Optical interference

Originates from superposition principle

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$$

$$I_{tot} = \frac{1}{\mu_0 c} \underbrace{\langle \vec{E}^2_{tot} \rangle}_{\text{time average}} = \frac{1}{\mu_0 c} \langle \vec{E}_1^2 + \vec{E}_2^2 + \overbrace{2\vec{E}_1 \vec{E}_2}^{\text{interference term}} \rangle$$

intensity of beam 1 — intensity of beam 2

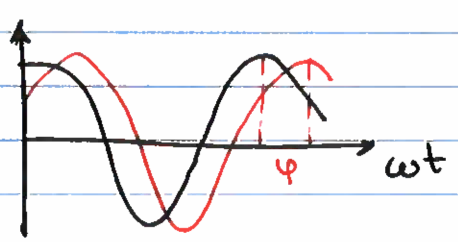
If $\vec{E}_1 \perp \vec{E}_2$ (two laser beams are orthogonally polarized) they do not interfere

If two fields have different frequencies
$E_1 E_2 \propto \cos\omega_1 t \cdot \cos\omega_2 t$ → oscillating term, averages to zero

Interference <u>most clear</u> if two laser fields have <u>same frequency</u> and <u>same</u> polarization

$$E_1(t) = E_1^{(0)} \cos \omega t$$

$$E_2(t) = E_2^{(0)} \cos(\omega t + \varphi)$$



φ characterizes the phase shift b/w two waves

$\varphi = 0, 2\pi\ldots$ — constructive

$\varphi = \pi, 3\pi\ldots$ — destructive

For example, if two fields originate from same point, but travel different distance Δx

$$\varphi = k\Delta x = \frac{2\pi}{\lambda} \cdot \Delta x$$

Math interlude:

Trig identity $\cos\omega t + \cos(\omega t + \varphi) = 2\cos(\omega t + \frac{\varphi}{2})\cos\frac{\varphi}{2}$

"Easy" to verify with complex numbers

$$\cos\omega t + \cos(\omega t+\varphi) = \mathrm{Re}\left[e^{i\omega t} + e^{i\omega t + i\varphi}\right] =$$

$$= \mathrm{Re}\left[e^{i\omega t + i\varphi/2}\underbrace{\left(e^{-i\varphi/2} + e^{i\varphi/2}\right)}_{2\cos\varphi/2}\right] =$$

$$= 2\cos\varphi/2 \; \mathrm{Re}\left[e^{i\omega t + i\varphi/2}\right] = 2\cos\varphi/2 \; \cos(\omega t + \varphi/2)$$

$$\left(E_1(t) + E_2(t)\right)^2 = \left(E_1^{(0)}\cos\omega t + E_2^{(0)}\cos(\omega t+\varphi)\right)^2$$

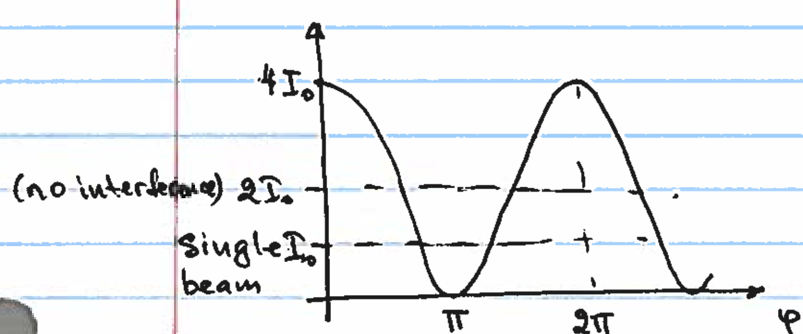
assume $E_1^{(0)} = E_2^{(0)} = E_0$

$$= E_0^2\left(\cos\omega t + \cos(\omega t+\varphi)\right)^2 = 4E_0^2\cos^2\frac{\varphi}{2}\;\cos^2(\omega t + \frac{\varphi}{2})$$

$$I = \frac{1}{\mu_0 c}\overline{\left(E_1(t)+E_2(t)\right)^2} = 4E_0^2\,\frac{1}{\mu_0 c}\cdot\cos^2\frac{\varphi}{2}\;\underbrace{\overline{\cos^2(\omega t + \frac{\varphi}{2})}}_{=1/2}$$

$$= 4\cdot\frac{E_0^2}{2\mu_0 c}\cdot\cos^2\frac{\varphi}{2} = 4\cdot I_0\cdot\cos^2\frac{\varphi}{2}$$

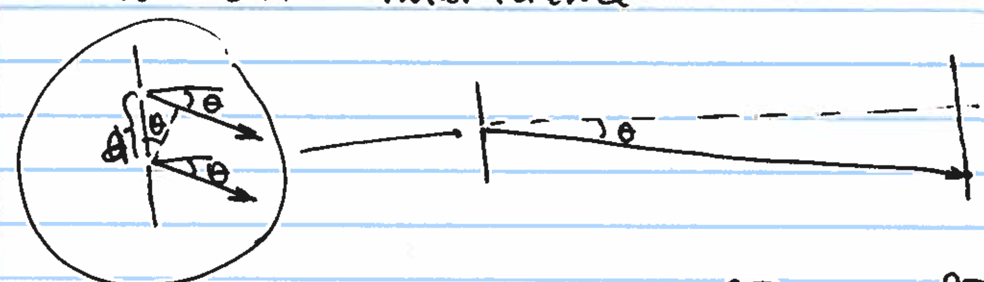
$I_0 = \frac{E_0^2}{2\mu_0 c}$ intensity of a single beam



Where we are likely to see interference phenomena?

1. Two-slit interference
2. Many-slit interference (diffraction grating)
3. Interferometers (optical devices intended to divide and ~~reconstruct~~ recombine ~~the~~ laser beams)
4. Thin-film interference (reflections off two near surfaces)
5. Diffraction on sharp edges

Two slit interference



$$\Delta x = d \sin\theta \qquad \varphi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi d}{\lambda}\sin\theta$$

Intensity on the screen at angle θ:

$$I(\theta) = 4\cdot I_0 \cos^2\left[\frac{2\pi d}{\lambda}\sin\theta\right] \underset{\sin\theta\approx\theta}{\approx} 4 I_0 \cos^2\left[\frac{2\pi d}{\lambda}\theta\right]$$

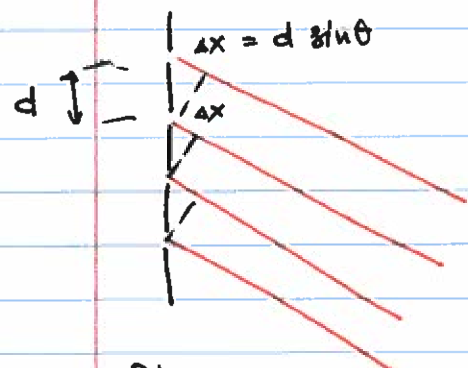
Constructive interference: $\frac{2\pi d}{\lambda}\theta_{max} = 2\pi\cdot m$ $\quad m=0,1,2..$

$$\theta_{max} = \frac{\lambda}{d}\cdot m$$

Destructive interference: $\frac{2\pi d}{\lambda}\theta_{min} = \pi + 2\pi m$

$$\theta_{min} = \frac{\lambda}{d}\left(m+\frac{1}{2}\right)$$

Diffraction grating — many slits



$$E_{tot} = E_0 e^{i\omega t} + E_0 e^{i\omega t + i\varphi} + E_0 e^{i\omega t + 2i\varphi} + \ldots E_0 e^{i\omega t + i(N-1)\cdot\varphi}$$

Geometric progression
$1 + r + r^2 + \ldots + r^{N-1} = \dfrac{1-r^N}{1-r}$

Phase added to each beam: $\varphi = \dfrac{2\pi}{\lambda}\Delta x = \dfrac{2\pi d}{\lambda}\sin\theta$

$$E_{tot} = E_0 e^{i\omega t}\left[1 + e^{i\varphi} + e^{i2\varphi} + \ldots e^{i(N-1)\varphi}\right] =$$

$$= E_0 e^{i\omega t}\,\frac{1 - e^{iN\varphi}}{1 - e^{i\varphi}}$$ geometrical progression

Total average intensity $I = I_0 \left(\dfrac{\sin\frac{N\Delta\varphi}{2}}{\sin\frac{\Delta\varphi}{2}}\right)^2$

$$I = I_0\,\frac{\sin^2\left[N\frac{\pi d}{\lambda}\sin\theta\right]}{\sin^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}$$

Global maximum is when both numerator and denominator approach zero

$$\lim_{x\to 0}\frac{\sin^2(Nx)}{\sin^2 x} = N^2$$

Happens when $\dfrac{\pi d \overset{\approx\theta}{\sin\theta}}{\lambda} = 0, \pi, 2\pi = m\cdot\lambda$

$$\theta_{\substack{\text{global}\\ \text{maxima}}} = \frac{m\lambda}{d} \qquad m = 0, 1, 2$$