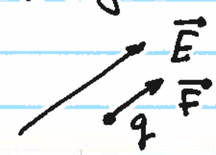
Magnetic force and magnetic field

Magnetic force acts <u>only</u> on moving charges (free particles or electric currents)

Electric field → any charged object (moving or stationary)

Electic force is straightforward

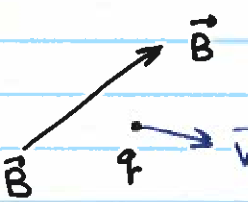
$$\vec{F}_{\vec{E}} = q \cdot \vec{E}$$

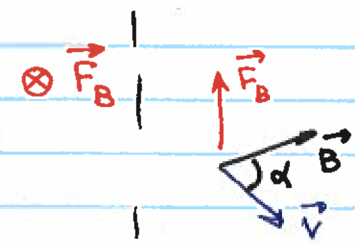


Magnetic field → only moving object

Magnetic force is round about

$$\vec{F}_{\vec{B}} = q \; \vec{v} \times \vec{B}$$



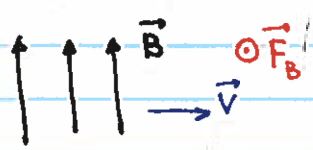


The magnetic field is perpendicular to both $\vec{v}$ and $\vec{B}$

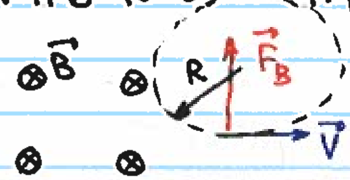
$$|\vec{F}_{\vec{B}}| = q \cdot v \cdot B \cdot \sin \alpha$$

Maximum force when $\vec{v} \perp \vec{B}$ $\quad |\vec{F}_B| = q v \cdot B$

Minimum (zero) force when $\vec{v} \parallel \vec{B}$ $(\alpha = 0)$ $\quad |\vec{F}_B| = 0$



$\Longleftrightarrow$



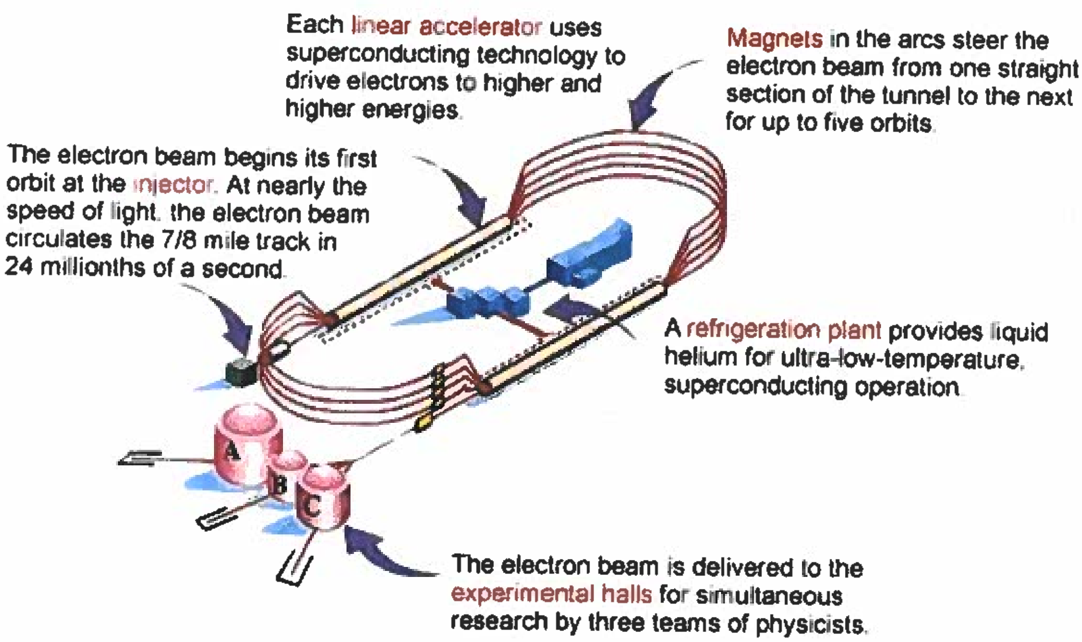
Acceleration due to magnetic force is perpendicular to the velocity → it only changes its direction, not magnitude!

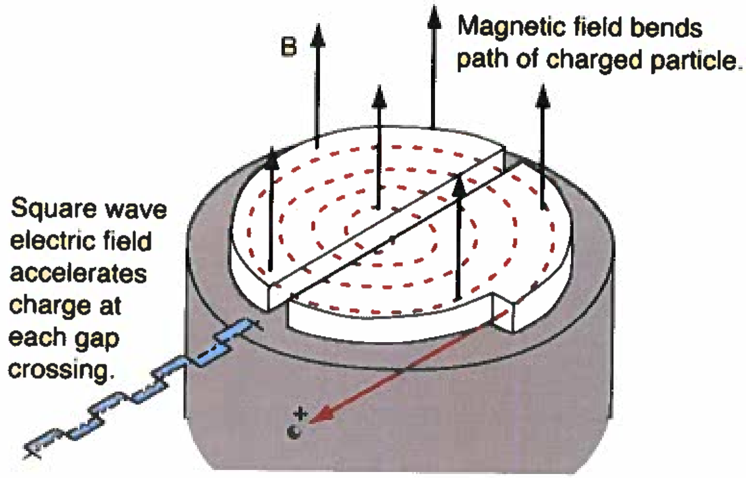
$$\vec{a}_B = \frac{\vec{F}_B}{m} \quad \text{– centripital acceleration}$$

cicrular motion

$$\frac{v^2}{R} = a_B = \frac{q \cdot v \cdot B}{m} \Rightarrow R = \frac{mv}{qB}$$

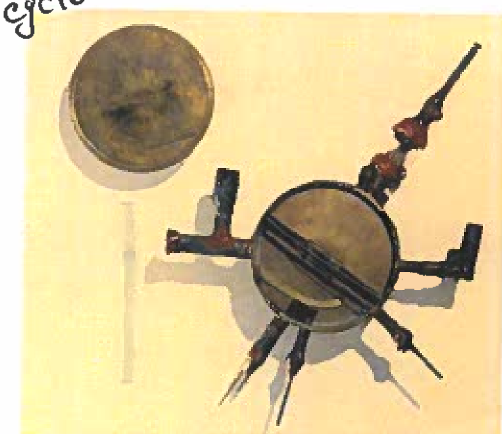
HOW CEBAF WORKS





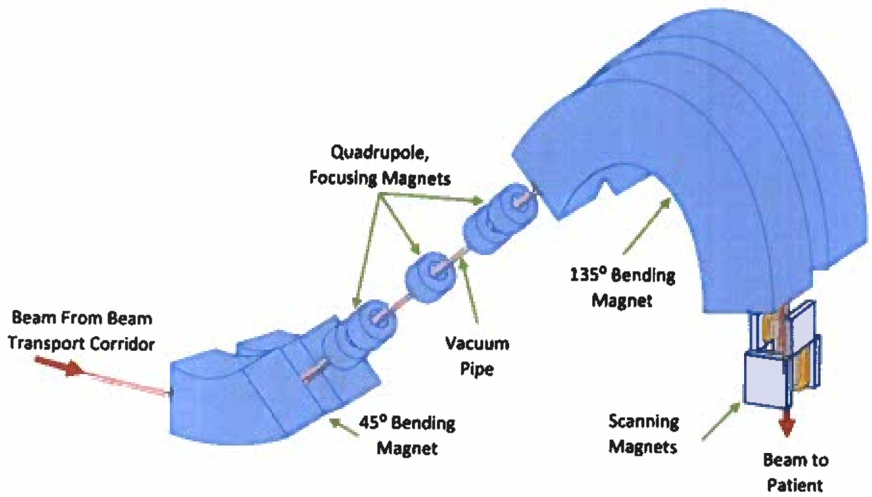
flips of the cyclotrone frequency

It always takes the same time for a particle to return to the gap, so it is always gaining energy

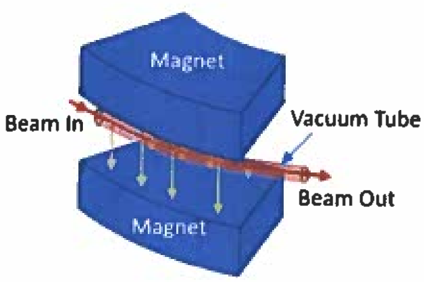


Proton therapy apparatus

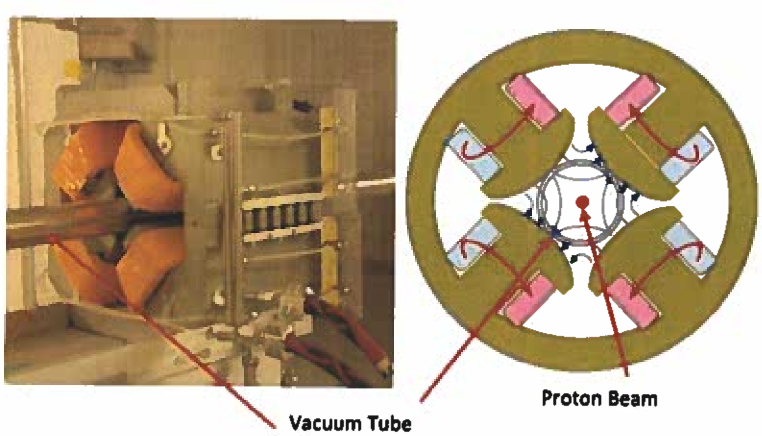
https://www.oncolink.org/healthcare-professionals/oncolink-university/proton-therapy-professional-education/oncolink-proton-education-modules/proton-therapy-delivery-the-equipment

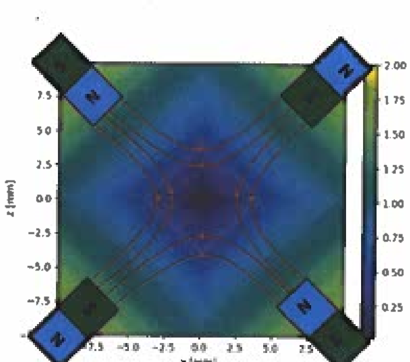


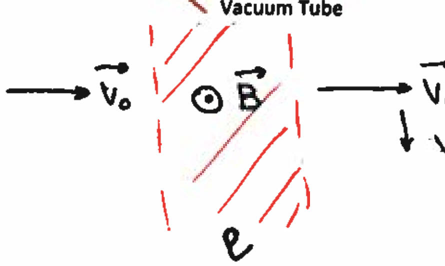
Bending magnets



Focusing magnets

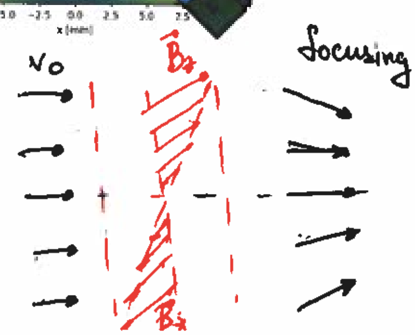






$$v_y = a_y \cdot t = \frac{e v_0 B}{m} \cdot \frac{\ell}{v_0}$$

$$v_y = e\ell B/m$$

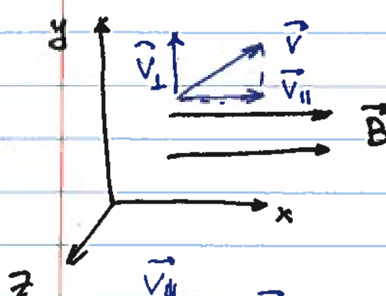


If a particle circulates in a constant orthogonal magnetic field, its period

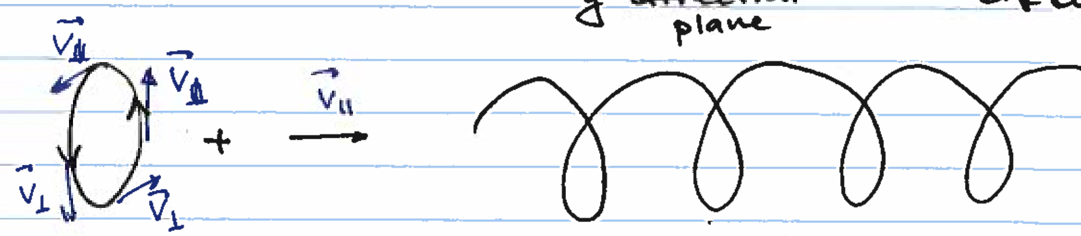
$T = \frac{2\pi R}{v} = 2\pi \frac{m}{qB}$, it depends on its mass, but not velocity

$\omega = \frac{2\pi}{T} = \frac{qB}{m}$ cyclotrone frequency

What if velocity is not perpendicular to $\vec{B}$



no acceleration in x direction
→ moving with constant speed
centripital acceleration in
y-$\vec{z}$ ~~direction~~ plane → circular motion



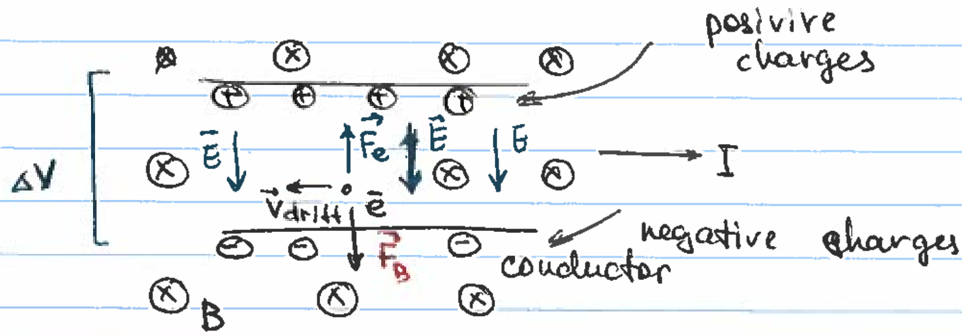
spiraling

What if both magnetic and electric fields are present?

Lorentz force $\vec{F}_L = q \cdot \vec{E} + q\vec{v} \times \vec{B}$

Applications: mass spectrometers
isotope separation
cyclotrone accelerators
Hall effect

Hall effect



① Magnetic force ~~bends~~ pushes electrons down, creating surface charge on top and bottom of the conductor

② The charge will continue accumulate until electric field due to these charges balances magnetic force

$$|\vec{F}_B| = e \cdot V_{drift} \cdot B \qquad E = \frac{\Delta V}{d} \qquad (d - \text{thickness of the conductor})$$

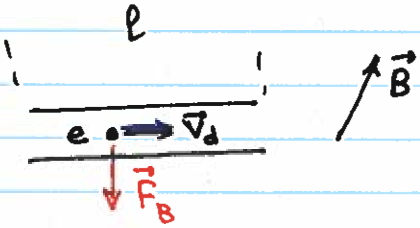
$$F_E = e \cdot E = \cancel{e} \frac{\Delta V}{d} = \cancel{e} \cdot V_{drift} \cdot B$$

If we know B, we can measure $V_{drift} = \frac{\Delta V}{d\,B}$

However, for a calibrated system (V_{drift} is known) we can use such ~~probe~~ Hall probe to measure magnetic field.

Magnetic force on a current-carrying conductor

Since electric current is a stream of moving charges, and magnetic field exerts a force on each, there will be a total force on the whole wire



$$\vec{F}_{total} = N_{charges} \cdot e\,\vec{V}_d \times \vec{B}$$

$$N_{charges} = n \cdot A \cdot \ell$$

n – charge density; A – area of the wire; ℓ – length of the wire

$$\vec{F} = nA\ell \cdot e\vec{V}_d \times \vec{B} = I\,\vec{\ell} \times \vec{B}$$

($nAe\,V_d = I$)

$\vec{\ell}$ – vector length of the current-carrying wire, direction is along the current