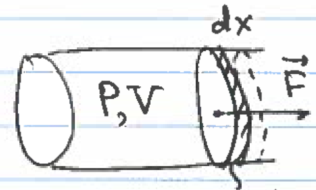
First law of thermodynamics

Let's get back to ideal gases (monoatomic)

Internal energy: $E_{int} = \frac{3}{2} N k_B T = \frac{3}{2} nRT$

E_{int} is a state variable (uniquely defined by temperature)

Gas can expand or contract, thus producing mechanical work

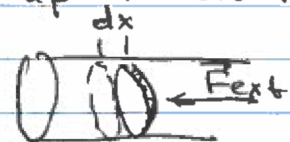


Force acting on a piston
$|\vec{F}| = P \cdot A$
If the piston moves by dx the amount of work done is $dW = F dx = P A dx = P dV$

Have to use differentials here, since pressure may change

Warning: read problem carefully, two variants of work are used intermittently

- work done by the gas $dW = P dV$
  when gas expands, this work is positive
  energy of the gas is used up to do the work



- work done on the gas
  $dW = -P dV$, work done by an external force, added energy

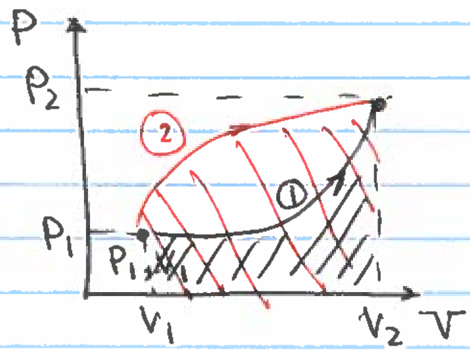
I will try consistently use the first definition

Total amount of work done when the gas changes from (P_1, V_1) to (P_2, V_2)

$$W = \int_{V_1}^{V_2} P(V)\, dV$$

This value depends on how this change happened!

P-V diagram

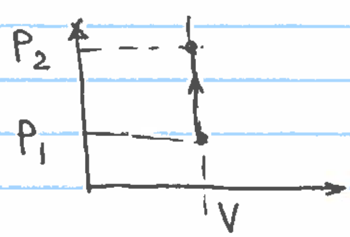


Work $\longrightarrow$ integral under P(V) curve

$W_1 < W_2$

Special processes

① Isovolumetric / isochoric $V = \text{const}$



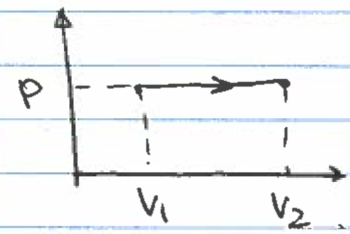
$\Delta V = 0 \qquad W = 0$

$$\Delta E_{int} = \frac{3}{2} nR(T_2 - T_1) =$$

$$= \frac{3}{2} nRT_2 - \frac{3}{2} nRT_1 = \frac{3}{2} P_2 V - \frac{3}{2} P_1 V \Rightarrow$$

$$\Delta E_{int} = \frac{3}{2} \Delta P \cdot V$$

2 Isobaric $P = const$

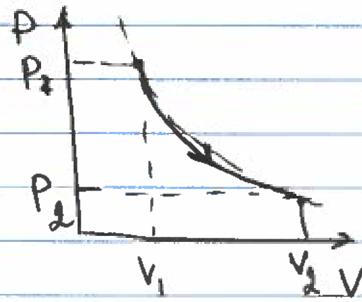


$P = const$ $\quad W = \int_{V_1}^{V_2} P\,dV = P\int_{V_1}^{V_2} dV = P\Delta V$

$$\Delta E_{int} = \frac{3}{2} nRT_2 - \frac{3}{2} nRT_1 =$$

$$= \frac{3}{2} P_2 \cdot V_2 - \frac{3}{2} P \cdot V_1 = \frac{3}{2} P\Delta V$$

3 Isothermal $T = const$

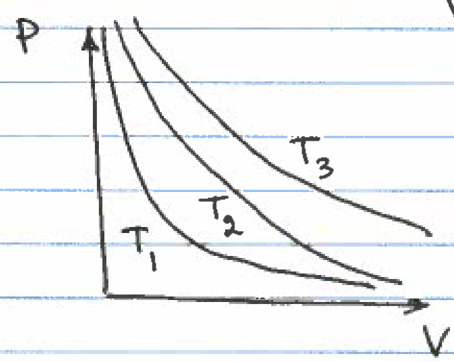


$PV = nRT = const \Rightarrow P = \frac{nRT}{V}$

$$W = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln \frac{V_2}{V_1}$$

$\Delta E_{int} = 0$ since $T = const$

On a P-V diagram the ~~points~~ states with same temperature are on the hyperbola line $P = \frac{nRT}{V}$



$T_1 < T_2 < T_3$

The first law of thermodynamics

The change in the internal energy of the system ΔE_{int} is

$$\Delta E_{int} = Q - W \qquad \text{energy conservation}$$

Isothermal process $\quad T = const \quad \Delta E_{int} = 0$

$$Q = W = nRT \ln V_2/V_1$$

Isochoric process $\quad V = const \quad W = 0$

$$Q = \Delta E_{int} = \frac{3}{2} \Delta P \cdot V = \frac{3}{2} nR\Delta T$$

Isobaric process $\quad P = const$

$$\underbrace{\frac{3}{2} P \cdot \Delta V}_{\Delta E_{int}} = Q - \underbrace{P \cdot \Delta V}_{W} \qquad Q = \frac{5}{2} P\Delta V$$

A gas doesn't have a well-defined specific heat capacity, since the amount of energy needed changes depending on a process!

Two special cases

$$V = const: \quad Q_V = \frac{3}{2} nR\Delta T \equiv n \cdot C_V \cdot \Delta T \qquad C_V = \frac{3}{2} R$$

$$P = const: \quad Q_P = \frac{5}{2} nR\Delta T \equiv n \cdot C_P \Delta T \qquad C_P = \frac{5}{2} R$$

Note that for the same temperature change

$$Q_P = Q_V + nR\Delta T \Rightarrow \boxed{C_P = C_V + R}$$

let's remember that