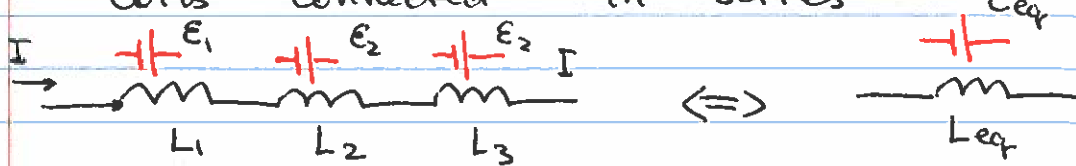
Coils connected in series



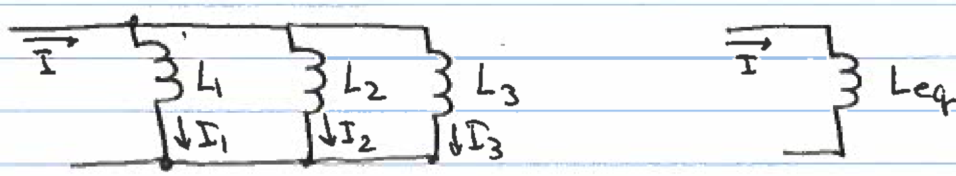
$$\mathcal{E}_1 = L_1 \frac{dI}{dt}, \quad \mathcal{E}_2 = L_2 \frac{dI}{dt}$$

$$\mathcal{E}_3 = L_3 \frac{dI}{dt}$$

$$\mathcal{E}_{tot} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 = \underbrace{(L_1 + L_2 + L_3)}_{L_{eq}} \frac{dI}{dt}$$

$L_{eq} = L_1 + L_2 + L_3$ for series connection

Parallel connection



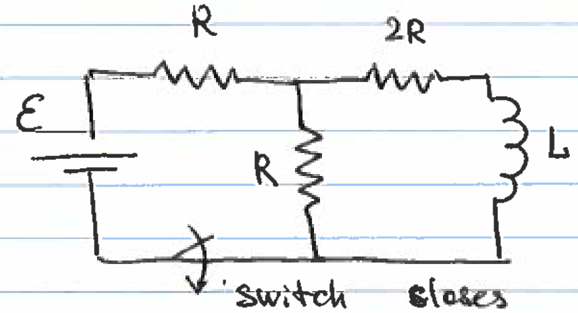
$$\mathcal{E}_{ind} = L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_3 \frac{dI_3}{dt}$$

$$\frac{dI_1}{dt} = \frac{\mathcal{E}}{L_1} \qquad \frac{dI_2}{dt} = \frac{\mathcal{E}}{L_2} \qquad \frac{dI_3}{dt} = \frac{\mathcal{E}}{L_3}$$

$$\frac{dI}{dt} = \frac{\mathcal{E}_{eq}}{L_{eq}} = \frac{d(I_1 + I_2 + I_3)}{dt}$$

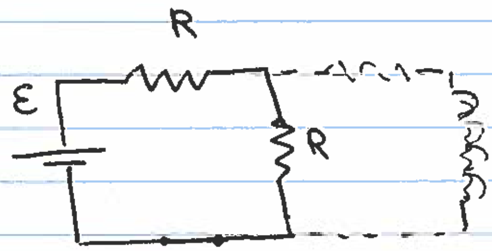
$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$ for parallel connection

Example: more complicated circuit



switch closes at $t=0$

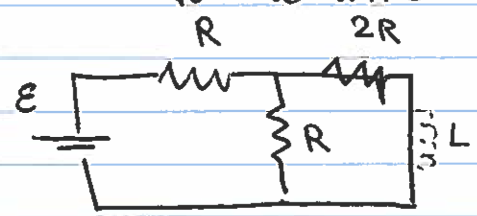
time $t = 0+s$ → no current flows through the inductance



$I_{coil} = 0$

$I_{switch} = \mathcal{E}/2R$

time $t \to \infty$ current is stable, L is equivalent to a wire

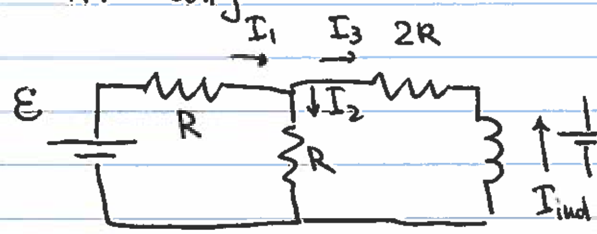


$$R_{eq} = R + \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{5}{3}R$$

$$I_{switch} = 3\mathcal{E}/5R$$

$$I_{coil} = I_{2R} = \frac{1}{3} I_{switch} = \mathcal{E}/5R$$

At any $t > 0$



$\mathcal{E}_{ind} = L\frac{dI_3}{dt}$

Kirchhoff's rules:
① $I_1 = I_2 + I_3$
② $\mathcal{E} - I_1 R - I_2 R = 0$
③ $\mathcal{E} - I_1 R - 2I_3 R - L\frac{dI_3}{dt} = 0$

$$I_2 = I_1 - I_3$$

$$\varepsilon - I_1 R - (I_1 - I_3)R = 0 \qquad RI_1 = (\varepsilon + RI_3)/2$$

$$\varepsilon - \frac{\varepsilon}{2} - \frac{RI_3}{2} - 2I_3 R - L\frac{dI_3}{dt} = 0$$

$$-L\frac{dI_3}{dt} - \frac{5}{2}RI_3 + \frac{\varepsilon}{2} = 0$$

We know the general solution form

$I_3(t) = A(1 - e^{-t/\tau})$ since $I_3(t=0) = 0$

moreover, since we know that $I_3(t\to\infty) = \varepsilon/5R$

we know that $A = \varepsilon/5R$, so we only

need to find τ

$$I_3 = \frac{\varepsilon}{5R}(1 - e^{-t/\tau})$$

$$\frac{dI_3}{dt} = \frac{\varepsilon}{5R}\frac{1}{\tau}e^{-t/\tau}$$

$$-\frac{L\varepsilon}{5R}\frac{1}{\tau}e^{-t/\tau} - \frac{5}{2}\frac{\varepsilon}{5}(1 - e^{-t/\tau}) + \frac{\varepsilon}{2} = 0$$

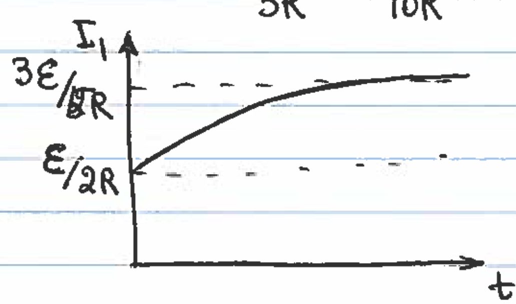
$$-\frac{L\varepsilon}{5R}\frac{1}{\tau}\cancel{e^{-t/\tau}} - \cancel{\frac{\varepsilon}{2}} + \frac{\varepsilon}{2}\cancel{e^{-t/\tau}} + \cancel{\frac{\varepsilon}{2}} = 0$$

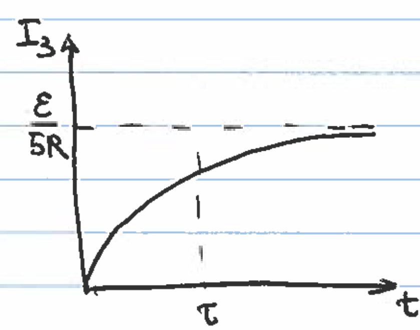
$$-\frac{L\cancel{\varepsilon}}{5R}\frac{1}{\tau} + \frac{\cancel{\varepsilon}}{2} = 0 \quad \Rightarrow \boxed{\tau = \frac{2L}{5R}}$$

$$I_3 = \frac{\varepsilon}{5R}(1 - e^{-t/\tau})$$

$$I_1 = \frac{\varepsilon}{2R} + \frac{I_3}{2} = \frac{\varepsilon}{2R} + \frac{\varepsilon}{10R} - \frac{\varepsilon}{10R}e^{-t/\tau} =$$

$$= \frac{3\varepsilon}{5R} - \frac{\varepsilon}{10R}e^{-t/\tau}$$





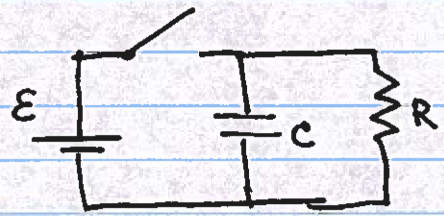
Basic circuit components

voltage drop across : a resistor $V_R = R \cdot I$

an inductor $V_L = -L \dfrac{dI}{dt}$

a capacitor $V_C = \dfrac{Q}{C} = \dfrac{1}{C} \displaystyle\int_{-\infty}^{t} I(t')\,dt$

RC circuit



$t<0 \quad V_C = \mathcal{E}, \quad I_0 = \mathcal{E}/R$

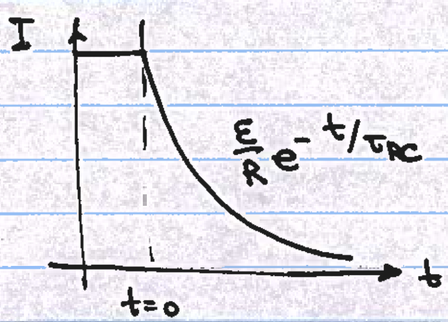
$$-IR + \frac{Q}{C} = 0 \qquad I = -\frac{dQ}{dt}$$

$$\frac{dQ}{dt} = -\frac{1}{RC} \cdot Q = 0$$

$$Q(t) = \mathcal{E} \cdot C\, e^{-t/\tau_{RC}}$$

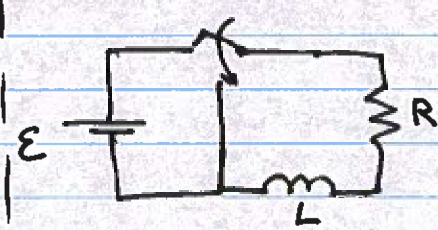
$$\tau_{RC} = RC$$

$$I(t) = -\frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/\tau_{RC}}$$



Charged capacitor works as a voltage source allowing current to run until it is discharged

RL circuit



$t<0 \quad V_L = 0, \quad I_0 = \mathcal{E}/R$

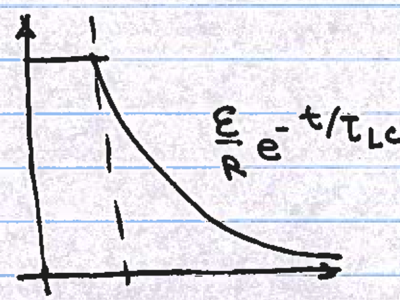
$t=0 \quad I_L = I_0$

$$-IR - L\frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = -R/L\, I$$

$$I = I_0\, e^{-t/\tau_{RL}}$$

$$\tau_{RL} = R/L$$



Magnetic energy stored in the coil is used to keep the current running for a while