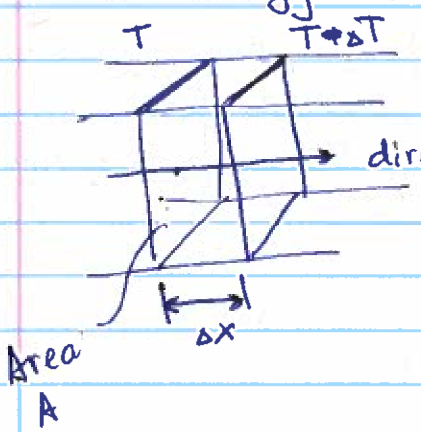
Heat transfer mechanisms

1. Convection – energy transferred by the movement of a warm substance

2. Thermal conduction – energy is transferred ~~b~~ through kinetic energy exchange between microscopic particles.

Reminder: energy flows (spontaneously) from hot to cold

The energy transfer rate (power) $P = k \cdot A \dfrac{dT}{dx}$



k – thermal conductivity

$$P = \frac{\Delta Q}{\Delta t} = k \cdot A \frac{\Delta T}{\Delta x}$$

The higher is k, the easier it is for the energy to flow

Max k – metals, as free electrons carry energy

The flowing heat will affect the temperature T gradient, in a sample $T(x,t)$ depends of time and x

Heat equation

$$\frac{\partial T(x,t)}{\partial t} = \frac{k}{c \cdot \rho} \frac{\partial^2 T(x,t)}{\partial x^2}$$

k – thermal conductivity
c – specific heat capacity
ρ – density

] material constants

Thermal Conductivities for Various Materials

Material	Thermal Conductivity (J / s•m•°C)
Diamond	1,600
Silver	420
Copper	390
Brass	110
Lead	35
Steel	14.0
Glass	0.80
Water	0.60
Body fat	0.20
Wood	0.15
Wool	0.04
Air	0.0256
Styrofoam	0.01

Material	Thermal conductivity at 25°C (W/m K)	Specific heat capacity	
		(J/kg K)	(J/m^3 K)
PA 6	0.25	1600	1824
PLA	0.13	1800	2340
Air	0.024	1005	1231
Wood (oak)	0.17	2000	1833
Steel	43	490	3822
Aluminium	205	870	2484
Water	0.58	4182	4190
Concrete	1.7	880	2122

Example : a bar of length L has fixed temperature at both ends

a) Steady state solution $\frac{\partial T}{\partial t} = 0$ $\quad \frac{\partial^2 T}{\partial x^2} = 0$

$T(x) = a + bx$

Boundary conditions: $T(0) = a$ $\quad T(L) = a + b \cdot L$

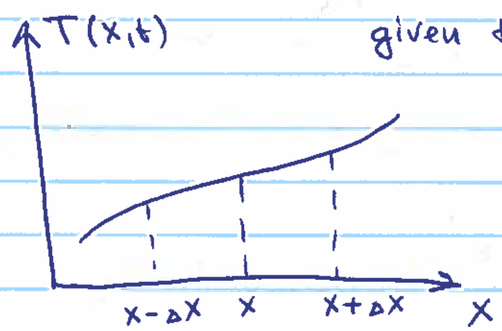
Solution: $T(x) = T(0) + x \cdot \frac{T(L) - T(0)}{L}$

b) Let's allow the temperature of one end to change in time $T(0,t)$

In this case we have to solve the heat equation, often numerically

$$\frac{\partial T(x,t)}{\partial t} = \underbrace{\frac{k}{c\rho} \cdot \frac{\partial^2 T(x,t)}{\partial x^2}}_{\text{rate of change in local temperature}}$$

given time t



Numerical derivative

$$\left.\frac{\partial T}{\partial x}\right|_- \rightarrow \frac{T(x) - T(x-\Delta x)}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_+ \rightarrow \frac{T(x+\Delta x) - T(x)}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\left.\frac{\partial T}{\partial x}\right|_+ - \left.\frac{\partial T}{\partial x}\right|_-}{\Delta x} = \frac{T(x+\Delta x) + T(x-\Delta x) - 2T(x)}{(\Delta x)^2}$$

$$\frac{\partial T(x,t)}{\partial t} \approx \frac{T(x,t+\Delta t) - T(x,t)}{\Delta t} = \frac{k}{c\rho} \left[\frac{T(x+\Delta x) + T(x-\Delta x) - 2T(x)}{(\Delta x)^2} \right.$$

$$T(x,t+\Delta t) = T(x,t) + \underbrace{\frac{k}{c\rho^2} \Delta t}_{\text{constant coefficient}} \underbrace{\left[\frac{T(x+\Delta x,t) + T(x-\Delta x,t) - 2T(x,t}{(\Delta x)^2} \right.}_{\text{digital derivative}}$$

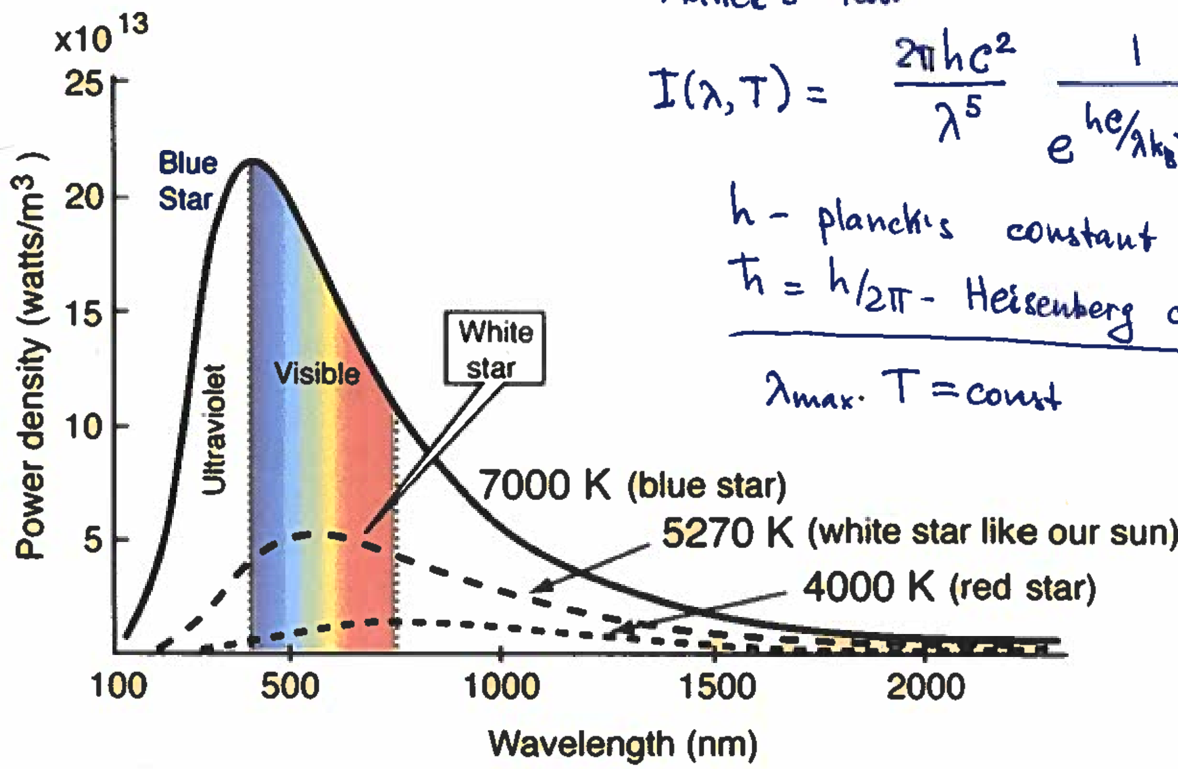
Planck's law

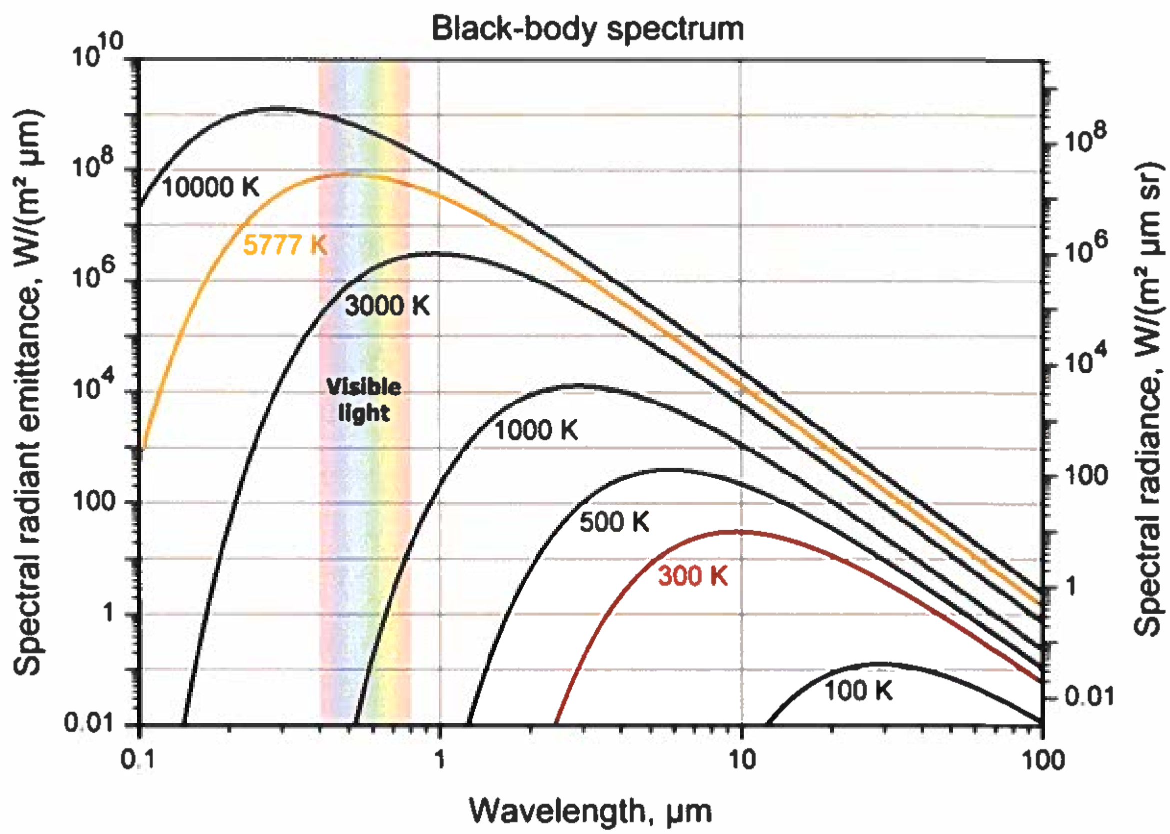
$$I(\lambda,T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

h - planck's constant

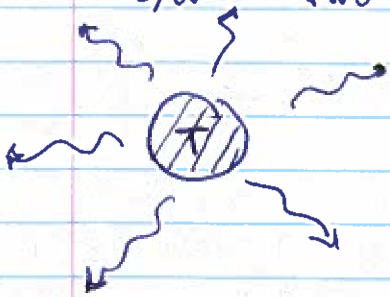
$\hbar = h/2\pi$ - Heisenberg constant

$\lambda_{max} \cdot T = \text{const}$





Radiation – energy is transferred through emission and absorption of electro-magnetic waves. Does not require any material contact b/w two objects



Emitted power $P = \sigma \cdot A \cdot e \cdot T^4$

$\sigma = 5.67 \cdot 10^{-8}\ W/m^2K^4$ Stephan constant

e – emissivity, describes the capacity of the object to absorb energy $0 < e < 1$

Such thermal radiation is often called <u>black body</u> radiation

Black body is an object that absorbs all radiation falling of its surface and then emits it with perfect emissivity $e = 1$

The origin of BB radiation is the microscopic motion of ~~atoms~~ ions and of electrons inside atoms / molecules, since every moving charged particle has to emit e-m waves