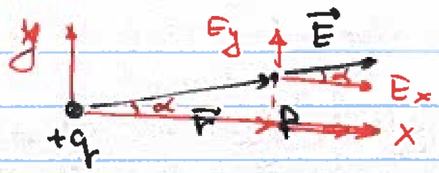
Electricity – review

Point electric charge



Electric field (vector)

$$\vec{E} = \frac{kq}{r^2}\,\hat{r} \qquad k = \frac{1}{4\pi\varepsilon} = 9\cdot 10^{9}\,\frac{N\cdot}{C}$$

$\cos\alpha = x/\sqrt{x^2+y^2}$

$\sin\alpha = y/\sqrt{x^2+y^2}$

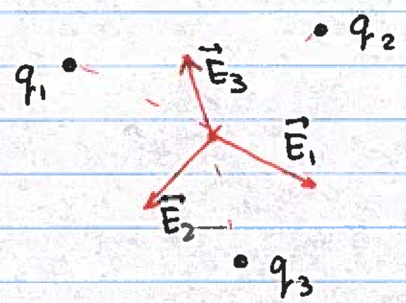
$$\vec{E} = \{E_x, E_y\} = \left\{\frac{kq}{r^2}\cos\alpha,\ \frac{kq}{r^2}\sin\alpha\right\}$$

$$= \left\{\frac{kqx}{(x^2+y^2)^{3/2}};\ \frac{kqy}{(x^2+y^2)^{3/2}}\right\}$$

Electrostatic potential (scalar) $V = \dfrac{kq}{r} = \dfrac{kq}{\sqrt{x^2+y^2}}$

in general $\vec{E} = -\nabla V = \left\{-\dfrac{\partial V}{\partial x},\ -\dfrac{\partial V}{\partial y},\ -\dfrac{\partial V}{\partial z}\right\}$

Combination of point charges



$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_{tot\,x} = E_{1x} + E_{2x} + E_{3x}$$

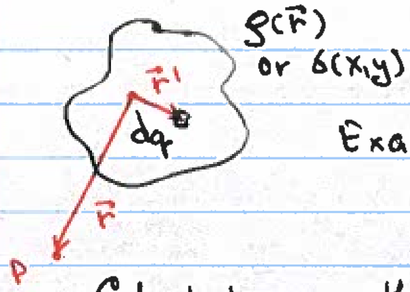
$$E_{tot\,y} = E_{1y} + E_{2y} + E_{3y}$$

$$V_{tot} = V_1 + V_2 + V_3$$

Force on a test charge
$\vec{F} = q_{test} \cdot \vec{E}$

$$W_{12} = \frac{kq_1 q_2}{r_{12}}$$

Continuous charge distribution : general approach



1. "Break" the volume into small pieces → "point" charges

Examples: $dq = \rho(\vec{r})\, dV$ (3d)

$dq = \sigma(x,y)\, dx\, dy$ (2d)

2. Calculate the components of the electric field or the value of potential in the point of interest

$$d\vec{E}(\vec{r}) = \frac{\rho(\vec{r}')\, dV}{|\vec{r}-\vec{r}'|^2} (\hat{r}-\hat{r}')$$

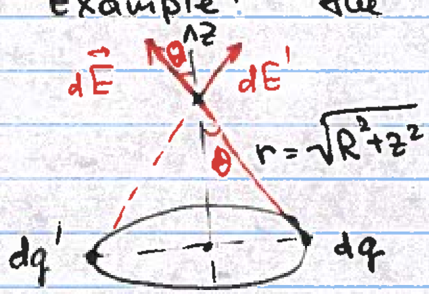
$$dV = \frac{\rho(\vec{r}')\, dV}{|\vec{r}-\vec{r}'|}$$

3. Integrate over the whole charge distribution

$$\vec{E} = \int_{\text{volume}} d\vec{E}(\vec{r}') \qquad V = \int_{\text{volume}} dV(\vec{r}')$$

For most problem you encounter there is usually ways to use the symmetry of the problem to simplify the calculation

Example: the electric field of ~~the~~ a ring



only vertical z- component not cancelled

$$dE_z = \frac{k\, dq}{r^2} \cos\alpha$$ (since I defined θ differently)

$$\cos\theta = \frac{z}{r}$$

$$dE_z = \frac{k\, dq \cdot z}{r^3}$$

Summing over all "elementary" charges $dq \to Q$

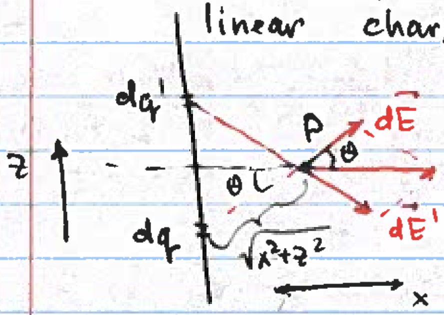
$$E_z = \frac{kQz}{(R^2+z^2)^{3/2}} \qquad E_x = E_y = 0$$

$$dV = \frac{k\,dq}{r} \quad \text{same for all } dq$$

$$V = \frac{kQ}{r} = \frac{kQ}{\sqrt{x^2+z^2}}$$

Similar approach

linear charge density $\lambda = Q/L$



here only x-component contributes

$$dE_x = \frac{k\,dq}{(x^2+z^2)}\cos\theta = \frac{k\,dq \cdot x}{(x^2+z^2)^{3/2}}$$

$dq = \lambda dz$

$$E_x = \int_{-\infty}^{+\infty} \frac{k\lambda \cdot x \; dz}{(x^2+z^2)^{3/2}} = k\lambda x \underbrace{\int_{-\infty}^{+\infty} \frac{dz}{(x^2+z^2)^{3/2}}}_{\text{would be provided}}$$

$$= \frac{2}{x^2}$$

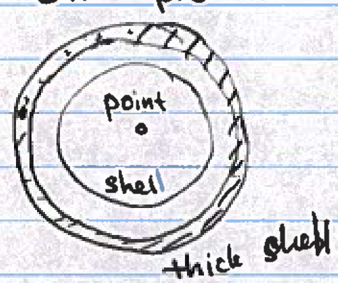
$$E_x = \frac{2k\lambda}{x}$$

Alternative approach to calculate Electric field : Gauss's law

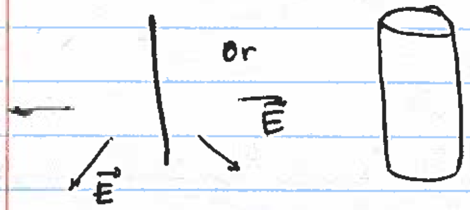
$$\oint \vec{E}\, d\vec{s} = \frac{q_{enc}}{\varepsilon_0}$$

If (using the symmetry considerations) we can
a) predict the direction of $\vec{E}$
b) predict a surface for which $|\vec{E}|$ is the same
we can use this Gaussian surface to find the electric field magnitude

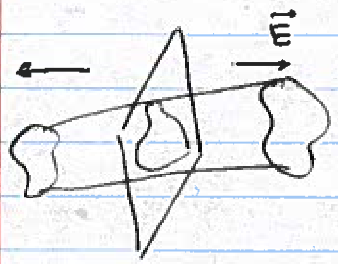
Example: spherical symmetry



For all these cases we can use a concentric sphere as a gaussian surface, $\vec{E}$ is always perpendicular



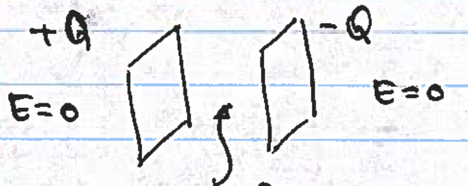
→ cylindrical surface
$\vec{E}$ is perpendicular
(or zero on top & bottom)



– plane → one can use any shape ~~the~~ prism with sides perpendicular to the surface since $\vec{E}$ must be perpendicular to the surface

$$2ES = \frac{\sigma \cdot S}{\varepsilon_0} \qquad E_{surface} = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Capacitors



$E = Q/A\varepsilon_0$

$$V = \frac{Q \cdot d}{A\varepsilon_0} = \frac{Q}{C} \qquad C = \frac{A\varepsilon_0}{d}$$

capacitance

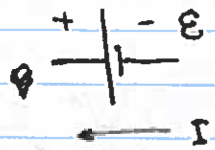
Energy stored $U_C = \frac{Q^2}{2C} = \frac{1}{2}VC^2 = \frac{V \cdot Q}{2}$

Conductors vs insulators

– No electric field inside a conductor (free moving charges will arrange themselves on the surface to have equal potential on the surface so that no more force make them move)

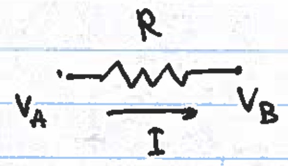
– In an insulator we can create any charge distribution, since the charges cannot move

Electric circuit components



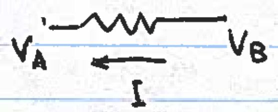
A battery provides an emf (constant voltage) to move charges in a closed loop

ideal wire — connect points with the same ~~to~~ potential

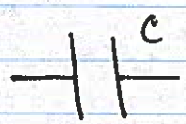


$V_B - V_A = -IR$ voltage drop

$U_R = I^2R = V^2/R = IV$ Power dissipated



$V_B - V_A = IR$ if the current direction is reversed



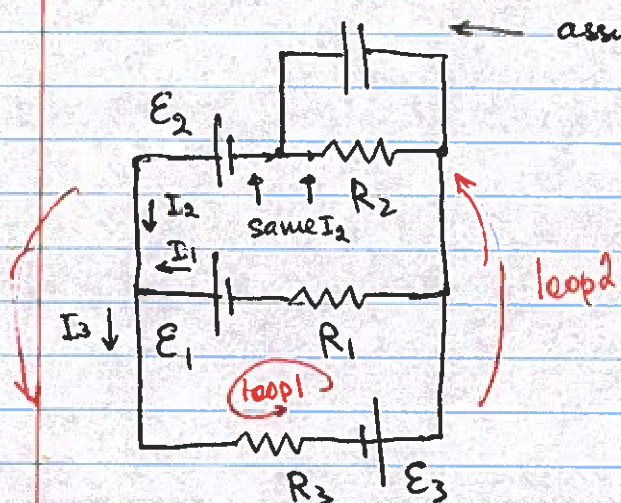
Charged capacitor $I_C = 0$ no current

Uncharged capacitor $V_C = 0$ no voltage drop

Kirchoff's rules

1. For each junction Σ currents in $=$ Σ currents out

2. For each loop $\Sigma V = 0$



← assume charged $I_C = 0$

$I_1 + I_2 = I_3$

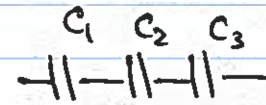
$-I_1R_1 + \mathcal{E}_1 - IR_3 + \mathcal{E}_3 = 0$

$-I_3R_3 + \mathcal{E}_3 - I_2R_2 + \mathcal{E}_2 = 0$

$V_C = I_2 \cdot R_2$ if needed

(can treat as an extra loop)

Series connection



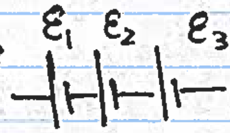
same current, $V_{tot} = V_1 + V_2 + V_3$

$R_{eq} = R_1 + R_2 + R_3$

same charge, $V_{tot} = V_1 + V_2 + V_3$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

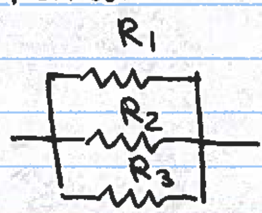
~~Paralle~~

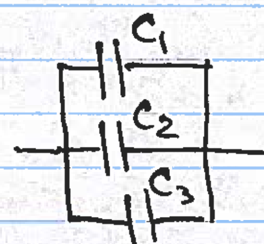


Battery stack

$\mathcal{E}_{tot} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3$

Parallel connection





same voltage, $I_{tot} = I_1 + I_2 + I_3$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

same voltage, $Q_{tot} = Q_1 + Q_2 + Q_3$

$C_{eq} = C_1 + C_2 + C_3$

(don't put batteries in series unless you know what you're doing)