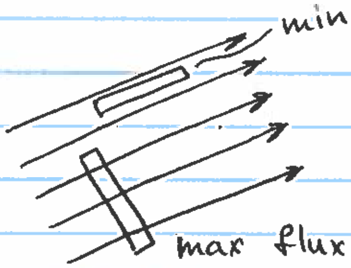
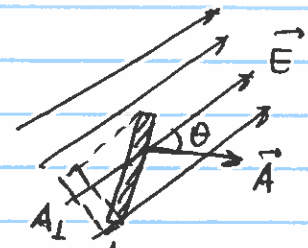
Gauss law for electric field

The concept of a flux



(electric) field lines

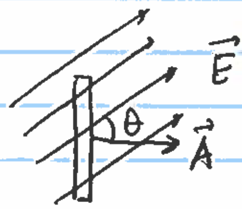


$\vec{A}$ - area vector

$|\vec{A}| = A$

direction - normal to the surface

$$\Phi = \vec{E}\cdot\vec{A} = |\vec{E}|\underbrace{|\vec{A}|\cos\theta}_{A_\perp}$$

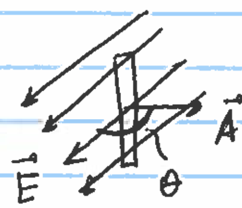


$\theta < 90^\circ$

$\Phi = |\vec{A}||\vec{E}|\cos\theta > 0$

$90^\circ < \theta < 180^\circ$

$\Phi = |\vec{A}||\vec{E}|\cos\theta < 0$



If electric field changes in space

$$d\Phi = \vec{E}\,d\vec{A}$$

$$\Phi_{tot} = \int_{\text{surface area}} \vec{E}\,d\vec{A}$$

If the surface is closed

$$\Phi_{tot} = \oint \vec{E}\,d\vec{A}$$

$d\vec{A}$ points outside

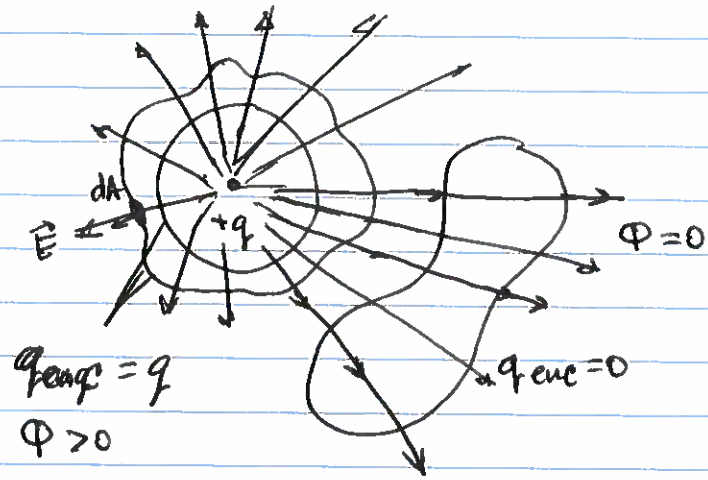
Gauss law

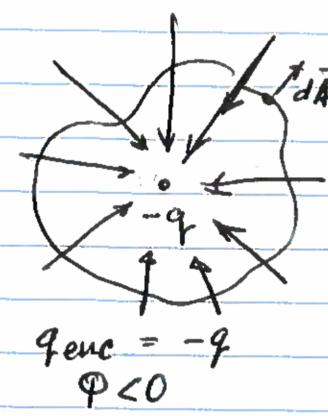
The flux of electric field $\vec{E}$ through any <u>closed</u> <u>surface</u> is equal to the net charge enclosed inside the surface, divided by ε_0.

$$\Phi = \oint \vec{E}\, d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$

ε_0 – dielectric permittivity of vacuum (purely historic artefact name)

$\varepsilon_0 = 8.84 \cdot 10^{-12}$ F/m (F – farad, wait till we get to capacitors)





Gauss law is always valid, and is the first out of four main equations of electromagnetic theory – Maxwell's equations.

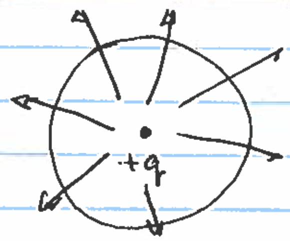
However, sometimes we can use the symmetry of the problem to find electric field. In this case we need to figure out a surface, along which E-field is constant, so then

$$\int \vec{E}\, d\vec{A} \rightarrow E \cdot \int dA = E \cdot A$$

Point charge

Electric field points radially away from positive charge (or into a negative charge)

$|\vec{E}|$ must be the same at the given distance



If we take a sphere, centered at q, $\vec{E}$ - field will be pointing perpendicular to this sphere everywhere, and E-field magnitude is constant

$$d\Phi = \vec{E}\, d\vec{A} = E \cdot dA$$

$$\Phi = \int d\Phi = E \underbrace{\int dA}_{\text{surface of the sphere}} = E \cdot A = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0} \frac{1}{4\pi r^2} = \overbrace{\frac{1}{4\pi\varepsilon_0}}^{k} \frac{q}{r^2} = \frac{kq}{r^2}$$

$$\vec{E} = \frac{kq}{r^2}\,\hat{r}$$

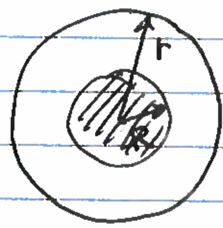
Here we used our physical intuition and the symmetry of the problem to provide extra information about $\vec{E}$ - field direction.

Uniformally charged sphere

Total charge $+Q$, radius R

volume charge density $\rho = Q \Big/ \frac{4\pi}{3}R^3 = \frac{3Q}{4\pi R^3}$

Two distinct regions



outside: $r > R$

$q_{enc} = Q$

$\Phi_{\substack{tot \\ outside}} = E \cdot 4\pi r^2 = \frac{Q}{4\pi\varepsilon_0 r^2}$

$E(r>R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ same as point charge

inside: $r > R$

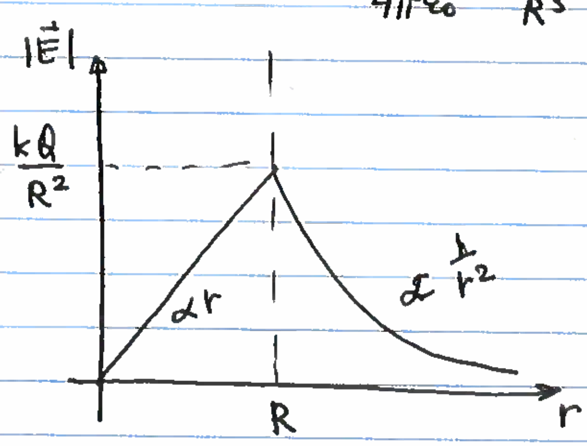
$q_{enc} < Q \qquad q_{enc} = \rho \cdot \frac{4\pi r^3}{3}$

$= Q \, \frac{4\pi}{3} r^3 \Big/ \frac{4\pi}{3} R^3 = Q \frac{r^3}{R^3}$

$\Phi_{inside} = |\vec{E}(r)| \cdot 4\pi r^2 = q_{enc}/\varepsilon_0 = \frac{1}{\varepsilon_0} Q \frac{r^3}{R^3}$

$|\vec{E}(\vec{r})| = \frac{1}{4\pi\varepsilon_0} \frac{r}{R^3}$

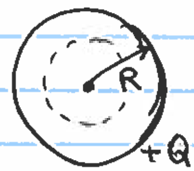
$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{r}}{R^3}$



Thin charged shell

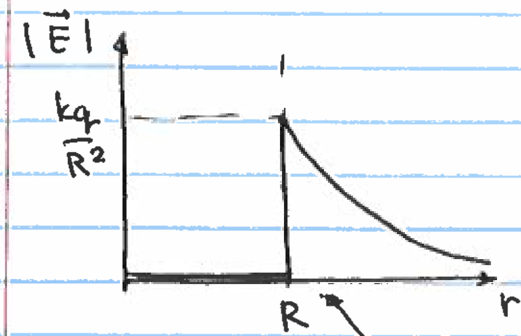
Outside $r > R$ $\quad q_{enc} = Q$

$$\vec{E}_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



Inside $r < R$ $\quad q_{enc} = 0$

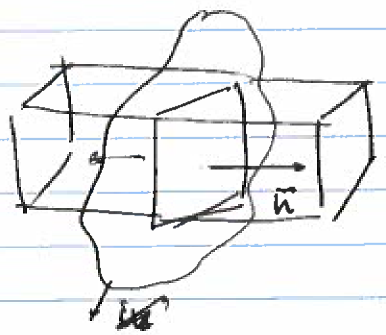
$$\vec{E}_{inside} = 0$$



the discontinuity indicates the location of the charge

Electric field inside any charged sphere is zero because of Gauss's law!

Uniformly charged plane



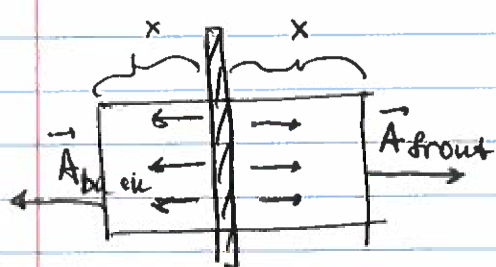
constant charge density σ

If the plane is infinite, $\vec{E}$ - field can only point perpendicular to the plane.

Good choice for Gaussian surface:

cube or cylinder with sides parallel to the plane

Top veiw



$q_{enc} = \sigma \cdot A$

$\Phi_{tot} = \Phi_{sides} + \Phi_{front} + \Phi_{back}$

$\Phi_{front} = \vec{E} \cdot \vec{A}_{front} = E \cdot A$

$\Phi_{back} = \vec{E} \cdot \vec{A}_{back} = E \cdot A$

$\Phi_{sides} = 0$

$$\Phi_{tot} = 2E \cdot \cancel{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma \cdot \cancel{A}}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

Note, this is the same electric field we calculated when near the surface of a charged disk.