Ideal gas → a gas of non-interacting atoms or molecules (ideal = simple to describe)

Thermodynamic description uses state variables

temperature T (in K) The amount of gas
Pressure P (# of moles n)
Volume V often assumed constant

These variables uniquely describe the state of the gas

Universal gas law $PV = nRT$
derived phenomenologically, from observations

(Robert Boyle)
Boyle's law (1627-1691) $V \propto 1/P$ @ constant T

(Jacque Charles)
Chalre's' law (1746 - 1823) $V \propto T$ @ constant P

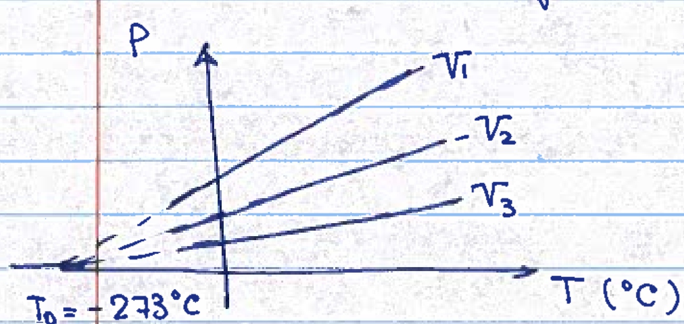
(Joseph Gay-Lussac)
Gay-Lussac's law (1778-1850) $P \propto T$ @ constant V

$R = 8.314\ J/mol \cdot K$ – universal gas constant
(needed to properly connect SI units)

$PV = nRT$ is valid only for T in Kelvin scale!

$$P = \frac{nR}{V} \cdot T$$

and can be used to measure absolute zero

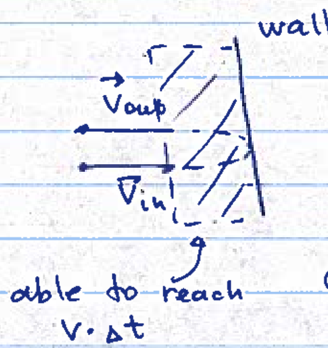


$P \propto T$ for a constant volume

Microscopic picture — the kinetic theory of gasses

Ideal gas:
- no internal structure
- large number of molecules N
- random motion
- all molecules are identical
- elastic collison with walls and each other
- molecule separation >> their size

First step → crude model : all molecules move with same velosity v in ~~random~~ one direction~~s~~



elastic collision $v_{in} = v_{out}$

Transferred momentum $\Delta p_1 = 2mv$

To calculate a force on the wall, we need to know how many collisions happen during time Δt

Particle density $\frac{N}{V}$, and molecules within $v \cdot \Delta t$ near the wall can hit it, ½ moves away

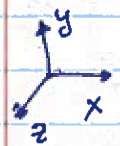
transferred momentum $= \Delta p_1 \cdot \frac{N}{V} \cdot v \cdot \Delta t \cdot A \cdot \frac{1}{2}$ (area)

$$= 2mv \cdot \frac{N}{V} \cdot v\Delta t \cdot A \frac{1}{2} = \cancel{2} m v^2 \cdot \frac{N}{V} \cdot \Delta t \cdot A.$$

Pressure $P = \frac{F}{A} = \frac{\text{transferred momentum}}{A \, \Delta t} = \frac{N}{V} \cdot \cancel{2} m v^2$

$$PV = N \cdot \cancel{2} m v^2$$ related to temperature

More realistic 3D model $v^2 \longrightarrow v_x^2$



also, all velocities are different

$v_x^2 \longrightarrow \langle v_x^2 \rangle$ average v_x

$$PV = N \, m \langle v_x^2 \rangle$$

Random motion $\rightarrow$ any dimension is equally possible

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$PV = N \, \frac{1}{3} m \langle v^2 \rangle = \frac{2}{3} N \underbrace{\left\langle \frac{mv^2}{2} \right\rangle}_{\text{average}} = \frac{2}{3} N \cdot k_B \cdot T$$

average kinetic energy

k_B - Boltzmann's constant $k_B = 1.38 \cdot 10^{-23}$ J/K

$R = k_B \cdot N_A$ $N_A = 6.022 \cdot 10^{23}$ # of atoms/molecule in one mole of a substance

Average kinetic energy depends <u>only</u> on its temperature

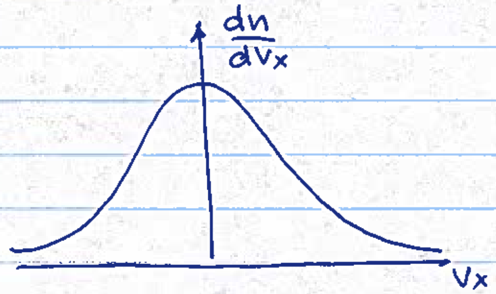
Do we know anything about velocity distribution for a given temperature?

Boltzmann law

a fraction of atoms with a given energe E
$\propto e^{-E/k_BT}$

One component $v_x \rightarrow \propto e^{-mv_x^2/2k_BT}$

$v_x = 0$ is most probable $dn(v_x) \propto N e^{-mv_x^2/2k_BT} dv_x$



symmetric distribution arond $v_x = 0$

3D : $dn(\vec{v}) = dn(v_x)\, dn(v_y)\, dn(v_z) \propto$

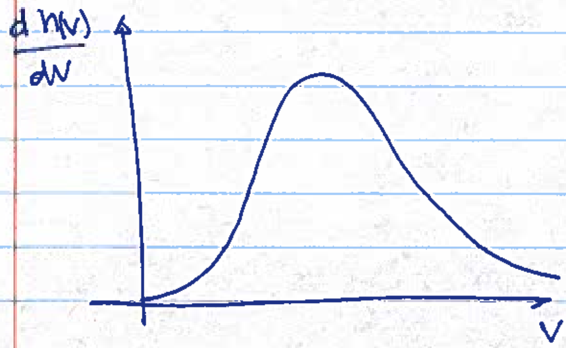
$$= N e^{-mv_x^2/2k_BT} e^{-mv_y^2/2k_BT} e^{-mv_z^2/2k_BT} \underbrace{dv_x\, dv_y\, dv_z}_{\text{can write in spherical coordinates}} =$$

$$= N e^{-mv^2/2k_BT} v^2 dv \underbrace{d\Omega}_{\text{solid angle}}$$

If we are not interested in the direction of motion, only velocity magnitude, we can average over all directions $d\Omega \rightarrow 4\pi$

$$dn(v) = 4\pi N \underbrace{\left(\frac{m}{2\pi k_BT}\right)^{3/2}}_{\text{normalization constant}} v^2 e^{-mv^2/2k_BT} dv$$

so that $\int_{v=0}^{\infty} dn(v) = N$



$$\frac{dn(v)}{dv} = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T}$$

small v: $\frac{dn(v)}{dv} \sim v^2$

large v: $\frac{dn(v)}{dv} \sim v^2 e^{-v^2/(2kT/m)}$

Most probable velocity $\left(\max \frac{dn}{dv}\right)$

$$V_{mp} = \sqrt{\frac{2k_B T}{m}} = 1.41\sqrt{\frac{k_B T}{m}}$$

Average velocity $V_{ave} = \langle V \rangle = \int_0^\infty V \cdot dn(v) = \sqrt{\frac{8k_B T}{\pi m}} = 1.6\sqrt{\frac{k_B T}{m}}$

Root mean square velocity

$$V_{RMS} = \sqrt{\langle V^2 \rangle} = \int_0^\infty V^2 \, dn(v) = \sqrt{\frac{3k_B T}{m}} = 1.73\sqrt{\frac{k_B T}{m}}$$