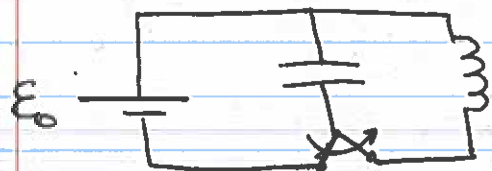
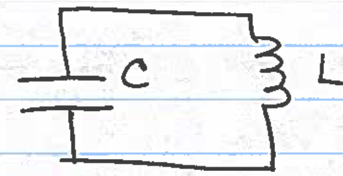
RC circuit



$t<0$ no current

$V_c = \mathcal{E}_0 \qquad Q = \mathcal{E}_0 \cdot C$

$t>0$



$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \qquad I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt} = 0$$

$$\frac{d^2Q}{dt^2} + LC\,Q = 0$$

$$Q(t) = \mathcal{E}_0 \cdot C \cos(\omega_{LC}\cdot t)$$

$$\frac{d^2Q}{dt^2} = -\omega_{LC}^2 \cdot \mathcal{E}_0 C\cos(\omega_{LC}\cdot t)$$

$$-\omega_{LC}^2 \cdot \cancel{\mathcal{E}_0 C\cos(\omega_{LC}t)} + \frac{1}{LC}\cdot \cancel{\mathcal{E}_0 C\cos(\omega_{LC}t)} = 0$$

$$\omega_{LC} = 1/\sqrt{LC}$$ oscillator frequency

$$Q(t) = \mathcal{E}_0 C\cos(\omega_{LC}\cdot t)$$

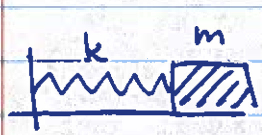
$$I(t) = -\frac{dQ}{dt} = \mathcal{E}_0 C\omega_{LC}\sin(\omega_{LC}t)$$

$$U_c = \frac{Q^2}{2C} = \frac{\mathcal{E}_0^2\cdot C}{2}\cos^2(\omega_{LC}t)$$

$$U_L = \frac{LI^2}{2} = \frac{\cancel{L}}{2}\frac{1}{\cancel{L}\cancel{C}}\mathcal{E}_0^2 C^{\cancel{2}}\sin^2(\omega_{LC}t)$$

$$= \frac{\mathcal{E}_0^2 C}{2}\sin^2(\omega_{LC}t)$$

$$U_L + U_c = \frac{\mathcal{E}_0^2 C}{2}$$ total energy is conserved



$\omega = \sqrt{k/m}$

$x = x_0 \cos\omega t$

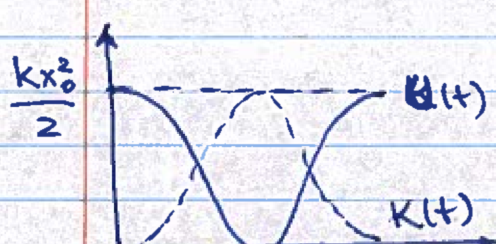
$v = x_0\omega \sin\omega t$

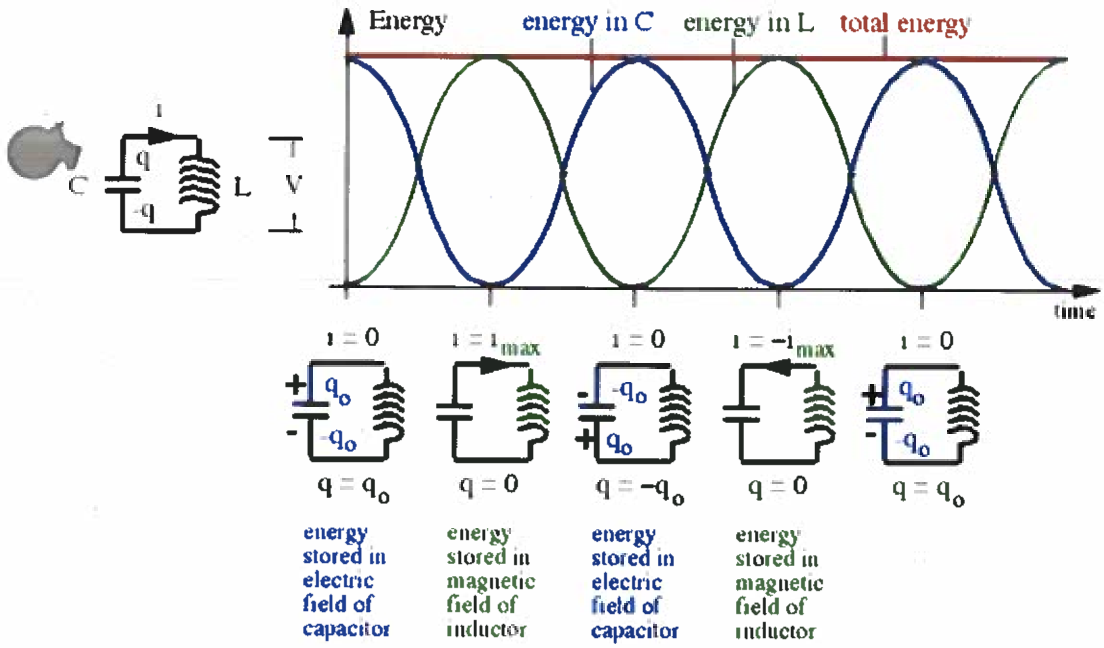
mechanical oscillator

Energy conservation

$K = \frac{mv^2}{2} = \frac{x_0\omega^2}{2m}\sin^2\omega t$

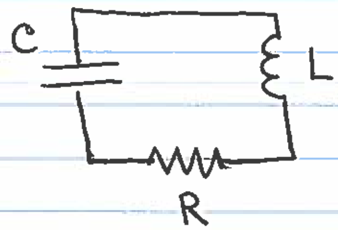
$U = \frac{kx^2}{2} = \frac{x_0\omega^2}{2m}\cos^2\omega t$





Energy
energy in C
energy in L
total energy
time
C
q
-q
L
V
i
i = 0
i = i max
i = 0
i = -i max
i = 0
q = q₀
q = 0
q = -q₀
q = 0
q = q₀
energy stored in electric field of capacitor
energy stored in magnetic field of inductor
energy stored in electric field of capacitor
energy stored in magnetic field of inductor

What if we add a resistance?

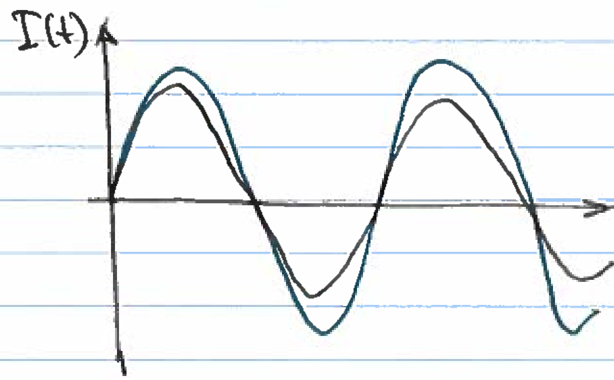


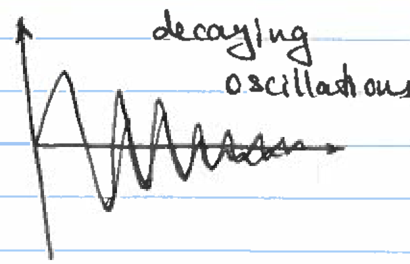
$$\frac{Q}{C} + IR + L\frac{dI}{dt} = 0 \qquad I = -\frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

oscillator with losses





If R is large, it dissipates energy too quickly, no oscillations can happen

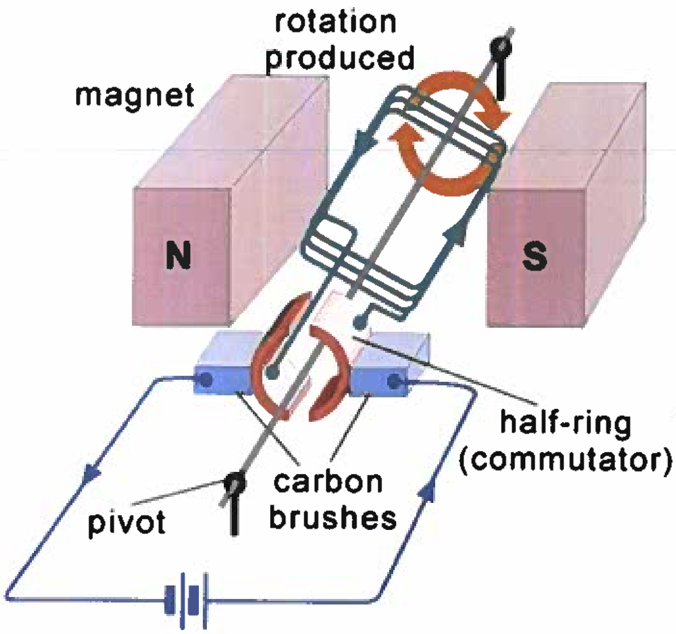
If R is not too large $R < \sqrt{4L/C}$

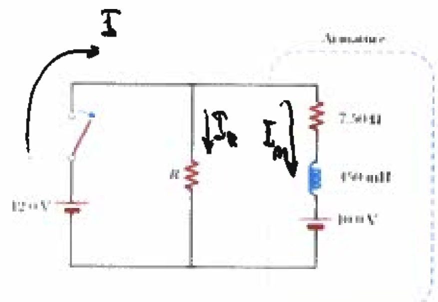
→ damped oscillations

$$Q = Q_0\, e^{-R/2L\, t} \cos(\omega' t + \varphi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\omega_{LC}^2 - R^2/4L^2}$$

$\omega_{LC} = \sqrt{1/LC}$ ideal LC circuit



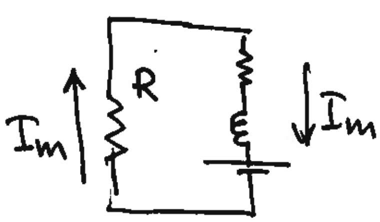


Q4 It may be helpful to first read the textbook section about how motors work, although you don't have to know that. Briefly, motors use electricity to produce torque (rotation) of the armature. Since here is a rotating current loop involved, an emf induced in the armature that prevents the magnetic flux changes, usually called "back emf". If the motor power source is disconnected, it can produce a high voltage drop across the gap creating arcing that may damage the armature (maybe you noticed occasional spark when unplugging a power supply? similar effect). To prevent such spontaneous arc, a discharge resistor is connected parallel to the armature, as shown in the figure. If the motor runs on 12V power, and the back emf is 10V when the motor is running, what is the maximum resistance R that limits the voltage across the armature to 80 V when the 12V power supply is unplugged?

Before the battery is disconnected

$12V - I_m \cdot 7.5\Omega - 10V = 0 \qquad I_m = \frac{2V}{7.5\Omega}$

After the battery is disconnected



$V_R = I_m \cdot R = 80V$

$R = \frac{80V}{I_m} = \frac{80V}{2V} \cdot 7.5\Omega$

$= 300\Omega$