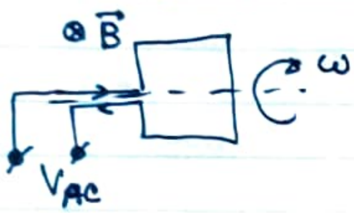
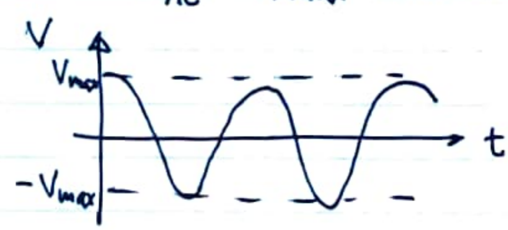
AC circuit

Up to now, our voltage source $-||-$ was assumed to be providing same voltage. This would be called a DC (or direct current) voltage source.

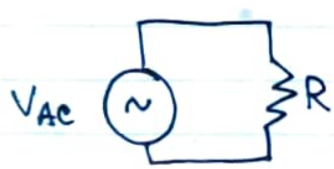
In practice, majority of the power supplies are AC – or alternating current. We've already discussed that such type of source can be produced by an electric turbine rotating in the magnetic field



$$V_{AC} = V_{max} \cos\omega t$$



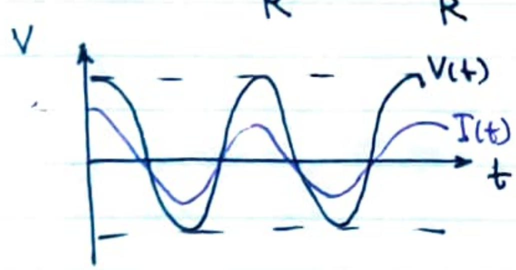
Question: ~~If~~ ~~the~~ what would be the current through a resistor, a capacitor and a ~~conductor~~ inductor, when they are hooked up to an AC voltage source?



$$V_{AC}(t) = V_{max} \cos\omega t$$

$$i(t) = ?$$

$$i(t) = \frac{V(t)}{R} = \frac{V_{max}}{R} \cos\omega t$$



Question: How much power on average will be converted to heat?

Instantaneous power: $P(t) = i(t) \cdot V(t) = \frac{V_{max}^2}{R} \cos^2 \omega t$

$\langle \cos^2 \theta \rangle = 1/2$

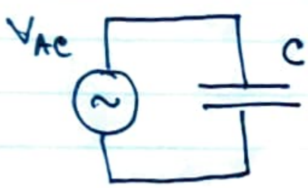
Averaged over many cycless

$P_{ave} = \frac{1}{2} P_{max} = \frac{V_{max}^2}{2R}$

Sometimes you can see labels V_{RMS} or I_{RMS} on measuring devices.
RMS trands for "root-mean-square", and represents the values of DC voltage and current that would produce the same amount of heat

$$V_{RMS} = \frac{V_{max}}{\sqrt{2}} \qquad I_{RMS} = \frac{I_{max}}{\sqrt{2}}$$

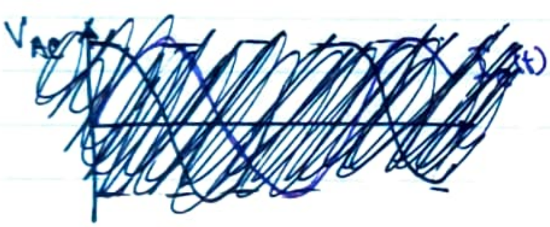
What about a capacitor or an inductor?



$V - Q/C = 0 \qquad Q(t) = V(t) \cdot C$

$Q(t) = V_{max} \cdot C \cos \omega t$

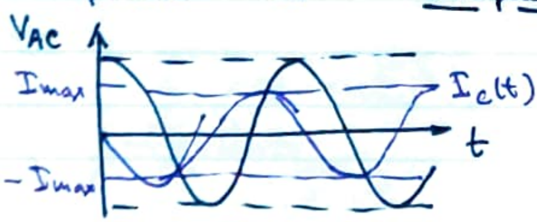
$I(t) = + \frac{dQ}{dt} = -V_{max} C \cdot \omega \sin \omega t$



Oscillating voltage and current are quater-cycle away from each other

$I_{max} = V_{max} \cdot C\omega$

The ratio b/w I_{max} and V_{max} now depends on frequency of oscillations.



I_c is a quater-cyclee ahead of the voltage

$$V(t) - L\frac{dI_L}{dt} = 0$$

$$\frac{dI_L}{dt} = \frac{V(t)}{L} = \frac{V_{max}}{L}\cos\omega t$$

$$I_L(t) = \frac{V_{max}}{\omega L}\sin\omega t$$

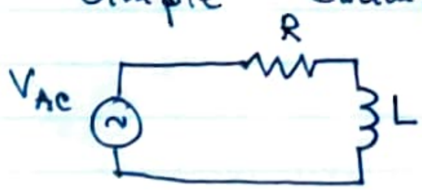
$$I_{max} = \frac{V_{max}}{\omega L}$$

I_L is a quater-cycle behind the voltage

Question: what is the relation between the voltage and the current if we have different types of components?

They are still be oscillating, but the delay b/w them will ~~be~~ depend on values of R, L, C.

Simple example



$V(t) = V_{max}\cos\omega t$

We assume that there is some unknown delay b/w the current, even though same current flows through all components

$i(t) = I_1\cos\omega t + I_2\sin\omega t$ $\qquad$ I_1 & I_2 - unknown constants

$$V(t) - IR - L\frac{di}{dt} = 0$$

$$V_{max}\cos\omega t = I_1 R\cos\omega t + \underbrace{I_2 R\sin\omega t - I_1 L\omega\sin\omega t}_{\text{must cancel each other}} + I_2 L\omega\cos\omega t$$

$$I_2 R = I_1 L\omega \qquad I_2 = \frac{L\omega}{R} I_1$$

$$V_{max} = I_1 R + I_2 L\omega = I_1\left(R + \frac{L^2\omega^2}{R}\right) \Rightarrow I_1 = \frac{V_{max} R}{R^2 + \omega^2 L^2}$$

$$I_2 = \frac{V_{max}\,\omega L}{R^2 + \omega^2 L^2}$$

Final answer:

$$i(t) = \frac{V_{max}}{R^2+\omega^2L^2}\left[R\cos\omega t + \omega L \sin\omega t \right]$$

or

$$i(t) = \frac{V_{max}}{\sqrt{R^2+\omega^2L^2}} \cancel{\sin}(\omega t + \varphi) \quad \text{where} \quad \cos\varphi = \frac{R}{\sqrt{R^2+\omega^2L^2}}, \quad \sin\varphi = \frac{L\omega}{\sqrt{R^2+\omega^2L^2}}$$