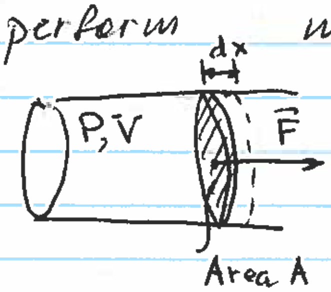
Ideal gasses: internal energy, heat, work
The first law of thermodynamics

Solids, liquids : $Q = mC\Delta T$
Heat leads to temperature change
(volume largely stays constant, apart from small thermal expansion)

Ideal gas → no interactions b/w atoms, only kinetic energy
Monoatomic gas → only linear motion $\langle K \rangle = \frac{3}{2} k_B T$

$$E_{int} = \frac{3}{2} N k_B T = \frac{3}{2} n \cdot RT$$

With gas, however, we must be more careful, as its volume can change, so the gas can perform mechanical work!



$|\vec{F}| = P \cdot A$

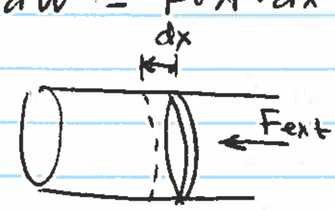
If the piston moves by small dx the work dW that gas performed

$dW = |\vec{F}| dx = P \cdot A \cdot dx = P\, dV$

You will have to be careful when reading problems :

— work done by the gas $dW = P\,dV$
when gas expands it does positive work

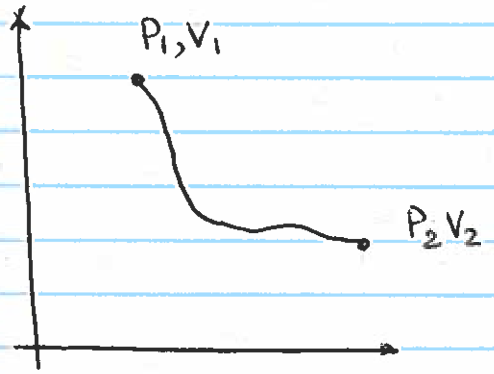
— work done on the gas (by some external force)
$dW = P \cdot A \cdot dx = -P\,dV$ since $dV < 0$



Balance $F_{ext} = P \cdot A$

I personally will try always use the work done by the gas

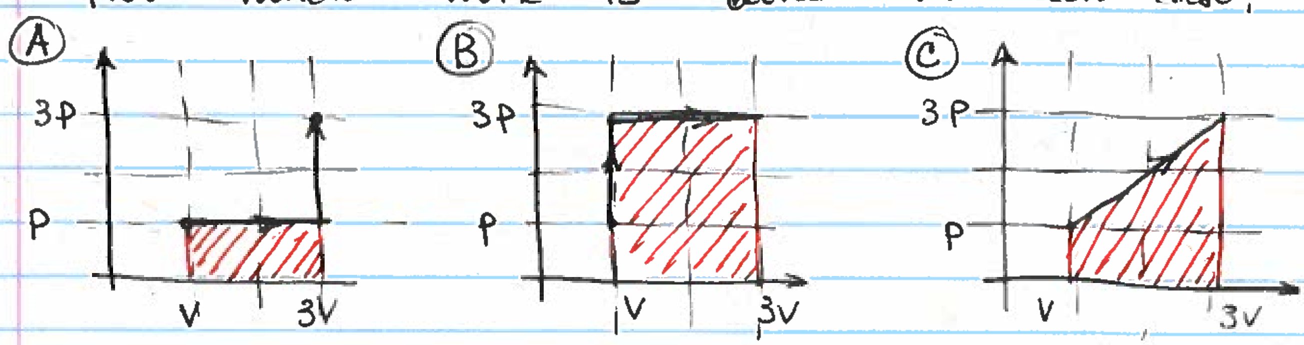
The amount of work depends on how the system evolves



$$W_{1\to 2} = \int dW = \int_{V_1}^{V_2} P\,dV$$

At any point of this curve $PV = nRT$, so $P = P(V,T)$

Example: one mole of an ideal gas warmed up slowly so that it goes from $(P,V) \rightarrow (3P, 3V)$ in three different ways. How much work is done in each case?



$W_A = P \cdot 2V = 2PV$ $\qquad$ $W_B = (3P)(2V) = 6PV$ $\qquad$ $W_C = 2P \cdot 2V = 4PV$

Change in the internal energy is the same

$T_{ini} = \frac{P \cdot V}{R}$ $\qquad$ $T_{fin} = \frac{9PV}{R}$

$$\Delta E_{int} = \frac{3}{2} R \Delta T = \frac{3}{2} \cancel{R} \cdot \frac{8PV}{\cancel{R}} = 12PV$$

The first law of thermodynamics

The change in the internal energy of the system $\Delta E_{int} = Q - W$

where Q is the heat exchange
W is the work done by the system

So in our three scenarios the amount of heat required in each case

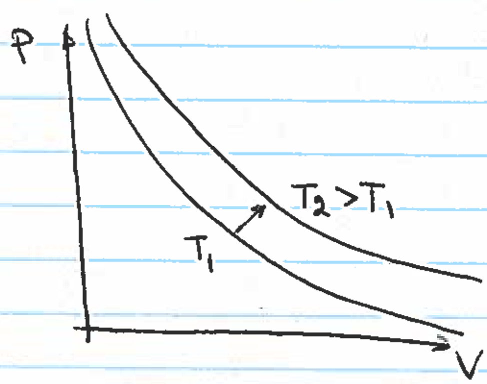
$Q_A = \Delta E_{int} + W = 12PV + 2PV = 14PV$

$Q_B = \Delta E_{int} + W = 12PV + 6PV = 18PV$

$Q_C = \Delta E_{int} + W = 12PV + 4PV = 16PV$

Special processes (iso = equal in Greek)

Isothermal $T = \text{const}$ $\Delta T = 0$ $\Delta E_{int} = 0$



$PV = \text{const} = nRT \Rightarrow P = \frac{nRT}{V}$

$$W = \int_{V_1}^{V_2} P\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} =$$

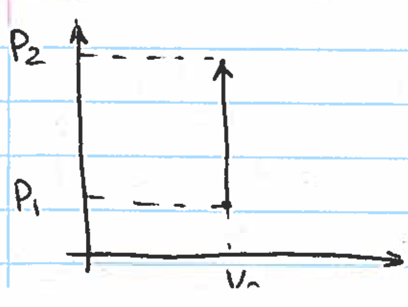
$$= nRT \ln V_2/V_1$$

$\Delta E_{int} = 0 = Q - W$

$Q = W$

All provided heat is transfered into work.

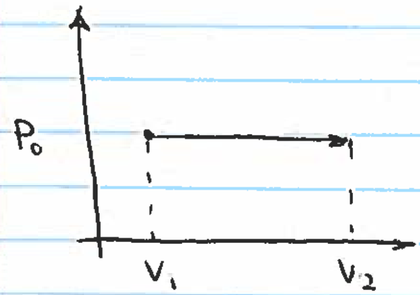
Isovolumetric / isochoric $\Delta V = 0$ no work! $W = 0$



$$Q = \Delta E_{int} = \frac{3}{2} nR (T_2 - T_1) =$$

$$= \frac{3}{2}(P_2 V_0 - P_1 V_0) = \frac{3}{2} \Delta P \cdot V_0$$

Isobaric $\quad P = \text{const} \quad \Delta P = 0$



$$W = P_0 (V_2 - V_1) = nR\,\Delta T$$

$$\Delta E_{int} = \frac{3}{2} nR\,\Delta T$$

$$Q = W + \Delta E_{int} = \frac{5}{2} nR\,\Delta T$$

Can we define specific heat capacity for a gas? Yes, but we have to be careful to define what type of process it is

$V = \text{const}$: $\quad Q = \frac{3}{2} nR\,\Delta T \overset{\text{define}}{=} n \cdot C_V \cdot \Delta T \Rightarrow C_V = \frac{3}{2} R$

$P = \text{const}$: $\quad Q = \frac{5}{2} nR\,\Delta T \overset{\text{define}}{=} n \cdot C_P \Delta T \Rightarrow C_P = \frac{5}{2} R$