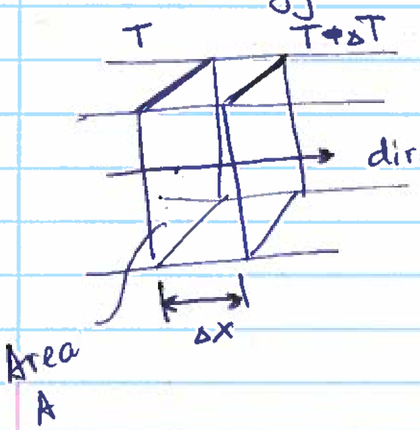
Heat transfer mechanisms

1. Convection – energy transferred by the movement of a warm substance

2. Thermal conduction – energy is transferred ~~b~~ through kinetic energy exchange between microscopic particles.

Reminder: energy flows (spontaneously) from hot to cold

The energy transfer rate (power) $P = k \cdot A \dfrac{dT}{dX}$



k – thermal conductivity

$$P = \frac{\Delta Q}{\Delta t} = k \cdot A \frac{\Delta T}{\Delta X}$$

The higher is k, the easier it is for the energy to flow

Max k – metals, as free electrons carry energ

The flowing heat will affect the temperature T gradient, in a sample $T(x,t)$ depends of time and x

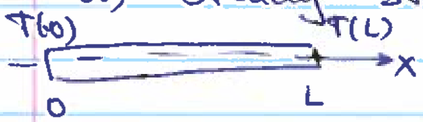
Heat equation

$$\frac{\partial T(x,t)}{\partial t} = \frac{k}{c \cdot \rho} \frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\left.\begin{array}{l} k - \text{thermal conductivity} \\ c - \text{specific heat capacity} \\ \rho - \text{density} \end{array}\right] \text{material constants}$$

Example: a bar of length L has fixed temperature at bothe ends

a) Steady state solution $\frac{\partial T}{\partial t} = 0$ $\quad \frac{\partial^2 T}{\partial x^2} = 0$



$T(x) = a + bx$

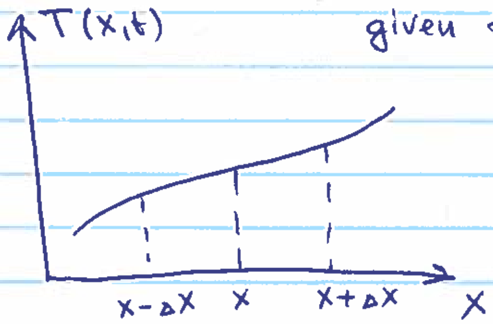
Boundary conditions: $T(0) = a$ $\quad T(L) = a + b \cdot L$

Solution: $T(x) = T(0) + x \cdot \frac{T(L) - T(0)}{L}$

b) Let's allow the temperature of one end to change in time $T(0,t)$

In this case we have to solve the heat equation, often numerically

$$\frac{\partial T(x,t)}{\partial t} = \underbrace{\frac{k}{c\rho} \cdot \frac{\partial^2 T(x,t)}{\partial x^2}}_{\text{rate of change in local temperature}}$$



Numerical derivative

$$\left.\frac{\partial T}{\partial x}\right|_{-} \rightarrow \frac{T(x) - T(x-\Delta x)}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{+} \rightarrow \frac{T(x+\Delta x) - T(x)}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\left.\frac{\partial T}{\partial x}\right|_{+} - \left.\frac{\partial T}{\partial x}\right|_{-}}{\Delta x} = \frac{T(x+\Delta x) + T(x-\Delta x) - 2T(x)}{(\Delta x)^2}$$

$$\frac{\partial T(x,t)}{\partial t} \approx \frac{T(x, t+\Delta t) - T(x,t)}{\Delta t} = \frac{k}{c\rho} \frac{[T(x+\Delta x) + T(x-\Delta x) - 2T(x)]}{(\Delta x)^2}$$

$$T(x, t+\Delta t) = T(x,t) + \underbrace{\frac{k}{c\rho^2}\,\Delta t}_{\text{constant coefficient}} \underbrace{\frac{[T(x+\Delta x,t) + T(x-\Delta x,t) - 2T(x,t)]}{(\Delta x)^2}}_{\text{digital derivative}}$$

Thermal Conductivities for Various Materials

Material	Thermal Conductivity (J / s•m•°C)
Diamond	1,600
Silver	420
Copper	390
Brass	110
Lead	35
Steel	14.0
Glass	0.80
Water	0.60
Body fat	0.20
Wood	0.15
Wool	0.04
Air	0.0256
Styrofoam	0.01

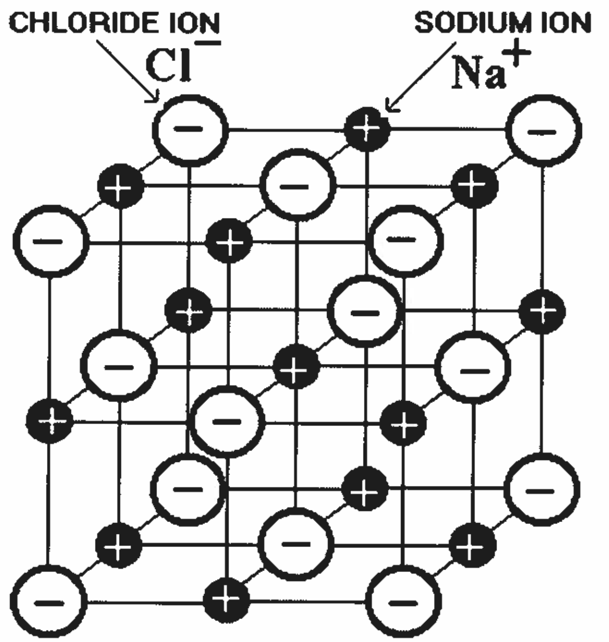
Material	Thermal conductivity at 25°C (W/m K)	Specific heat capacity	
		(J/kg K)	(J/m^3 K)
PA 6	0.25	1600	1824
PLA	0.13	1800	2340
Air	0.024	1005	1231
Wood (oak)	0.17	2000	1833
Steel	43	490	3822
Aluminium	205	870	2484
Water	0.58	4182	4190
Concrete	1.7	880	2122

TABLE 20.1 Electrical Resistivity and Thermal Conductivity of Copper and Other Pure Commercial Metals at 293 K.

(Metal 100)	Electrical Resistivity at 293 K, $\mu\Omega$cm	Thermal Conductivity $Wm^{-1}k^{-1}$	Relative Electrical Conductivity (Copper = 100)	Relative Thermal Conductivity (Copper = 100)
Silver	1.63	419	104	106
Copper	1.694	397	100	100
Gold	2.2	316	77	80
Aluminum	2.67	238	63	60
Beryllium	3.3	194	51	49
Magnesium	4.2	155	40	39
Tungsten	5.4	174	31	44
Zinc	5.96	120	28	30
Nickel	6.9	89	24	22
Iron	10.1	78	17	20
Platinum	10.58	73	16	18
Tin	12.6	73	13	18
Lead	20.6	35	8.2	8.8
Titanium	54	22	3.1	5.5
Bismuth	117	9	1.4	2.2

Adapted from Brandes, E. A., Ed., *Smithells Metals Reference Book*, Sixth Edition, Butterworth, Inc. 1983. (Used by permission.)

Insulator



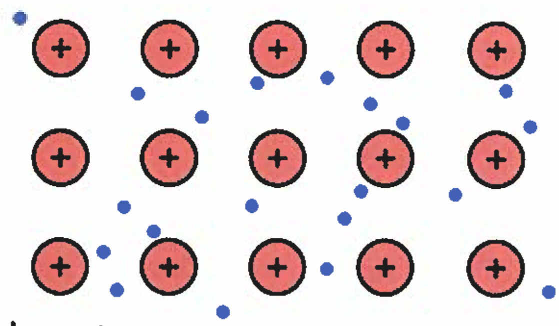
SODIUM CHLORIDE LATTICE STRUCTURE

All electrons are captured, very few free electrons

~~Any adder~~

Energy travels in a form of lattice oscillations that are slower

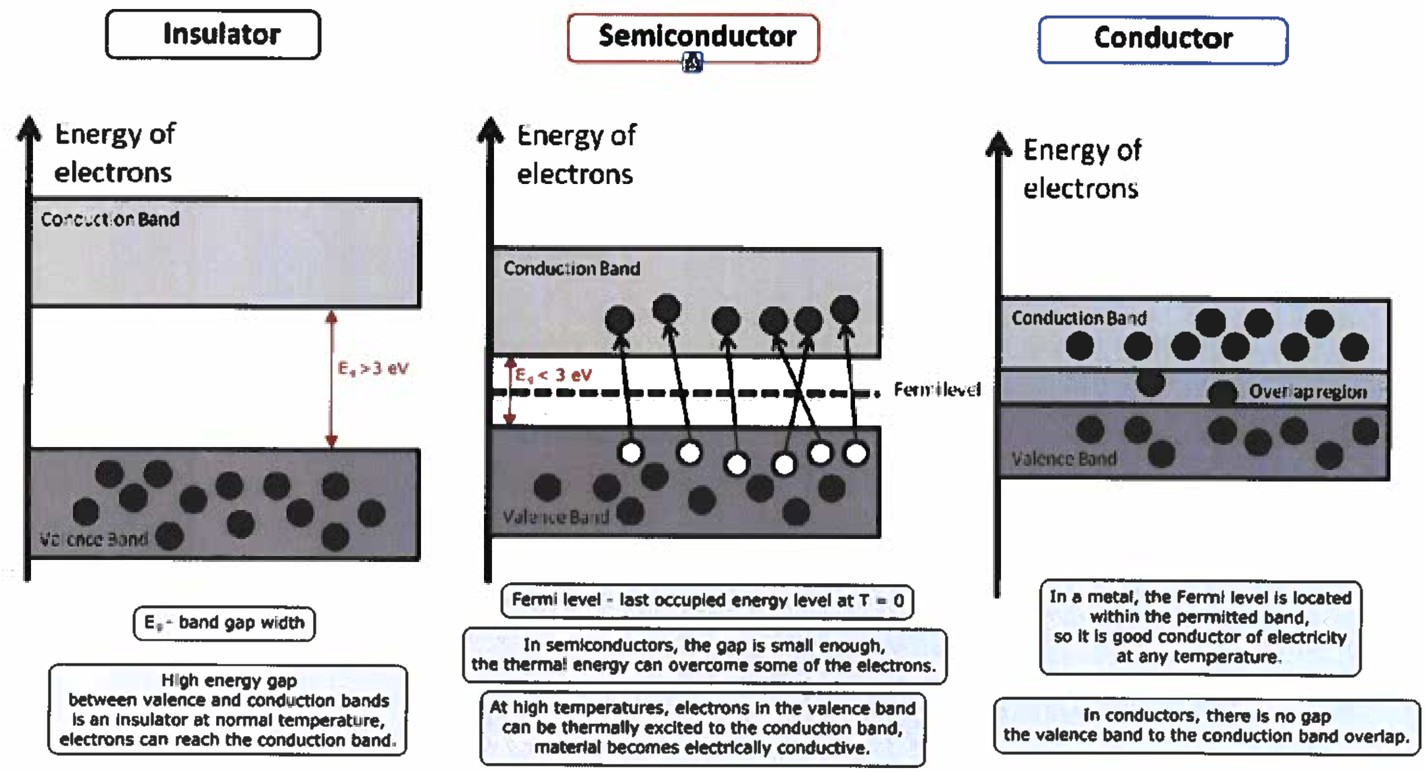
Conductor



(Valence) Electrons are free to move.

Can be modeled as ideal gas

Any heat flow first transfer the energy to the electrons, that quickly spread it throughout the volume, then thermalize with heavier ions.


Insulator
Energy of electrons
Conduction Band
E_g >3 eV
Valence Band
E_g - band gap width
High energy gap between valence and conduction bands is an insulator at normal temperature, electrons can reach the conduction band.
Semiconductor
Energy of electrons
Conduction Band
E_g < 3 eV
Fermi level
Valence Band
Fermi level - last occupied energy level at T = 0
In semiconductors, the gap is small enough, the thermal energy can overcome some of the electrons.
At high temperatures, electrons in the valence band can be thermally excited to the conduction band, material becomes electrically conductive.
Conductor
Energy of electrons
Conduction Band
Overlap region
Valence Band
In a metal, the Fermi level is located within the permitted band, so it is good conductor of electricity at any temperature.
In conductors, there is no gap the valence band to the conduction band overlap.

Radiation – energy is transferred through emission and absorption of electro-magnetic waves
Does not require any material contact b/w two objects

Emitted power $P = \sigma \cdot A \cdot e \cdot T^4$
$\sigma = 5.67 \cdot 10^{-8}\ W/m^2K^4$ Stephan constant

e - emissivity, describes the capacity of the object to absorb energy $0 < e < 1$

Such thermal radiation is often called <u>black body</u> radiation

Black body is an object that absorbs all radiation falling of its surface and then emits it with perfect emissivity $e = 1$

The origin of BB radiation is the microscopic motion of ~~atoms~~ ions and of electrons inside atoms / molecules, since every moving charged particle has to emit e-m waves

Planck's law

$$I(\lambda, T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

h - planck's constant

$\hbar = h/2\pi$ - Heisenberg constant

$\lambda_{max} \cdot T = const$