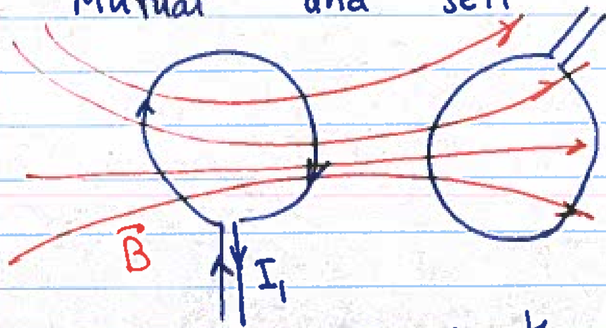
Mutual and self inductance

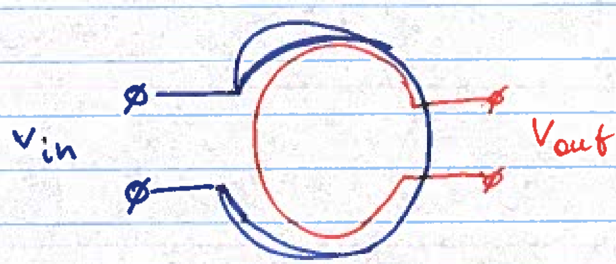


if I_1 is increasing, the flux through the second loop is increasing as well $\quad \Phi = A \cdot B \propto I_1(t)$

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} \propto -\frac{dI_1(t)}{dt}$$

Mutual inductance

Changing current in one loop creates Emf/ /voltage in the other. and vice versa.



$\Phi_{in} = \Phi_{out}$

for one loop

$$V_{in}\Big|_{\text{one loop}} = -\frac{d\Phi_{in}}{dt}$$

$$\parallel$$

$$V_{out}\Big|_{\text{one loop}} = -\frac{d\Phi_{out}}{dt}$$

Φ ~~Φ~~ flux for a single loop

Transformer

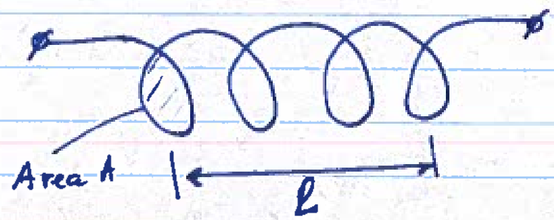
If we have different number of turns

$$V_{in} = -N_{in}\frac{d\Phi}{dt} \qquad V_{out} = -N_{out}\frac{d\Phi}{dt}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{N_{out}}{N_{in}}}$$

by changing the number of wraps, we can adjust output voltage

Induction coil / solenoid



$$B = \mu_0 \frac{N}{\ell} \cdot I$$

N – # of turns
ℓ – length

Flux through the solenoid $\Phi = N \cdot A \cdot B$

$$\Phi = \underbrace{\frac{\mu_0 N^2 \cdot A}{\ell}}_{\text{set-up of the coil}} \cdot I \quad = \quad L \cdot I$$

$$L = \frac{\mu_0 N^2 \cdot A}{\ell} \quad \text{inductance} \qquad L = \frac{\Phi}{I}$$

Changing current $I(t)$ changes the flux through the coil, producing additional EMF

$$\mathcal{E}_{ind} = \mathcal{E}_L = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

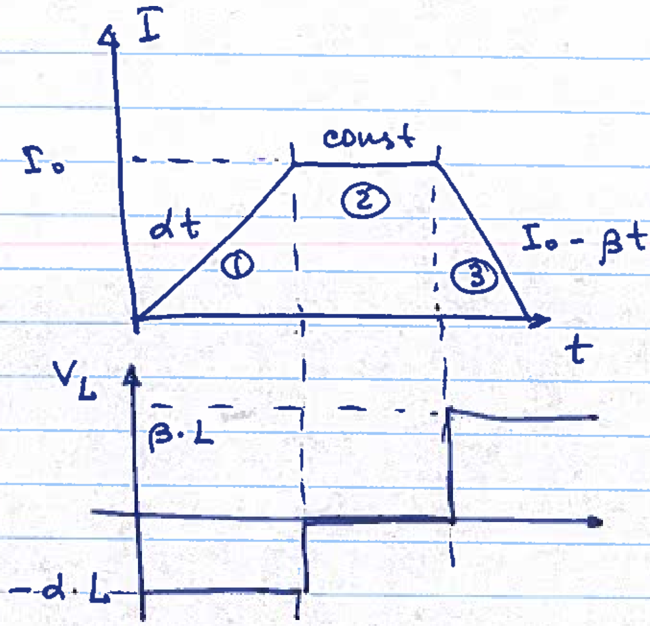
↑ Lenz rule again

EMF induced in a coil will oppose changes in current:

Current increases → induced $\mathcal{E}_L$ will ~~try~~ work against it

Current drops → induced $\mathcal{E}_L$ will work to prop it up

Current constant → the coil behaves as a long piece of wire ($\mathcal{E}_L = 0$)

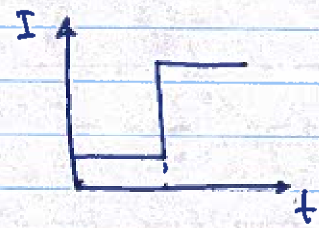


① $\mathcal{E} = -L \frac{dI}{dt} = -\alpha L$ $\quad (I = \alpha t)$

② $\mathcal{E} = -L \frac{dI_0}{dt} = 0$

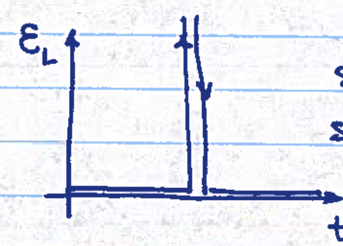
③ $\mathcal{E} = -L \frac{d(I_0 - \beta t)}{dt} = \beta L$

It is impossible to change electric current through ~~#~~ any inductor instantaneously



$\mathcal{E}_L = -\frac{dI}{dt}$

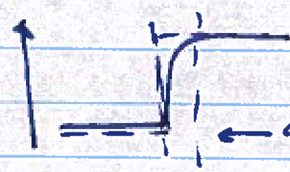
$\Longrightarrow$



supershort superstrong voltage pulse

(that's why we need surge protectors

Realistically, if ~~ever~~ anything changes in the circuit, the current won't change instantaneously, but develops EMF to maintain the previous current (for a moment), and then gradually adjusts.



$\leftarrow$ action of $\mathcal{E}_L$

(Unlike a resistor that reacts to voltage change instantly $\quad V_R = R \cdot I$)

As time goes by $\mathcal{E}_{ind}(t) - I(t)\cdot R = 0$

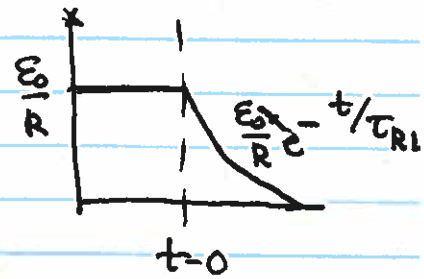
$$-L\frac{dI}{dt} - I\cdot R = 0$$

$$\frac{dI}{dt} = -\frac{R}{L} I$$

Exponential solution $I(t) = I_0 e^{-t/\tau_{RL}}$

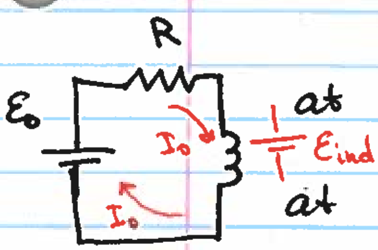
$$\frac{dI}{dt} = -\frac{1}{\tau_{RL}} \cancel{I_0 e^{-t/\tau_{RL}}} = -\frac{R}{L} \cancel{I_0 e^{-t/\tau_{RL}}}$$

$$\tau_{RL} = \frac{L}{R}$$



Current $I(t) = \frac{\mathcal{E}_0}{R} e^{-R/L\cdot t}$

Switching from ~~2 to~~ position 2 to position 1

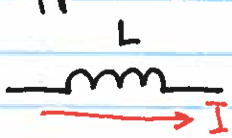


at $t=0$ $I=0$, at $t\to\infty$ $I = I_0 = \mathcal{E}_0/R$

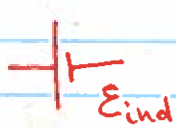
at any time $\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$

$I(t) = I_0 - I_0 e^{-t/\tau_{RL}}$ $\tau_{RL} = L/R$

The direction of the induced EMF always opposeds the change of the current



Current increases



value $\mathcal{E}_{ind} = \left| -L\frac{dI}{dt} \right|$

Current decreases