac{1}{d_i d_o R}$$

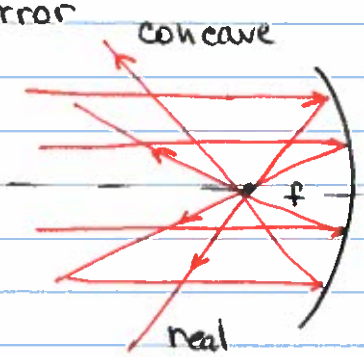
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{R}$$

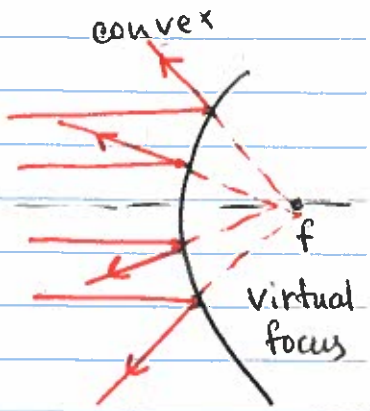
$$f = \frac{R}{2}$$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

Focus of the mirror





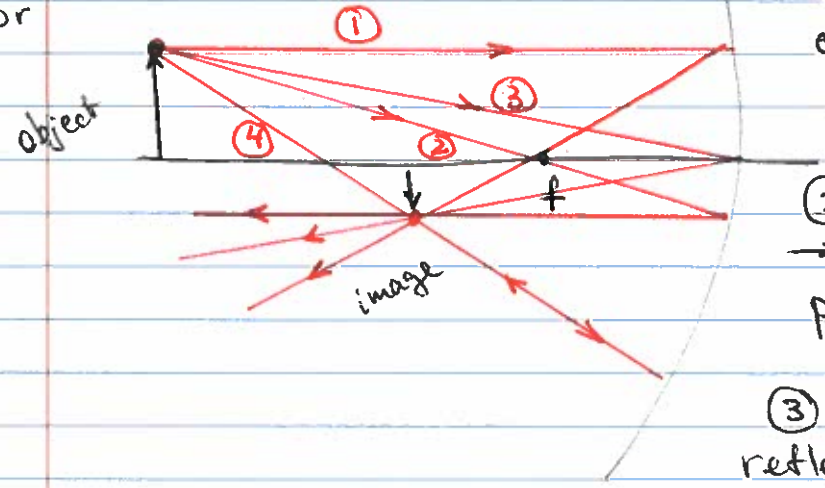


$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Real image	$d_i > 0$	Real object	$d_o > 0$
Virtual image	$d_i < 0$	Virtual object	$d_o < 0$
Concave mirror	$f > 0$	Erect image	$M > 0$
Convex mirror	$f < 0$	Inverted image	$M < 0$

Selection of principle rays
(you need two, but others are good for testing)

Concave mirror



① Parallel to optical axes → pass through focal point

② Passes through the focus → reflected off to be parallel to optical axis

③ Hits the center → reflected at the symmetric angle

④ The ray reflected at normal (passes through the center of the sphere) → reflects back ward

$d_o = 3f$

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ $\qquad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f}$

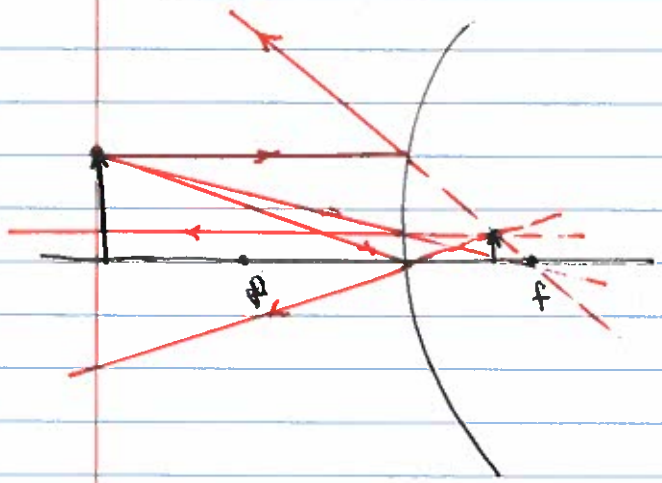
$d_i = \frac{3}{2} f$

Magnification: $\frac{|h'|}{h} = \frac{d_i}{d_o} = \frac{3/2 f}{3f} = \frac{1}{2}$

Inverted image $M = -\frac{1}{2}$ $\qquad h'$ – assigned to be negative

$M = \frac{h'}{h} = -\frac{d_i}{d_o}$

Convex mirror $\qquad f<0$



$d_o = 2f$

$\frac{1}{2f} + \frac{1}{d_i} = -\frac{1}{f}$ $\qquad$ virtual ↓

$\frac{1}{d_i} = -\frac{3f}{2f}$ $\qquad d_i = -\frac{2f}{3}$

$M = -\frac{d_i}{d_o} = \frac{2f/3}{2f} = \frac{1}{3}$

erect