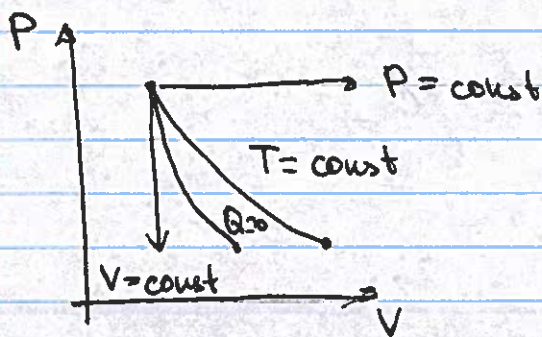


Let's summarize what have we learned about thermodynamics of various processes

Process	ΔE_{int}	Work	heat Q
Isothermal $T = \text{const}$	0	$\int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln \frac{V_2}{V_1}$	$nRT \ln \frac{V_2}{V_1}$
Isochoric $V = \text{const}$	$\frac{3}{2} nR \Delta T$ (monoatomic gas) $n C_V \Delta T$ (general)	0	$n C_V \Delta T$
Isobaric $P = \text{const}$	$\frac{3}{2} nR \Delta T$ (monoatomic gas) $n C_V \Delta T$	$P \Delta V = nR \Delta T$	$n C_V \Delta T + nR \Delta T = n(C_V + R) \Delta T = n C_P \Delta T$
Adiabatic	$n C_V \Delta T$	$\int_{V_1}^{V_2} P dV =$	0

These processes on PV diagram



In an adiabatic process all state variables are changing

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

and $P_1 V_1 = nRT_1$ and $P_2 V_2 = nRT_2$

At any point of the adiabatic process:

$$P V^\gamma = P_1 V_1^\gamma \Rightarrow P = \frac{P_1 V_1^\gamma}{V^\gamma} \leftarrow \text{constant}$$

$$W = \int_{V_1}^{V_2} \frac{P_1 V_1^\gamma}{V^\gamma} dV = -\frac{1}{\gamma-1} P_1 V_1^\gamma \left(\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right) =$$

$$= \frac{1}{\gamma-1} (P_1 V_1 - P_2 V_2)$$

Equipartition of ~~Energy~~ energy


Recall that we calculated the average value of the kinetic energy of an ideal gas without internal structure


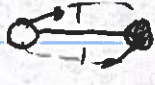
$$\langle K \rangle = \frac{3}{2} k_B T \quad \text{for 3D motion. Each dimension "contributed" } \frac{1}{2} k_B T$$

Single atom \rightarrow only linear motion in 3 independent direction
i.e., we need three parameter to describe completely the motion of an atom

Molecules are more complex!

Diatomic molecule: linear motion (like an atom) + internal motion

Vibrational motion  one possible motion, but contributes twice in energy (kinetic + potential)

Rotation  or 

Each independent way of moving = degree of freedom

Single atom has 3 degrees of freedom

Diatomic molecule has $3 + 2 + 2 = 7$ max degrees of freedom

Each degree of freedom contributes $\frac{1}{2} k_B T$ into the internal energy

$$\Delta E_{\text{int}} = N \cdot 7 \cdot \frac{1}{2} k_B T = \frac{7}{2} N k_B T = \frac{7}{2} n R T$$

Strangely enough, the value of the internal energy for a diatomic (and more complex) molecules is not always the same, because depending on the temperature, not all degrees of freedom are activated!

Rotational and vibrational motion of molecules is quantized, i.e. it requires a certain amount of energy - a quantum - to get excited.

If characteristic frequency is ω , we need at least $\hbar\omega$ of energy to start this motion (\hbar - Planck's constant $\hbar = 10^{-34}$ J.s)

Rotational frequencies are smaller and thus easier to activate than vibrational. In fact, at room temperature usually vibrational degrees of freedom are not activated.

Then there are 5 degrees of freedom

$$\Delta E_{int} = \frac{5}{2} N k_B T = \frac{5}{2} n R T$$

In this case for a diatomic gas:

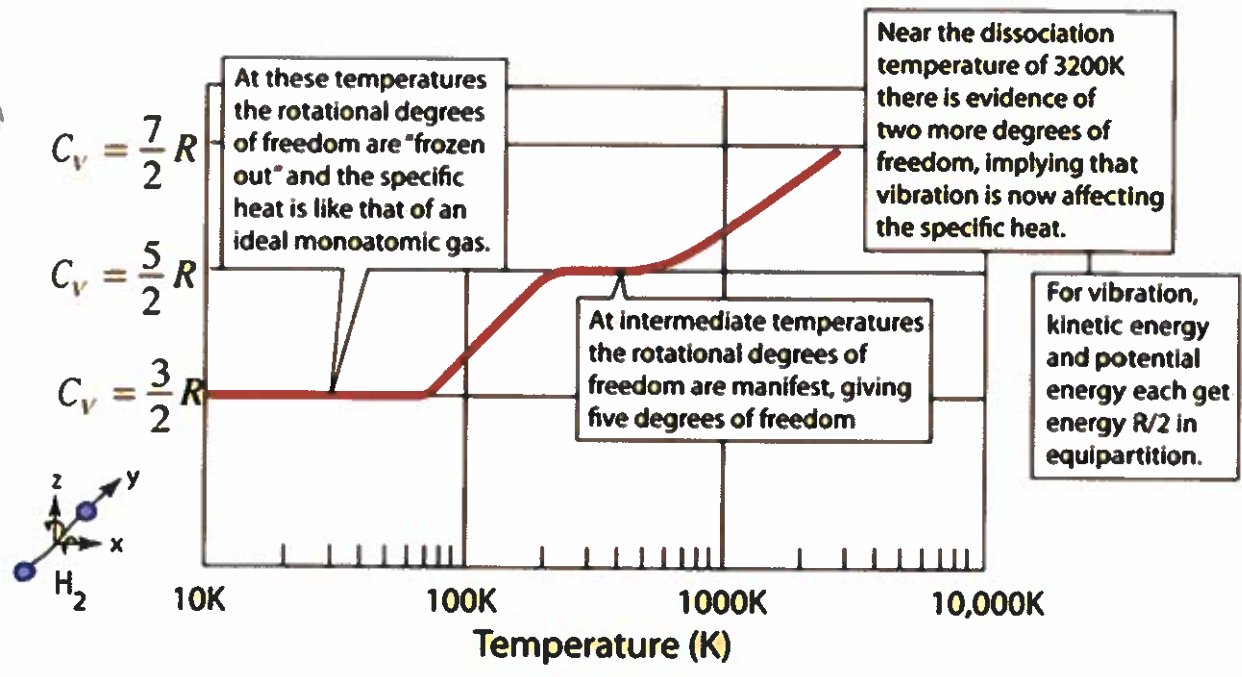
$$C_v = \frac{3}{2} R \quad (\gamma = \frac{5}{3}) \text{ for low temp, no rotations or vibrations}$$

$$= \frac{5}{2} R \quad (\gamma = \frac{7}{5}) \text{ for intermediate temp, only rotation}$$

$$= \frac{7}{2} R \quad (\gamma = \frac{9}{7}) \text{ for high temp, rotation + vibration}$$

In any case $C_p = C_v + R$ and

$$\gamma = \frac{C_p}{C_v}$$



Hydrogen molecule heat capacity (theory and experiment)

