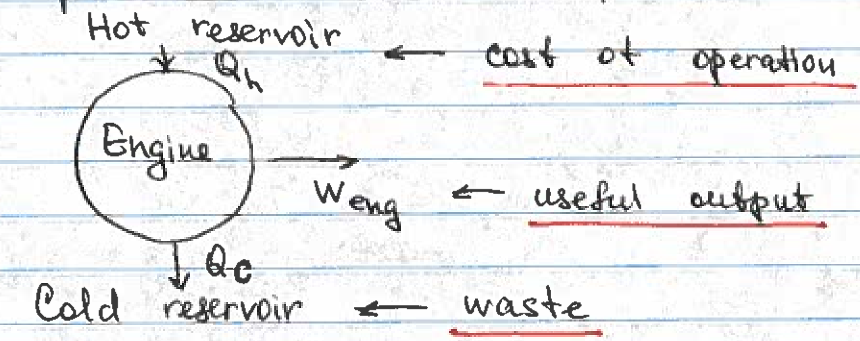
Heat engines and thermodynamic cycles

Heat engine – a device that takes energy from heat and, operating in a cycle, produces useful work.



First law of thermodynamics: $\Delta E_{int} = -W_{eng} + Q$

In a cycle $\Delta E_{int} = 0 \Rightarrow W_{eng} = Q = Q_h - |Q_c|$

Thermal efficiency $e = \dfrac{W_{eng}}{Q_h} = 1 - \dfrac{|Q_c|}{Q_h}$

$\boxed{e < 1}$

Second law of thermodynamics
(Kelvin – Planck form)

It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than converting all the input energy from heat into the equal amount of work.

Example of a heat engine cycle

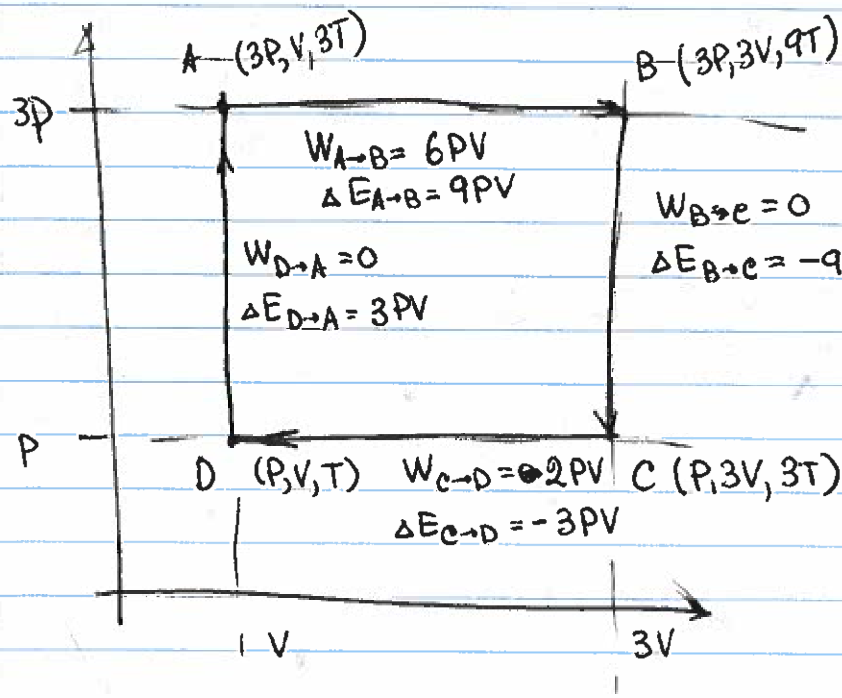
Useful equations:

Ideal gas law $\quad PV = nRT$

Work done by the gas $\quad W = \int_{V_i}^{V_f} P\,dV$

Change in the internal energy $\quad \Delta E_{int} = \frac{3}{2} nR(T_f - T_i)$

First law of thermodinqmics $\quad Q = \Delta E_{int} + W$



$W_{eng} = W_{AB} + W_{CD} = 4PV$ (total over the cycle)

$Q_H =$ ~~[illegible]~~

$Q_{A\to B} = 15PV$; $Q_{B\to C} = -9PV$; $Q_{CD} = -5PV$; $Q_{DA} = 3PV$

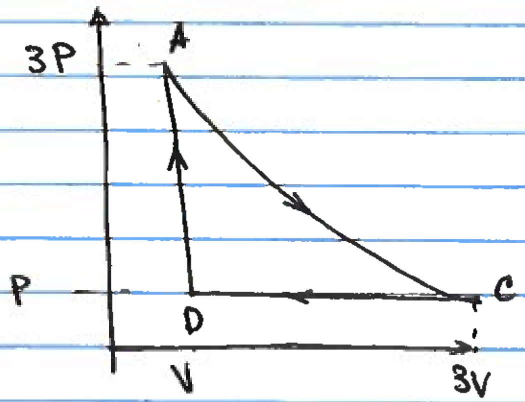
from hot reservoir $\quad$ from hot reservoir

$Q_H = Q_{AB} + Q_{DA} = 18PV$

Thermal efficiency

$$e = \frac{W_{eng}}{Q_H} = \frac{4PV}{18PV} \approx 22\%$$

Potentially a more efficient cycle?



Now we keep the same temperature going from $A \to C$

$\Delta E_{int} = 0$

$$W = \int_{V}^{3V} P\,dV = \int_{V}^{3V} \frac{3nRT}{V}\,dV$$

$$= 3nRT \ln 3 = 3PV \ln 3$$

$$W_{eng} = \underbrace{3\ln 3\, PV}_{W_{A\to C}} - \underbrace{2PV}_{W_{C\to D}}$$

$$Q_H = Q_{AC} + Q_{D\to A} = 3PV \ln 3 + 3PV$$

$$e = \frac{3\ln 3 - 2}{3\ln 3 + 3} \approx 20\%$$

Another "classic" process: adiabatic

$\Delta Q = 0$ (no heat exchange)

$\Delta E_{int} = -W$

all state variables (P, V, T) are changing

$$dE_{int} = \frac{3}{2} nR\, dT = nC_V\, dT$$

$$dW = P\,dV$$

$$PV = nRT \Rightarrow dP\cdot V + P\cdot dV = nR\,dT$$

$$dT = \frac{dP\cdot V + P\cdot dV}{nR}$$

$$dE_{int} = nC_V\cdot dT = -dW = +\left(\frac{dP\cdot V + P\,dV}{nR}\right)\cdot nC_V = -P\,dV$$

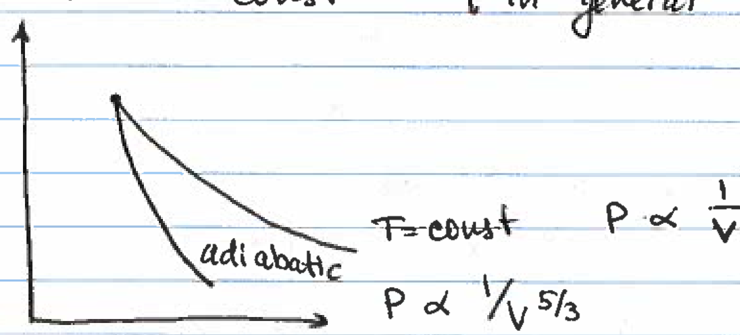
$$\underbrace{(R + C_V)}_{C_P} P\cdot dV + C_V\cdot V\,dP = 0 \qquad \gamma = \frac{C_P}{C_V}$$

$$\gamma P\,dV + V\cdot dP = 0$$

$$\gamma \frac{dV}{V} + \frac{dP}{P} = 0 \Rightarrow P\cdot V^{\gamma} = \text{const}$$

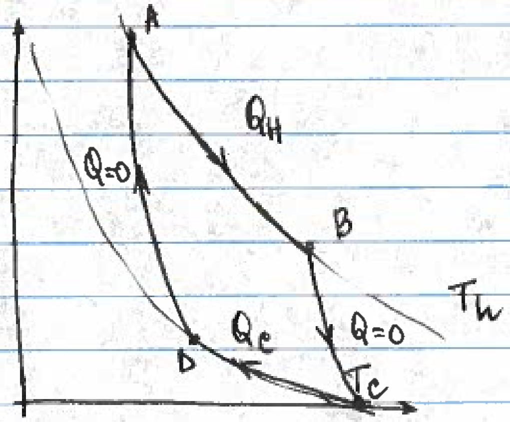
For ideal gas $\gamma = {}^5\!/_2 R \,/\, {}^3\!/_2 R = 5/3$

$PV^{5/3} = \text{const}$ (in general $PV^{\gamma} = \text{const}$)



Carnot engine

Carnot cycle consists of two <u>isothermal</u> processes, and two adiabatic processes



Heat only flows <u>into</u> the system $A \to B$

$$Q_H = W_{AB} = nRT_h \ln \frac{V_B}{V_A}$$

Heat only extracted $C \to D$

$$|Q_c| = -W_{CD} = nRT_c \ln \frac{V_C}{V_D}$$

For adiabatic process

$PV^{\gamma} = \text{const}$ but $PV = nRT$ $\Rightarrow$ $TV^{\gamma-1} = \text{const}$

$T_h V_B^{\gamma-1} = T_c V_C^{\gamma-1}$ and $T_c V_D^{\gamma-1} = T_H V_A^{\gamma-1}$

$$\frac{T_h}{T_c} = \left(\frac{V_C}{V_B}\right)^{\gamma-1} \qquad \frac{T_h}{T_c} = \left(\frac{V_D}{V_A}\right)^{\gamma-1}$$

$$\frac{V_C}{V_B} = \frac{V_D}{V_A} \Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$$

$$e_{carno} = 1 - \frac{T_c}{T_H}$$

Carnot theorem: no real heat engine operating b/w two energy reservoirs can be more efficient than a Carno engine operating b/w the same reservoirs

In our previous examples: if $T_H = 9T_c \Rightarrow e_{carnot} = \frac{8}{9} \approx 88\%$

if $T_H = 3T_c \Rightarrow e_{carnot} = \frac{2}{3} \approx 66\%$