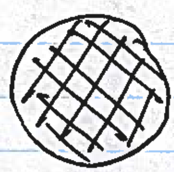
Charge distribution ~~at~~ in insulators vs conductors

Uniformly charged sphere



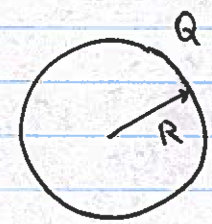
$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$

$$\vec{E} = \begin{cases} \dfrac{kQ}{r^2}\hat{r} & r > R \\ \dfrac{kQ}{R^3}\vec{r} & r < R \end{cases}$$

$$E_r = -\frac{dV}{dr}$$

$$V(r) = \begin{cases} \dfrac{kQ}{r} & r > R \\ -\dfrac{kQ}{2R^3}r^2 + \dfrac{3kQ}{2R} & r < R \end{cases}$$

Thin shell



$$\vec{E} = \begin{cases} \dfrac{kQ}{r^2}\hat{r} & r > R \\ 0 & r < R \end{cases}$$

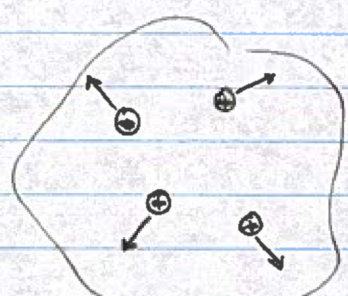
$$E_r = -\frac{dV}{dr}$$

$$V(r) = \begin{cases} \dfrac{kQ}{r} & r > R \\ \dfrac{kQ}{R} & r < R \end{cases}$$

constant

Conducting sphere — charges are free to move.

Bulk: all elementary charges (electrons) will push away from each other until the electric field inside is zero (otherwise there still be a force pushing charges around).



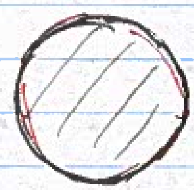
All charges will end up on a surface as ~~far away~~ ~~from~~ such that there is no force pushing them along the surface.

$V|_{surface} = \text{const}$

$\vec{E}$ is always perpendicular to the surface

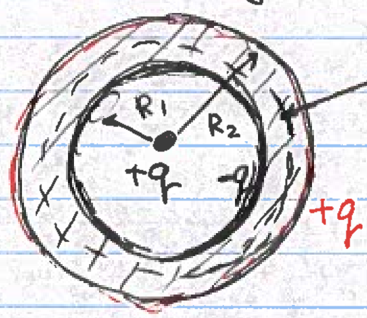
Important takeaways: in static (no moving charges) electric field inside a conductor is <u>zero</u>. Any added ~~charges~~ or intrinsic charges will arrange themselves to make it happen.

Solid conducting sphere → no charges inside, uniformly ~~&~~ surface charge



Same $\vec{E}$ and V as a hollow shell.

A charge inside a conducting shell

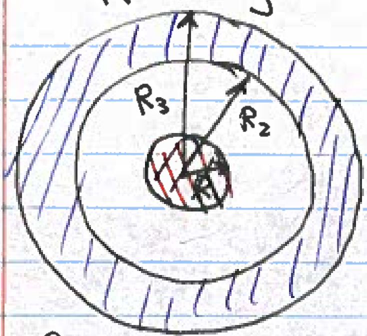


E=0
so total charge inside the dashed spher must be zero!
So even though the total charge of the shell is zero, there will be negative charge on the inner surface, and positive charge on the outer surface

~~lets see what Electric field of concentric shell~~

$$E(r) = \begin{cases} \dfrac{kq}{r} & r < R_1 \qquad q_{enc} = +q \\ 0 & R_1 < r < R_2 \qquad q_{enc} = +q - q = 0 \\ \dfrac{kq}{r} & r > R_2 \qquad q_{enc} = +q - q + q = 0 \end{cases}$$

Oppositely charged nested shells



$+Q$ on an inner ~~shell~~ sphere

$-Q$ on the outer shell

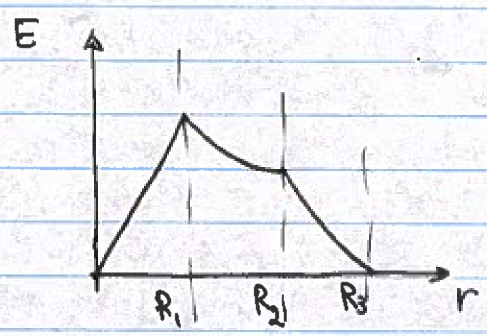
Insulator

$$\rho(r) = \begin{cases} \dfrac{Q}{\frac{4\pi}{3} R_1^3} & r < R_1 \\ \dfrac{-Q}{\frac{4\pi}{3}(R_3^3 - R_2^3)} & R_2 < r < R_3 \\ 0 & \text{elsewhere} \end{cases}$$

Gaussian surface - sphere

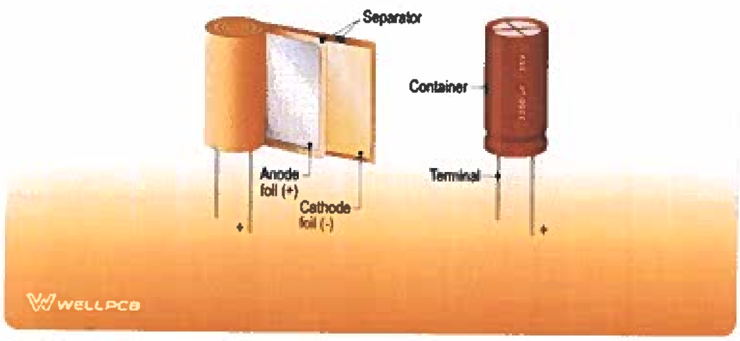
$\Phi = E \cdot 4\pi r^2$

$$E_r = \begin{cases} \dfrac{kQ}{R^3}\, r & r < R_1 \\ \dfrac{kQ}{r} & R_1 < r < R_2 \\ \dfrac{1}{r^2}\left[kQ - kQ\, \dfrac{(r^3 - R_2^3)}{R_3^3 - R_2^3}\right] & R_2 < r < R_3 \\ 0 & r > R_3 \end{cases}$$

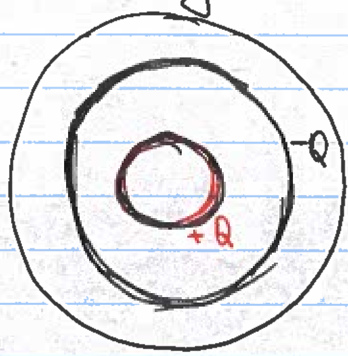


- **Spanish:** "condensador" (most common translation, though "capacitor" is also understood)
- **French:** "condensateur"
- **German:** "Kondensator"
- **Italian:** "condensatore"
- **Portuguese:** "condensador"
- **Chinese (traditional):** "電容器" (literally translates to "electric container")

CAPACITOR



Conducting shells



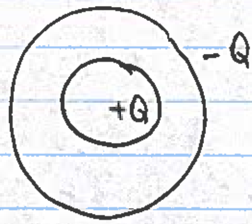
+Q - outside of the inner sphere

-Q - inside of the shell

$$E = \begin{cases} \dfrac{kQ}{r} & R_1 < r < R_2 \\ 0 & \text{elsewhere} \end{cases}$$

electric field is ~~so~~ concentrated b/w two surfaces.

This is one possible version of a <u>capacitor</u> (can simplify to two thin shells)



Potential difference

$V_+ = kQ/R_1$

$V_- = kQ/R_2$

$$V_+ - V_- = \frac{kQ}{R_1} - \frac{kQ}{R_2} = Q\left[\frac{k}{R_1} - \frac{k}{R_2}\right] = \frac{Q}{C}$$

$$C_{\text{sphere}} = 4\pi\varepsilon_0 \frac{R_2 \cdot R_1}{R_2 - R_1}$$ depends on <u>geometry</u>

More standard capacitor — parallel planes

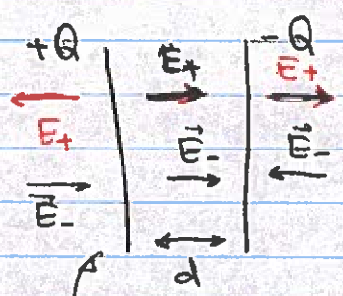


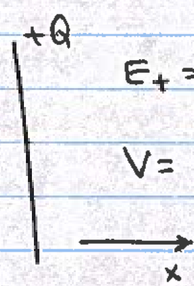
plate area A

Outside E-field from two plates cancel each other

Inside it doubles

$$E = 2 \cdot \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$V_+ = 0 \qquad V_- = E \cdot d = -\frac{Qd}{\varepsilon_0 A}$



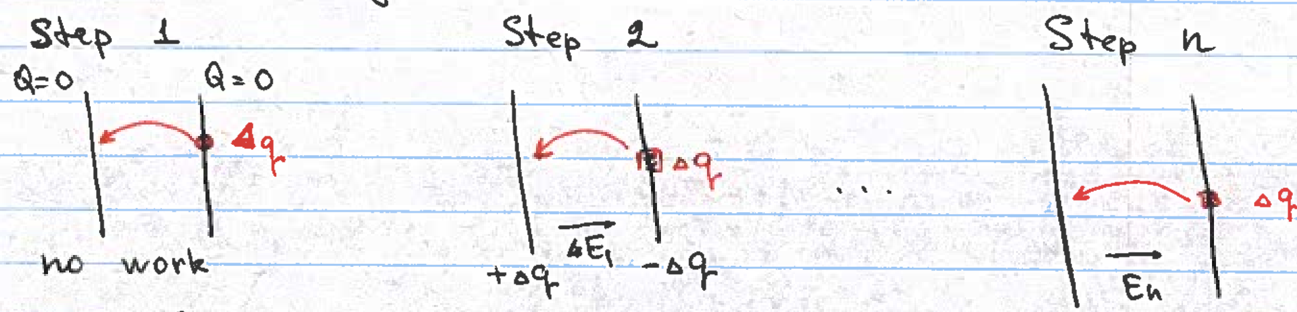
$E_+ = \frac{\sigma}{2\varepsilon_0}$

$V = -\int E_+ \, dx = E_+ x$

$$V_+ - V_- = \frac{Qd}{\varepsilon_0 A} = \frac{Q}{C}$$

$$C_{\text{plate}} = \frac{\varepsilon_0 A}{d}$$

Energy stored in a capacitor = work we must do to charge it



$\Delta W_1 = 0$

$\Delta W_2 = (\Delta q \cdot \underbrace{\Delta E_1) \cdot d}_{V_+ - V_- = \frac{\Delta q}{C}}$

$\Delta W_n = \Delta q \cdot \underbrace{E_n \cdot d}_{V_+ - V_- =}$

At each step $\Delta W = \frac{q}{C} \Delta q$

Total work $W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$

Commonly used notation: potential difference $V_+ - V_-$ $\Updownarrow$ voltage V

For a capacitor $V \equiv V_+ - V_- = \frac{Q}{C}$ or $Q = VC$

Energy stored in a capacitor

$$U = \frac{Q^2}{2C} = \frac{V^2 \cdot C}{2} = \frac{QV}{2}$$

We can also relate this to electric field strength

$$U = \frac{1}{2} V^2 \cdot C = \frac{1}{2} Q^2/C = \frac{1}{2} \frac{Q^2 \cdot d}{\varepsilon_0 \cdot A}$$

Energy density $u = \frac{U}{V} = \frac{1}{2} \frac{Q^2 \cdot \cancel{d}}{\varepsilon_0 \cdot A} \frac{1}{\cancel{d} A} = \frac{1}{2} \left[\frac{Q}{\varepsilon_0 A}\right]^2 \cdot \varepsilon_0 = \frac{\varepsilon_0}{2} E^2$