Let's gather together relevant information about different cycles
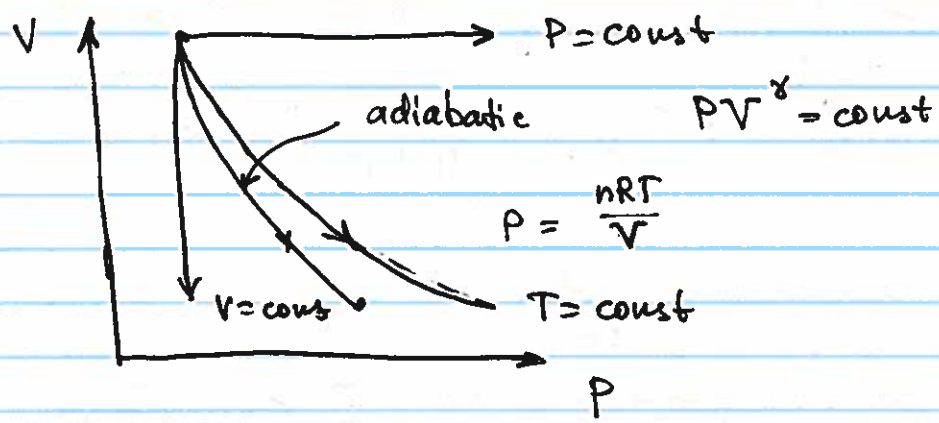
	ΔE_{int}	W	Q
Isothermal $T = $ const	0	$\int_{V_1}^{V_2} P\,dV = nRT \ln \frac{V_2}{V_1}$	$nRT \ln \frac{V_2}{V_1}$
Isobaric $P = $ const	~~$\cancel{}$~~ $nC_V \Delta T$ $[\frac{3}{2}nR\Delta T]$	$P\Delta V = nR\Delta T$	$\Delta E_{int} + W = nC_p \Delta T$ $nC_V\Delta T + nR\Delta T = n(C_V + R)\Delta T$
Isochoric $V = $ const	$nC_V \cdot \Delta T [\frac{3}{2}nR\Delta T]$	0	$n C_V \Delta T$
Adiabatic $Q = $ ~~const~~ 0 $PV^\gamma = $ const	$nC_V \cdot \Delta T [\frac{3}{2}nR\Delta T]$	$\int_{V_1}^{V_2} P\,dV = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$	0

for monoatomic gas

Adiabatic process: $\quad W = \int_{V_1}^{V_2} P\,dV = P_1 V_1^\gamma \int_{V_1}^{V_2} \frac{dV}{V^\gamma} =$

$$= P_1 V_1^\gamma \left(-\frac{1}{\gamma - 1}\right) \frac{1}{V^{\gamma - 1}} \Big|_{V_1}^{V_2} = P_1 V_1^\gamma \frac{1}{\gamma - 1} \left[\frac{1}{V_1^{\gamma - 1}} - \frac{1}{V_2^{\gamma - 1}}\right] =$$

$$= \frac{1}{\gamma - 1}\left[\frac{P_1 V_1^\gamma}{V_1^{\gamma - 1}} - \boxed{\frac{P_1 V_1^\gamma}{V_2^{\gamma - 1}}} \; {\color{red}P_2 V_2^\gamma}\right] = \frac{1}{\gamma - 1}\left[P_1 V_1 - P_2 V_2\right]$$

P = const

adiabatic $PV^\gamma = $ const

$P = \frac{nRT}{V}$

V = const   T = const

Equipartition of energy

Recall we calculated that for an ideal gas without internal structure (monoatomic gas) the average kinetic energy of an atom

$$\langle K \rangle = \frac{3}{2} k_B \cdot T$$

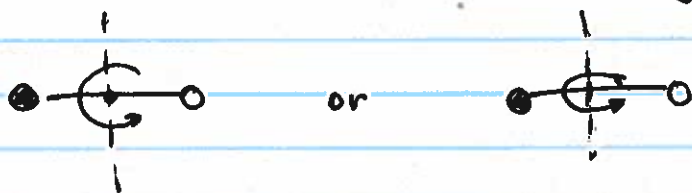because we live in a 3D world 3 possible independent types of motion (along x, y, z)

Scientific name: degrees of freedom

Single atom ⟶ 3 degrees of freedom

Molecules ⟶ more possible motions!

Diatomic molecule : ① linear motion → 3 degrees of freedom

② Rotational motion : 2 degrees of freedom

●━⊕━○   or   ●━⊕━○

Vibrations   ●∿∿○   two more degrees of freedom (kinetic + potential energy)

Max total # deg number of degrees of freedom
3 + 2 + 2 = 7.

Equipartition of energy : each degree of
freedom contributes $\frac{1}{2} k_B T$ into
the internal energy of a molecule

If all motions are activated, for a
diatomic gas

$$E_{int} = \frac{7}{2} k_B \cdot T \cdot N = \frac{7}{2} nRT$$

$$C_v = \frac{\Delta E_{int}}{n \Delta T} = \frac{7}{2} R \qquad C_p = C_v + R = \frac{9}{7} R$$

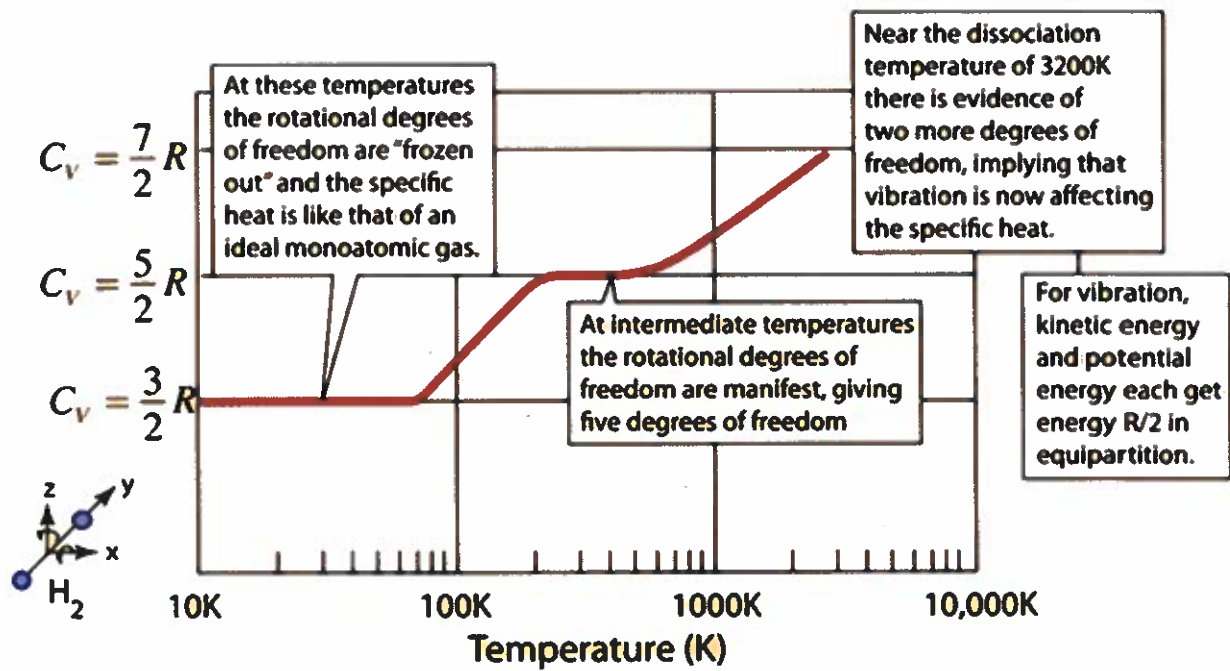adiabatic facto $\qquad \gamma = C_p / C_v = {}^9/_7 \approx 1.3$

However, if you look up C_v of atmospheric
gasses, it is $\quad C_v = \frac{5}{2} nRT!$ What is wrong?

Quantum mechanics!
Rotational and vibrational energy is
quantized $\rightarrow$ i.e., you must provide
certain amount of energy to initiate
this motion.
At room temperature molecules have
enough energy to rotate, but not
enough energy to vibrate!

$$C_v = \frac{7}{2}R$$

At these temperatures the rotational degrees of freedom are "frozen out" and the specific heat is like that of an ideal monoatomic gas.

Near the dissociation temperature of 3200K there is evidence of two more degrees of freedom, implying that vibration is now affecting the specific heat.

$$C_v = \frac{5}{2}R$$

At intermediate temperatures the rotational degrees of freedom are manifest, giving five degrees of freedom

For vibration, kinetic energy and potential energy each get energy R/2 in equipartition.

$$C_v = \frac{3}{2}R$$

z y x

H_2

10K   100K   1000K   10,000K

Temperature (K)

Hydrogen molecule heat capacity (theory and experiment)

C_V, Calories per mol per Degree

8

6

4

2

0

$\frac{7}{2}\mathcal{R}$

Vibration

$\frac{5}{2}\mathcal{R}$

Rotation

$\frac{3}{2}\mathcal{R}$

Translation

10   25   50 75 100   250   500 750 1000   2500   5000

Absolute Temperature, °K