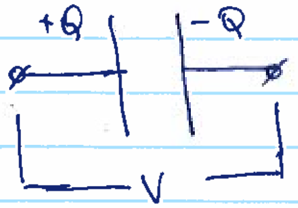
Capacitors and their connections

Reminder: a capacitor consists of two conducting objects with opposite charges, used to store electric energy (charge)



Capacitance: $C = \frac{Q}{V}$

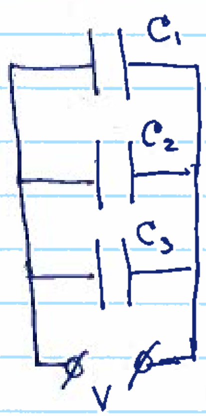
~~Plate~~ Parallel-plate capacitor:

$C = \frac{\varepsilon_0 d}{A}$ ← distance b/w plates; ← area of the plates

Capacitance unit – F, Farad.

Capacitor connections

Parallel connection: same voltage across each capacitor



$V_1 = V_2 = V_3 = V$

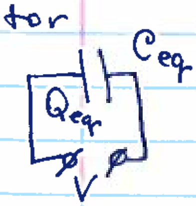
$Q_1 = C_1 \cdot V$, $Q_2 = C_2 \cdot V$, $Q_3 = C_3 \cdot V$

Equivalent single capacitor

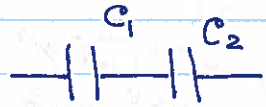
$C_{eq} = C_1 + C_2 + C_3$

$Q_{eq} = Q_1 + Q_2 + Q_3$

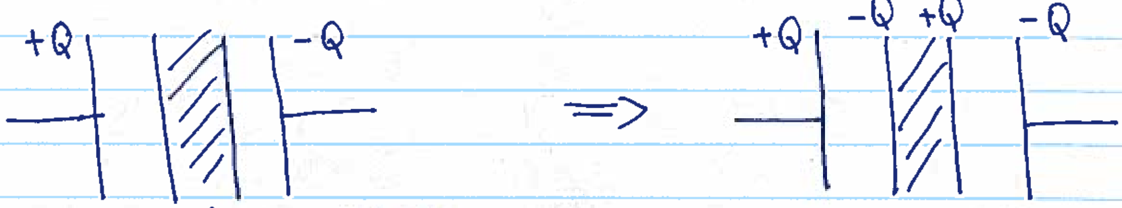
$V_{eq} = V$



Series connection

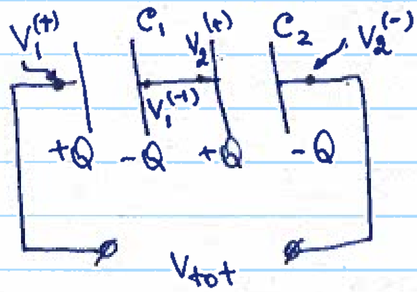


Small detour: a chunk of metal inside → a charged capacitor



↑ no electric field inside, so free moving charges inside the metal will move to the sides to compensate for the external electric field

This is the same as two capacitors in series



$$Q_1 = Q_2 = Q$$

$$V_1 = Q/C_1 \qquad V_2 = Q/C_2$$

Total voltage drop

$$V_{tot} = V_1^{(+)} - V_2^{(-)} = V_1^{(+)} - V_1^{(-)} + V_2^{(+)} - V_2^{(-)} =$$

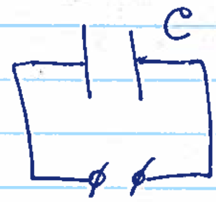
$$= V_1 + V_2$$

Equivalent capacitance $C_{eq} = \dfrac{Q}{V_{tot}}$ or $\dfrac{1}{C_{eq}} = \dfrac{V_{tot}}{Q}$

$$\frac{1}{C_{eq}} = \frac{Q/C_1 + Q/C_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Charging capacitors

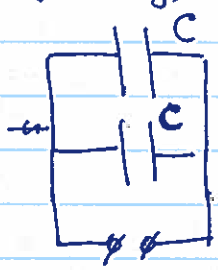


Voltage V_0
source
(battery)

Voltage across capacitor – V_0
Charge on the capacitor

$Q = C \cdot V_0$

$C_{eq} = 2C$

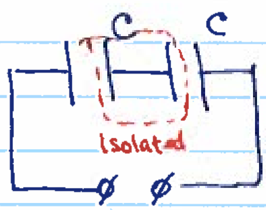


V_0

~~Charge~~ V_0 - Voltage across each capacitor
Charge across each capacitor $Q = C \cdot V_0$

Total charge passed through the battery
$Q_{tot} = 2Q = 2C \cdot V_0 = C_{eq} \cdot V_0$

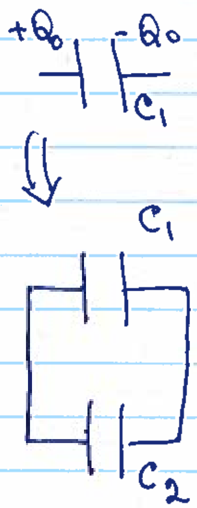
$C_{eq} = \frac{C}{2}$



V_0

Voltage across each capacitor $\frac{V_0}{2}$
Charge on each capacitor $Q = C \cdot \frac{V_0}{2}$
Total charge passed through the battery $Q_{tot} = \frac{CV_0}{2} = C_{eq} \cdot V_0$

What if we charged one capacitor, disconnected it from the battery, and then connected to another one?



$V_0 = Q_0 / C_1$

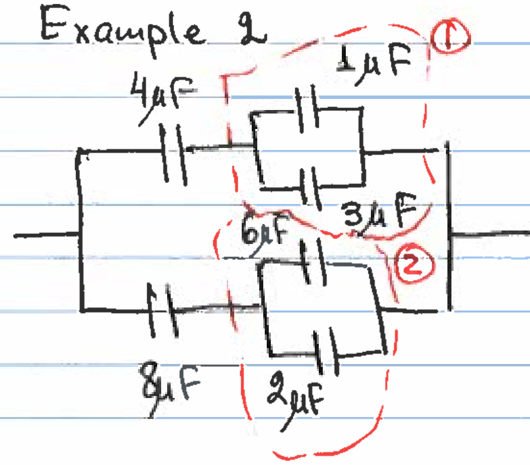
Once we reach a steddy state
Voltages across each capacitor are equal $V_1 = V_2 = V_{new}$

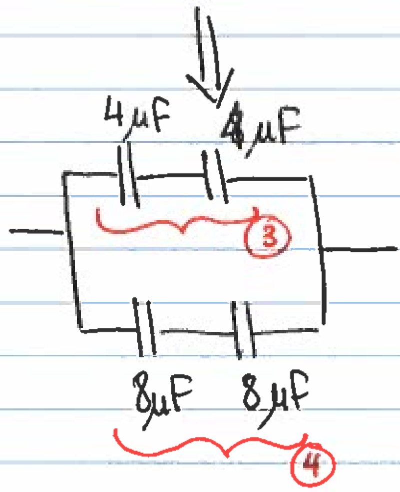
Original charge is redistributed
$Q_1 + Q_2 = Q_0$

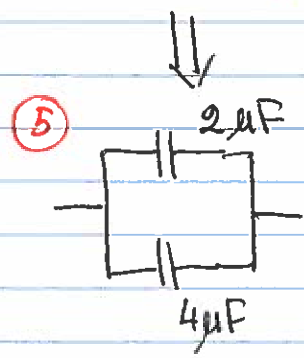
$V_1 \cdot C_1 + V_2 \cdot C_2 = V_0 \cdot C_1$

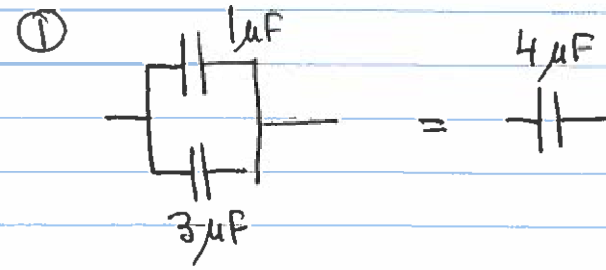
$V_{new}(C_1 + C_2) = V_0 C_1 \Rightarrow V_{new} = \frac{C_1}{C_1 + C_2} V_0$

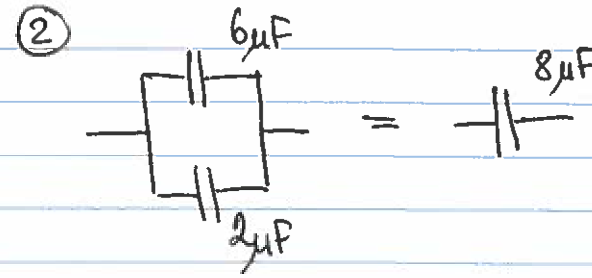
Example 2

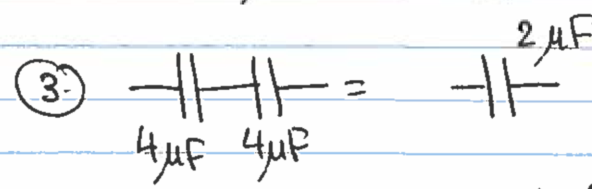


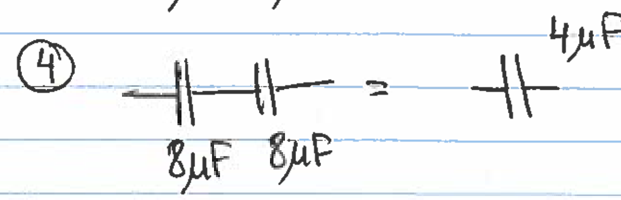


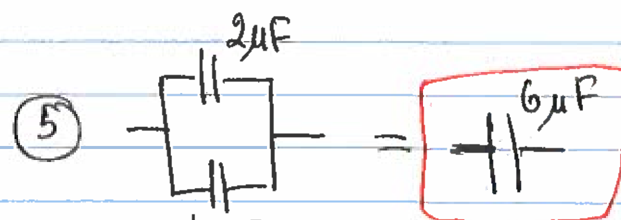










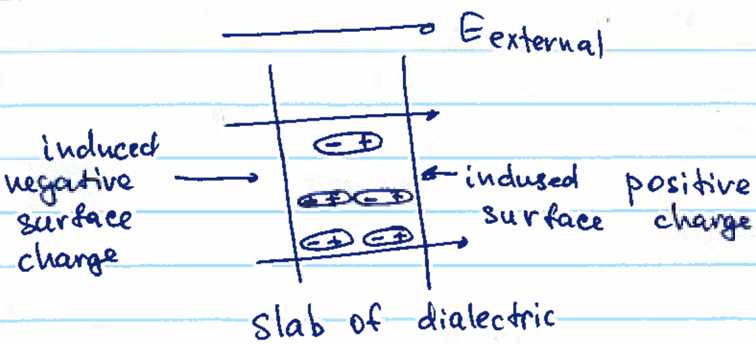


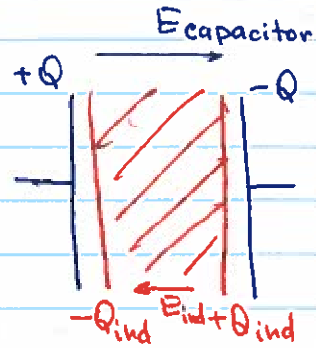
final answer

Capacitors with dielectric inside

~~Dia~~

Dielectric material — no free charges, but consist of polar molecules (dipoles) that can be aligned in the electric field (since they are randomly organized without external field)





$$E_{tot} = E_{cap} - E_{ind} = \frac{E_{cap}}{K}$$

K – dielectric constant depends on material

Empty capacitor

$$V_{empty} = E_{empty} \cdot d = \frac{Q}{\varepsilon_0 \cdot A} \cdot d$$

$$C_{empty} = \frac{Q}{V_{empty}} = \frac{\varepsilon_0 A}{d}$$

Filled with dialectric K

$$V_{filled} = E_{tot} \cdot d = \frac{Q}{K \cdot \varepsilon_0 A} \cdot d$$

$$C_{filled} = \frac{K \varepsilon_0 A}{d} = K \cdot C_{empty}$$

A capacitor with dialectric filling can store more energy for the same voltage drop

$$U_{empty} = \frac{C V^2}{2} \Rightarrow U_{filled} = \frac{K \cdot C_{empty} \cdot V^2}{2} = K \cdot U_{empty}$$

Material	Dielectric constant
Air (dry)	1.0
Bakelite	4.9
Mylar	3.2
Nylon	3.4
Paper	3.7
Paraffin-impregnated paper	3.5
Polypropylene	2.2
Polystyrene	2.6
Polyvinyl chloride	3.4
Porcelain	6.0
Pyrex glass	5.6
Strontium titanate	233.0
Water	80.0

Source: Jewett and Serway (2008)