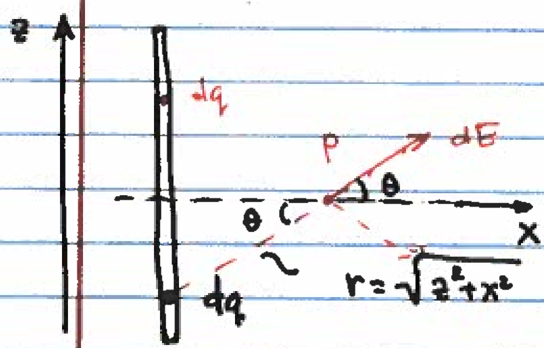
More applications of Gauss law for calculating electric field

A line of charges - analytical calculation



Assuming symmetric position only dE_x will contribute

$$dE_x = \frac{k\,dq}{r^2}\cos\theta = \frac{k\,dq \cdot x}{(z^2+x^2)^{3/2}}$$

$\lambda = Q/L$

$\cos\theta = x/\sqrt{z^2+x^2}$

$$dE_x = \frac{k\lambda\,dz \cdot x}{(x^2+z^2)^{3/2}}$$

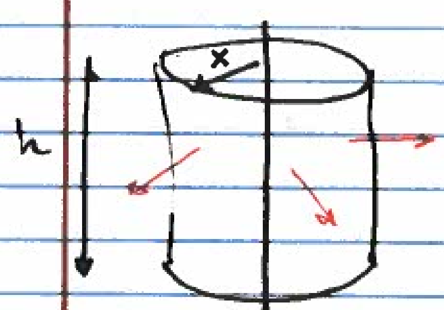
$$E_{tot_x}(x) = \int_{-L/2}^{L/2} dE_x = k\lambda \cdot x \int_{-L/2}^{L/2} \frac{dz}{(x^2+z^2)^{3/2}}$$

$$= k\lambda x \cdot \frac{1}{x^2} \left.\frac{z}{\sqrt{z^2+x^2}}\right|_{-L/2}^{L/2} = \frac{\lambda k L}{x\sqrt{x^2+L^2/4}}$$

if $L \gg x$ (don't see the ends, infinite rods)

$$E_{tot_x}(x) \approx \frac{\lambda k}{x} \lim_{L\to\infty} \frac{L}{\sqrt{L^2/4+x^2}} \approx \frac{2\lambda k}{x}$$

Calculation using Gauss law



Gaussian surface - cylinder surrounding the wire

$$\Phi = E \cdot A_{side} = E \cdot 2\pi x \cdot \cancel{h} =$$

$$= \frac{1}{\varepsilon_0} q_{enc} = \frac{1}{\varepsilon_0} \cdot \lambda \cdot \cancel{h}$$

$A_{side} = 2\pi x \cdot h$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} = \frac{2k\lambda}{x}$$

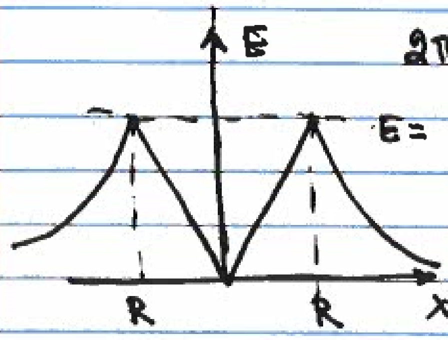
Uniform charged cylinder of radius R

$$E_{outside} = \frac{2k\lambda}{x}$$ (as for a line charge)

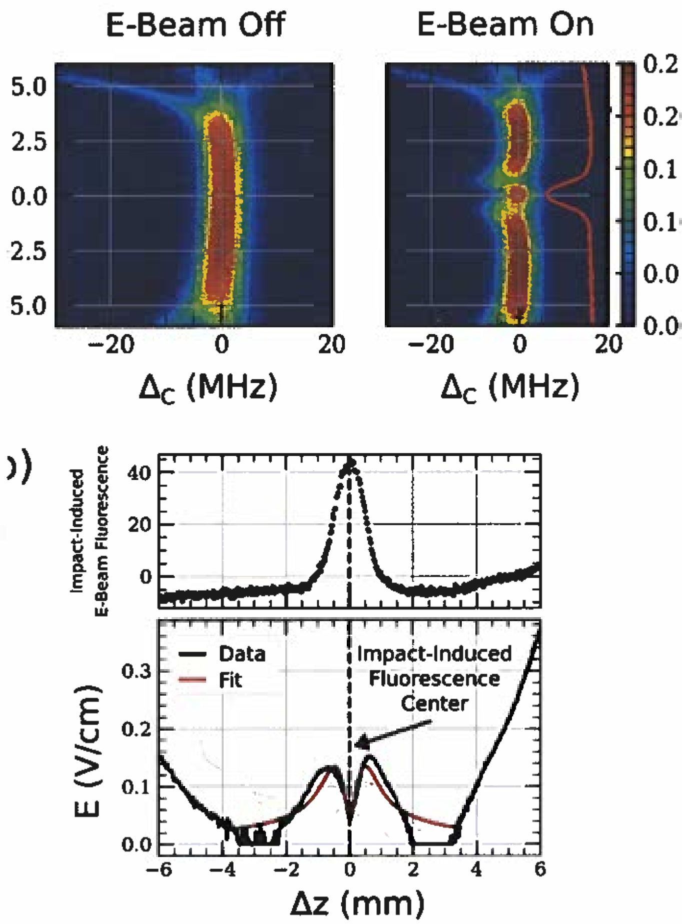
$$\Phi = E \cdot A_{side} = 2\pi x h E = \frac{1}{\varepsilon_0} q_{enc}$$

$$q_{enc} = \lambda \cdot h \cdot \frac{\cancel{\pi} x^2}{\cancel{\pi} R^2} = \lambda \cdot h \cdot \frac{x^2}{R^2}$$

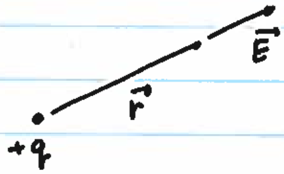
$$2\pi x \cancel{h} E = \frac{1}{\varepsilon_0} \lambda \cdot \cancel{h} \frac{x^2}{R^2} \qquad E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda \cdot x}{R^2}$$



$$E = \frac{2k\lambda}{R}$$


E-Beam Off
E-Beam On
5.0
2.5
0.0
2.5
5.0
−20
0
20
Δ_C (MHz)
0.2
0.1
0.0
)
Impact-Induced
E-Beam Fluorescence
40
20
0
Data
Fit
Impact-Induced
Fluorescence
Center
E (V/cm)
0.3
0.2
0.1
0.0
−6
−4
−2
0
2
4
6
Δz (mm)

Electric potential and potential difference



Electric field of a point charge

$$\vec{E}(\vec{r}) = \frac{kq}{r^2}\hat{r}$$

Electric potential

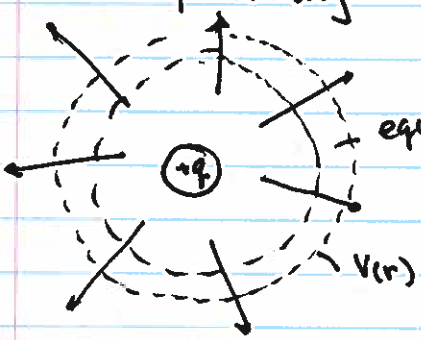
$$V(\vec{r}) = \frac{kq}{r}$$

Many charges: $\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \ldots$ vector sum

$V_{tot} = V_1 + V_2 + \ldots$ scalar sum

In general $\vec{E} = \left\{ -\frac{\partial V}{dx}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right\}$

Spherically symmetric potential

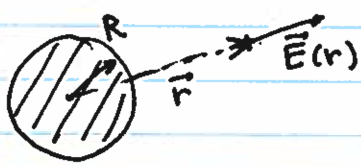


equipotential surface

$$E_r = -\frac{dV(r)}{dr}$$

In these cases when we used Gauss law to calculate electric field, we can then use it

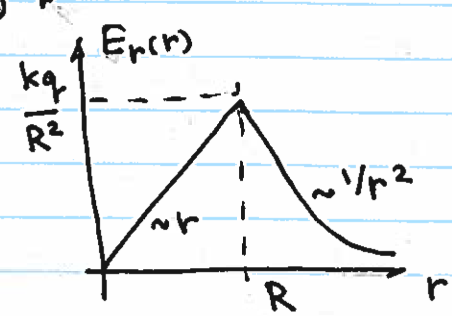
Solid charged sphere



$$\vec{E}(r) = \begin{cases} \frac{kq}{r^2}\hat{r} & r > R \\ \frac{kq}{R^3}\vec{r} & r < R \end{cases}$$

if $E_r(r) = -\frac{dV}{dr}$ $\quad V(r) = -\int E_r(r)dr + \text{const}$

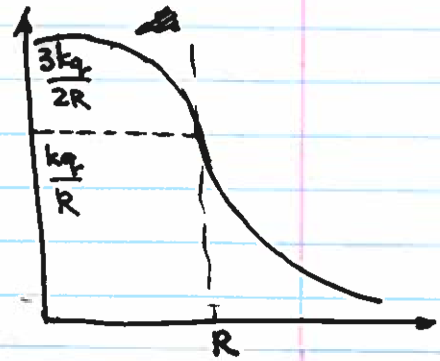
$$V(r) = \begin{cases} \frac{kq}{r} & r > R \\ -\frac{kq}{2R^3}r^2 + \text{const} & r < R \end{cases}$$



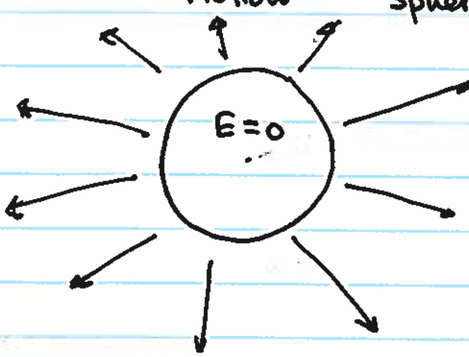
Since potential V represent energy, it must be continuous, so

at $r=R$ $\quad \frac{kq}{R} = -\frac{kq}{2R} + \text{const} \Rightarrow \text{const} = \frac{3kq}{2R}$

$$V(r) = \begin{cases} \frac{kq}{r} & r>R \\ -\frac{kq}{2R^3}r^2 + \frac{3kq}{2R} & r<R \end{cases}$$



Hollow sphere



Outside $V(r) = \frac{kq}{r}$ $\quad r>R$

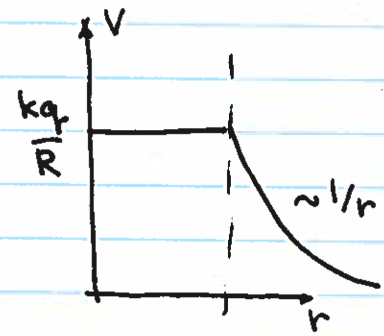
Inside: $E=0 \Rightarrow V = \text{const}$

to be continuous

$V(R) = kq/R$

so anywhere inside $V = \frac{kq}{R}$

$$V(r) = \begin{cases} \frac{kq}{r} & r>R \\ \frac{kq}{R} & r<R \end{cases}$$



That makes sense: if E inside is zero, it takes no work to move charges around, so their electrostatic potential energy is constant.

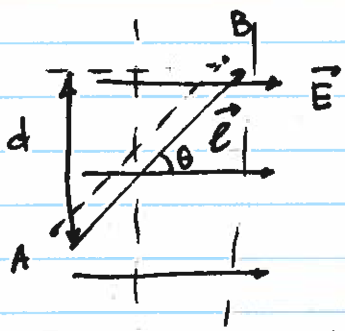
Electrostatic potential is defined up to a constant (basically, we deside where it is zero

More physically useful thing — potential difference (voltage). It represents the amount of work done when moving a test charge from point A to point B

$$W_A = W_B + [\text{work}] \Rightarrow V_A = V_B + \frac{\text{work}}{q_{test}}$$

$$\underline{\Delta V_{AB} = V_A - V_B}$$

Constant electric field



$$W_{on} = \vec{F}\cdot\vec{\ell} = q_{test}\cdot\vec{E}\cdot\vec{\ell}$$

$$V_A - V_B = \frac{\text{work}}{q_{test}} = \vec{E}\cdot\vec{\ell} =$$

$$= E\cdot\ell\cos\theta = E\cdot d$$

equipotential plane

Conductors — electric field inside must be zero, since otherwise free charges would move freely until it is zero.

Moreover, the surface of a conductor must have the same potential value, so that there is no force making charges move along the surface. We can have force / electric field working perpendicular to the surface, since charges cannot leave the surface.