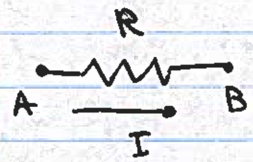
# Quick summary of circuit elements (so far)

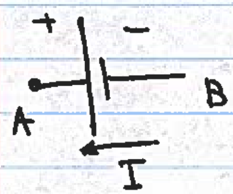
—— wire, no resistance, connects points with same potential




Ohm's law $V_A - V_B = IR$ (voltage drop)

Power converted to heat

$$P = I^2 \cdot R = I \cdot V = V^2/2R$$

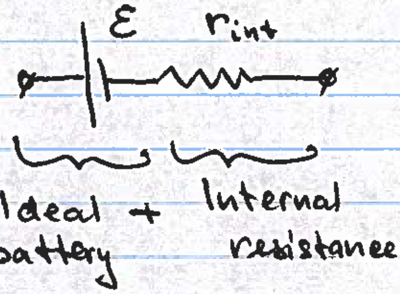



Ideal battery $V_A - V_B = \mathcal{E}$ emf

Source of constant potential difference

Side note: real battery has non-zero resistance

In a circuit we present it as




Actual voltage drop

$$V_{battery} = \mathcal{E} - I \cdot r_{int}$$

When a battery starts "dying", its internal resistance grows, effectively reducing voltage drops.

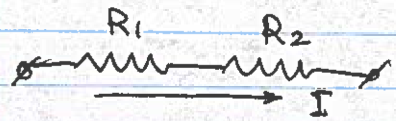
# Electric circuit : parallel and serial connections

Important rules

① Points connected by an (ideal) ~~wire~~ wire have the same potential

② If elements are connected in series, the same current flows through all of them

Series connection of resistors



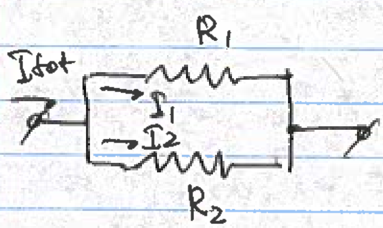

$R_{eq} = R_1 + R_2 \; (+R_3 ...)$

Voltage drop $V_{R_1} = I \cdot R_1$

$V_{R_2} = I \cdot R_2$

$V_{tot} = V_1 + V_2$

Parallel connection




$V_{R_1} = V_{R_2} = V$

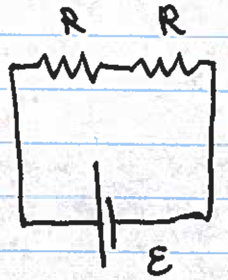
$V_{R_1} = I_1 V$

$V_{R_2} = I_2 V$

$I_{tot} = I_1 + I_2$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \; (+ \frac{1}{R_3} ...)$

PollEv questions




$I = \frac{\mathcal{E}}{2R}$

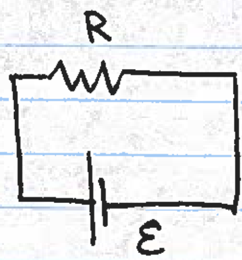
Power ~~on~~ emitted by one bulb (R)

$P_1 = I^2 \cdot R = \mathcal{E}^2/4R$

Total light power $P = 2P_1 = \mathcal{E}^2/2R$

Alternatively $P_{tot} = I^2 \cdot R_{eq} = \left(\frac{\mathcal{E}}{2R}\right)^2 \cdot 2R = \mathcal{E}^2/2R$

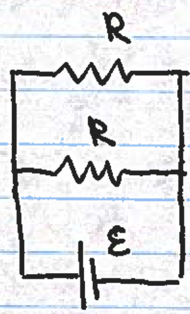
If the second bulb is shorted




$I = \mathcal{E}/R \qquad P = I^2 \cdot R = \mathcal{E}^2/R$

twice brighter than before

Parallel connection




Current through each resistor $I_1$

$\mathcal{E} = I_1 \cdot R \implies I_1 = \mathcal{E}/R$

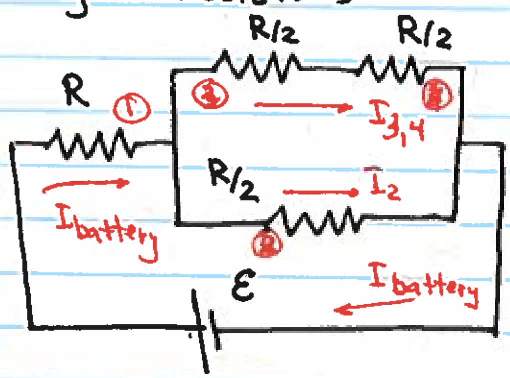
Power dissipated ~~for~~ at each resistor

$P_1 = I_1^2 \cdot R = \mathcal{E}^2/R$

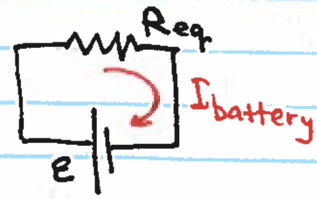
$P_{tot} = 2P_1 = 2\mathcal{E}^2/R$

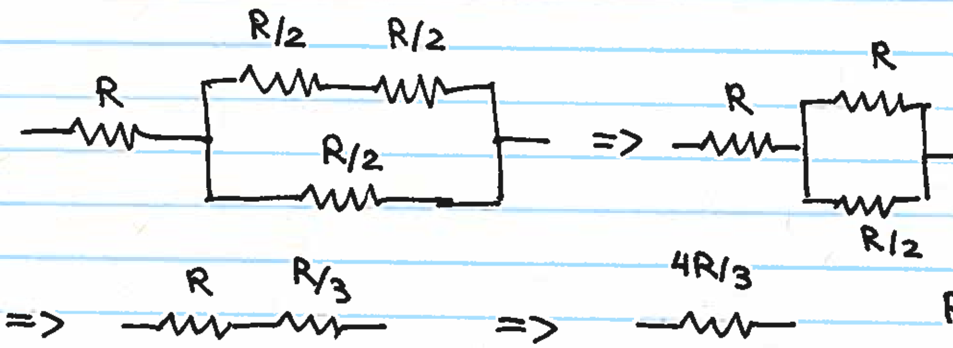
Alternatively $R_{eq} = R/2$ $\quad P = \mathcal{E}^2/R_{eq} = 2\mathcal{E}^2/R$

## Circuits with one battery and many resistors




1. Lets find $I_{battery}$ by calculating the equivalent resistance




$R_{eq} = 4R/3$

$$I_{battery} = \mathcal{E}/R_{eq} = \frac{3\mathcal{E}}{4R}$$

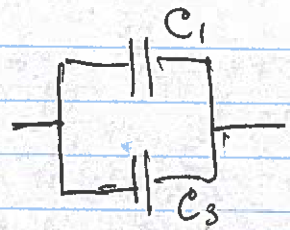
What is the current and the voltage drop on ~~the~~ each of the resistors?

2.

| Resistor | Current | Voltage |
| --- | --- | --- |
| 1 | $I_1 = I_{battery}$ | $V_1 = R \cdot I_{battery} = 3\mathcal{E}/4$ |
| 2 | $I_2 = \frac{V_2}{R/2} = \frac{\mathcal{E}}{2R}$ | $V_2 = \mathcal{E} - 3\mathcal{E}/4 = \mathcal{E}/4$ |
| 3,4 | $I_{3,4} = I_{battery} - I_2 = \frac{\mathcal{E}}{4R}$ | $V_3 = V_4 = I_{3,4} \cdot \frac{R}{2} = \frac{\mathcal{E}}{8}$ |

We can use similar logic to figure out the equivalent capacitance

Parallel connection




$V_{C_1} = V_{C_2} = V_{tot}$

$Q_1 = V_{C_1} \cdot C_1$ , $Q_2 = V_{C_2} \cdot C_2$

Total charge drawn from the battery: $Q_{tot} = Q_1 + Q_2$

Equivalent capactans:

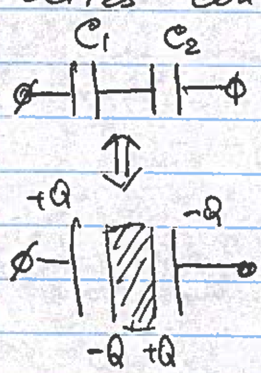
$$V_{tot} = \frac{Q_{tot}}{C_{eq}} \Rightarrow C_{eq} = \frac{Q_{tot}}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$$

~~$C_{eq} = Q_1 + Q_2 + (\ldots)$~~

$$C_{eq} = C_1 + C_2 + (C_3 + \ldots)$$

If originally the capacitors are not charge, each draws enough charge to ensure that voltages are the same.

Series connection



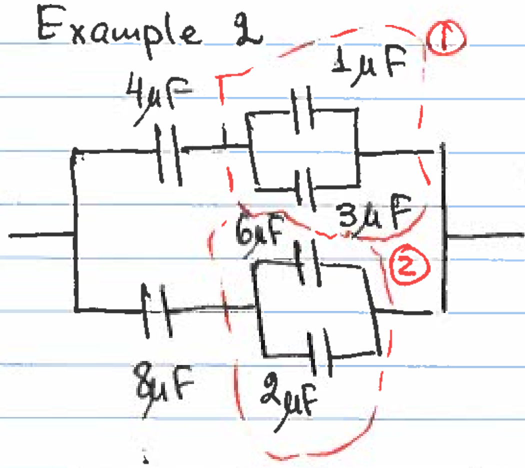

$V_{tot} = V_1 + V_2$

$Q_{C_1} = Q_{C_2} = Q$

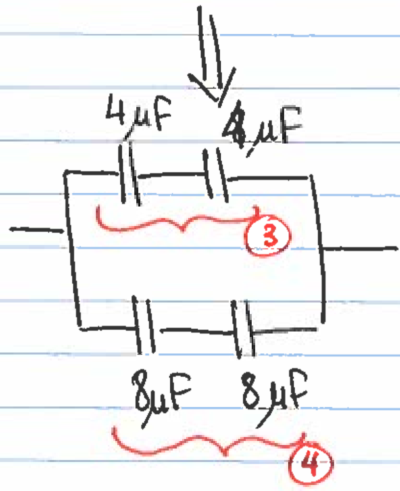
$$V_{tot} = \frac{Q}{C_{eq}} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

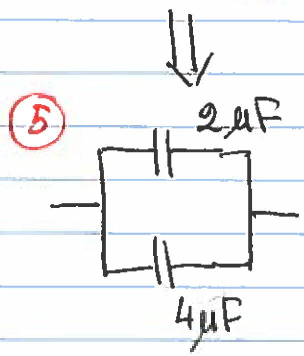
# Example 2



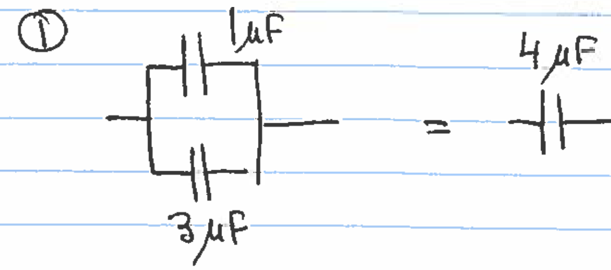
4μF
1μF
①
3μF
6μF
②
8μF
2μF

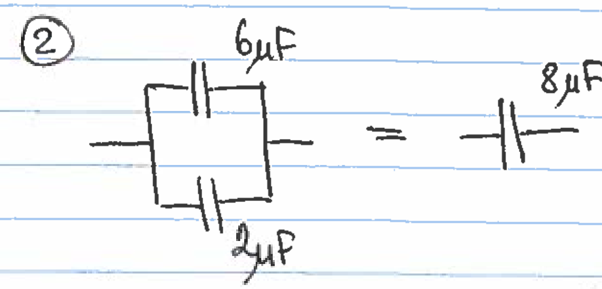
4μF
4μF
③
8μF
8μF
④

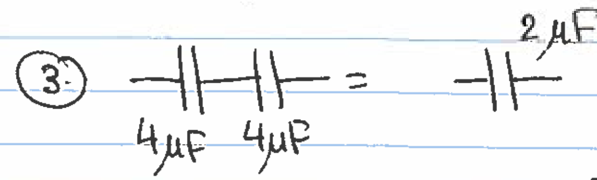
⑤
2μF
4μF

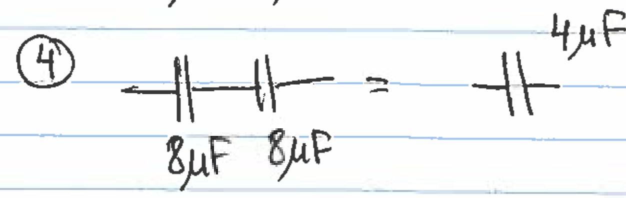
①
1μF
3μF
=
4μF

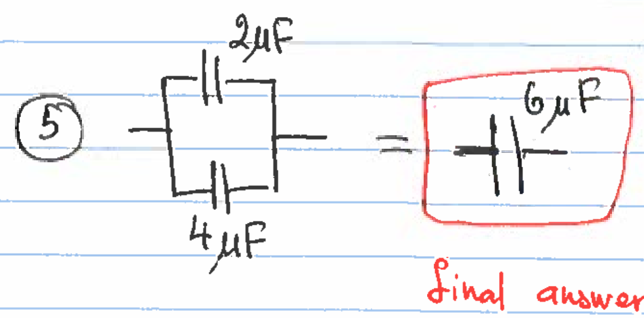
②
6μF
2μF
=
8μF

③
4μF
4μF
=
2μF

④
8μF
8μF
=
4μF

⑤
2μF
4μF
=
6μF
final answer