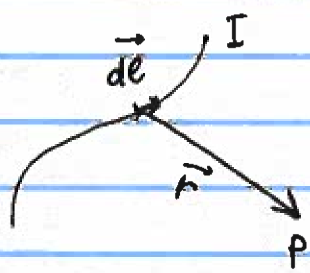
1. Magnetic force on moving charges and currents

$$\vec{F}_B = q \cdot \vec{v} \times \vec{B} \qquad \vec{F}_B = I\, \vec{l} \times \vec{B}$$

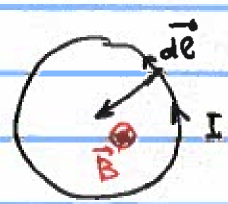
2. Generation of magnetic field by electric current

2.1 Bio-Savart law



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \hat{r}}{r^2}$$

Magnetic field of a ring / arc



$$|\vec{dl} \times \hat{r}| = dl \qquad |r| = R$$

$$dB = \frac{\mu_0}{4\pi} I \frac{dl}{R^2} \xrightarrow{\Sigma} \text{circuference}$$

$$B_{tot} = \frac{\mu_0}{4\pi} I \frac{2\pi R}{R^2} = \frac{\mu_0 I}{2R}$$

Straight pieces will not generate $\vec{B}$ because $d\vec{l} \times \hat{r} = 0$

$$dB = \frac{\mu_0}{4\pi} I \frac{dl}{R^2} \xrightarrow{\Sigma} \frac{1}{2} \text{ circumference } \pi R$$

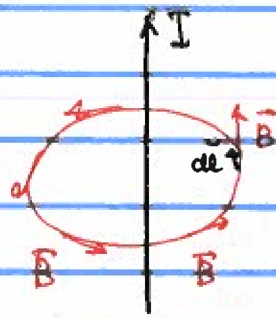
$$B_{semicircle} = \frac{\mu_0 I}{4R}$$

2.2. Ampere's law

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$

closed path

2.2.1

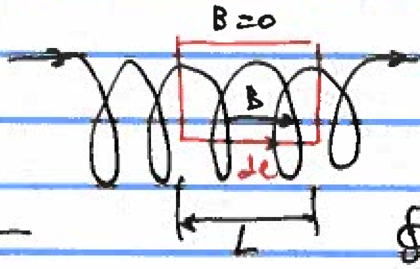


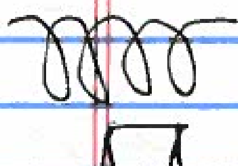
- If we choose a circle around the wire

$\oint \vec{B}\, d\ell = B \cdot 2\pi r = \mu_0 I$

$B_{wire} = \frac{\mu_0}{2\pi} \frac{I}{r}$

2.2.2 Solenoid / coil





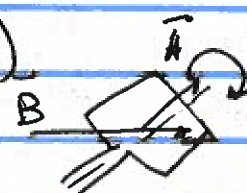
B is constant inside any point of the coil and it is zero outside

$\oint \vec{B}\, d\vec{\ell} = B \cdot L = \mu_0 \cdot I \cdot N$

$B_{solenoid} = \mu_0 I \cdot \frac{N}{L}$

3. Electromagnetic induction

$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B \cdot A \cdot \cos\theta)$

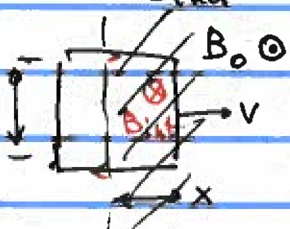


3.1 Electric motor / generator

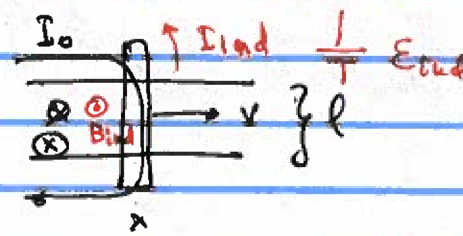
$\theta = 2\pi f \cdot t$

$\mathcal{E}_{ind} = -\frac{d}{dt}(B \cdot A \cdot \cos 2\pi f \cdot t) = B \cdot A \cdot 2\pi f \sin 2\pi f t$

3.2



or



$A = x \cdot \ell$

$\frac{dA}{dt} = \frac{dx}{dt} \cdot \ell = v \cdot \ell$

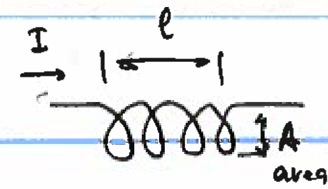
if $\theta = 90°$

$\mathcal{E}_{ind} = -\frac{d}{dt}(B \cdot A)$

$= -B v \ell$

Lenz law: $\mathcal{E}_{ind}$ tries to maintain Φ

4. (Self) inductance



$B = \mu_0 I \frac{N}{\ell}$

$U_{magn} = \frac{1}{2} L I^2$

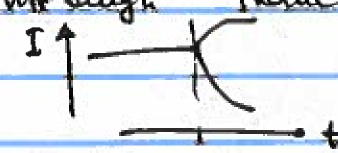
$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -\frac{d(A\cdot N\cdot B)}{dt} =$$

$$= -\underbrace{\boxed{A\cdot N\cdot \mu_0 \frac{N}{\ell}}}_{=L} \frac{dI}{dt} = -L\frac{dI}{dt}$$

L inductance

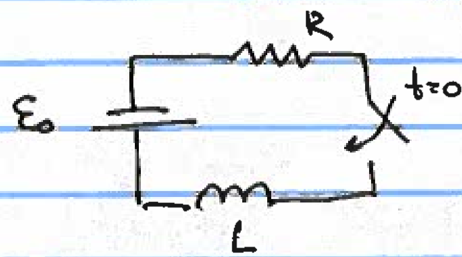
Special things about L when in a circuit

1. A current through inductance cannot be discontinuous



Current through L stays the same immediately after a change

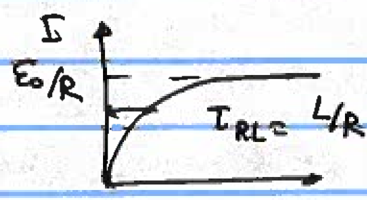
2. After a long time (in a new steady state) there is no voltage drop across L (~~to~~ it is just a long wire)

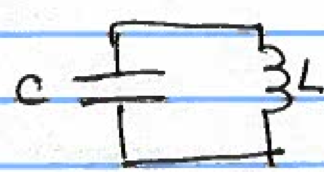


$I_L = 0$ at $t = +0$

If $t \to \infty$ $I = \mathcal{E}_0/R$

$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$





$\omega_{LC} = 1/\sqrt{LC}$

$I = I_0$ ~~cos~~ $\sin \omega_{LC} t$