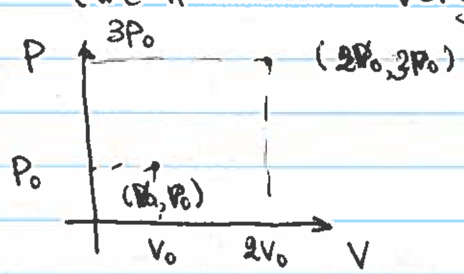
Graphic representation of thermodynamic processes (for ideal gasses)

Ideal gas law $PV = nRT = Nk_BT$

In most cases the amount of atoms N (or # of moles $n = N/N_A$) is fixed.

Then if we know (P, V, T) we know exact state of the system, and can predict how one parameter will affect others

However, if we want to represent this graphically, ~~we consider~~ historically it is done using 2D plots → most often P-V (we'll see very soon why)



Initial point $(P_0, V_0) \Rightarrow T_{ini} = \frac{P_0V_0}{nR}$

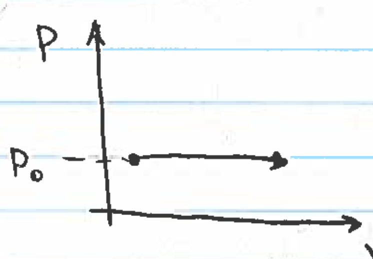
Final point $(3P_0, 2V_0) = T_{fin} = \frac{6P_0V_0}{nR}$

$T_{fin} = 6T_{ini}$

How to get to the final point? There are many possible ways.

Most often (at least in our course) we try to keep one variable constant

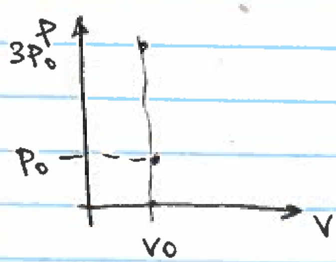
1. Pressure is constant $P = \text{const}$ (isobaric process)



$T \propto V$

if $V_0 \to 2V_0$ $\quad T \to 2T_0$

2. Volume is constant {$V = \text{const}$ (isochoric or isovolumetric)

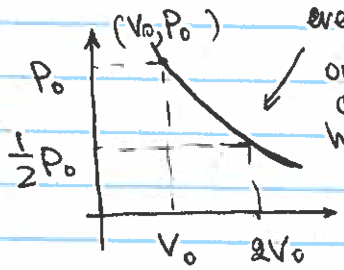


$P \propto T$

$P_0 \to 3P_0 \qquad T \to 3T_0$

3. Temperature is constant $T = \text{const}$ (isothermic)

$PV = \text{const}$

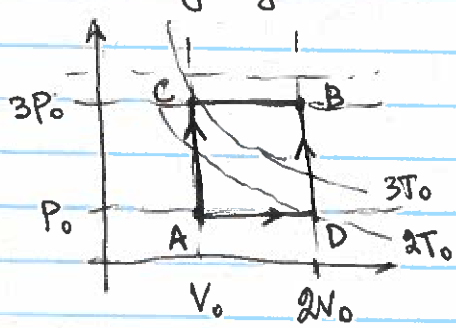


every point on this curve has the same temperature

$V_0 \to 2V_0 \Rightarrow P_0 \to \frac{1}{2}P_0$

Isotherm $\quad P = \dfrac{(nRT)}{V}$

So going back to our example



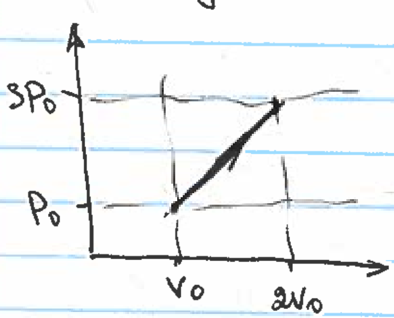
Initial point A: V_0, P_0, T_0

Final point B: $2V_0, 3P_0, 6T_0$

Intermediate point D&C – which one is hotter?

C: $V_0, 3P_0, 3T_0$ vs D: $2V_0, P_0, 2T_0$

Of course, we can do anything we want, for example, we can make pressure grow linearly with volume



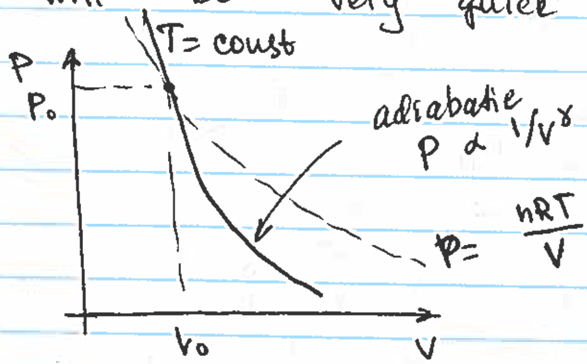
Equation: slope $= \dfrac{\Delta P}{\Delta V} = \dfrac{2P_0}{V_0}$

$$\frac{P - P_0}{V - V_0} = \frac{2P_0}{V_0}$$

$$P = \frac{2V}{V_0}P_0 + P_0 = P_0\left(1 + \frac{2V}{V}\right)$$

$$T = \frac{PV}{nR} = \frac{P_0}{nR} V\left(1 + \frac{2V}{V}\right)$$ quadratic

Let's check one more process – adiabatic
We will soon encounter it when the changes ~~wi~~ in the thermodynamical state will be very quick $PV^{\gamma} = \text{const}$ $(\gamma = 3/2 \text{ or } 5/2)$



How's temperature changes during adiabatic process?

Initial V_0, P_0 ← known $T_0 = \frac{P_0 V_0}{nR}$

Final V_f, P_f

$P_f \cdot V_f^{\gamma} = P_0 V_0^{\gamma}$ at any ~~[illegible]~~ moment $P = \frac{P_0 V_0^{\gamma}}{V^{\gamma}}$

$PV = nRT \Rightarrow T = \frac{1}{nR} PV = \frac{1}{nR} \frac{P_0 V_0^{\gamma}}{V^{\gamma - 1}} = T_0 \left(\frac{V_0}{V}\right)^{\gamma - 1}$

and if we track P : $V^{\gamma} = V_0^{\gamma}\, P_0/P \Rightarrow V = V_0 \left(P_0/P\right)^{1/\gamma}$

$T = \frac{1}{nR} PV = \frac{1}{nR} V_0 \cdot P \left(P_0/P\right)^{1/\gamma} = T_0 \left(P_0/P\right)^{1/\gamma - 1}$