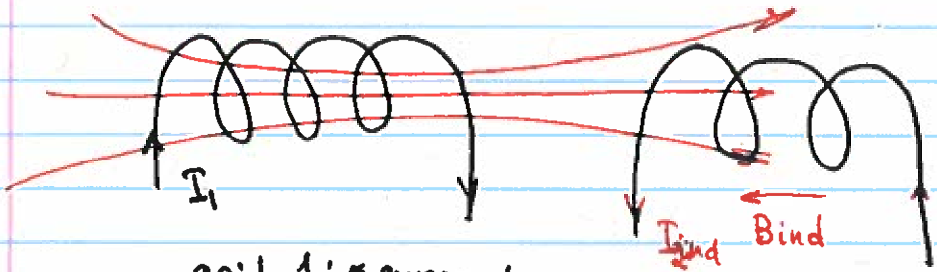
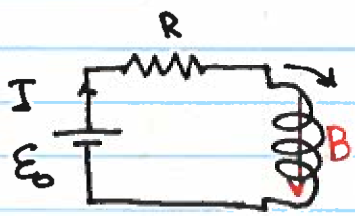
Mutual induction: one changing current creates varying magnetic field that creates current in another circuit



$$I_2 \propto \frac{dB}{dt} \propto \frac{dI_1}{dt}$$

coil 1: current increasing $\Rightarrow B \propto I_1$

However, this occures even in a single coil!



$$B = \mu_0 \frac{N}{\ell} \cdot I$$

Total flux through the solenoid

$$\Phi = N \cdot \Phi_{\text{single loop}} = N \cdot B \cdot A = \frac{\mu_0 N^2 A}{\ell} \cdot$$

(A: area)

Coil length $\rightarrow \ell$
area $\rightarrow A$

The ratio b/w the flux and current depends only on the geometry of the circuit. It is reffered as <u>inductance</u>

$$L = \frac{\Phi}{I}$$

For a coil $L = \dfrac{\mu_0 N^2 A}{\ell}$

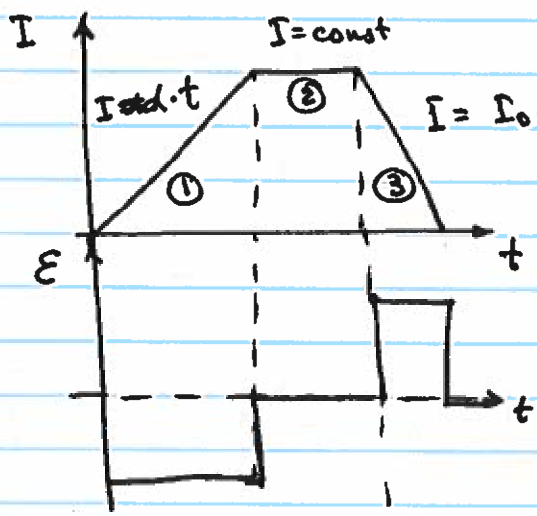
What if the current is changing? That changes the flux, so an additional emf will be induced in the coil

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

Current increasing $\rightarrow$ induced $\mathcal{E}_{ind}$ is against $\mathcal{E}_0$
decreasing $\rightarrow$ induced $\mathcal{E}_{ind}$ is along $\mathcal{E}_0$

Since the behavior of the inductance depends on how the electric current changes, it only manifest itself when something is changing.

Constant current $\longrightarrow$ any inductance is just a long wire, no role

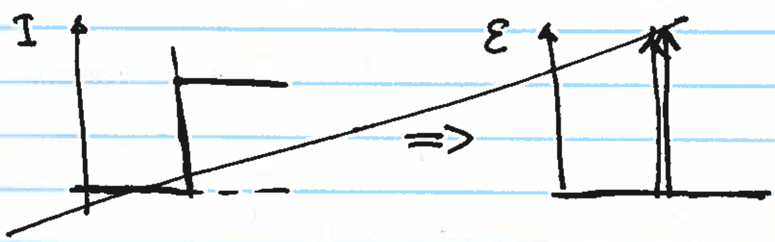


① $\mathcal{E}_1 = -L \frac{dI}{dt} = -\alpha \cdot L$

② $\mathcal{E}_2 = -L \frac{dI}{dt} = 0$

③ $\mathcal{E}_3 = -L \frac{dI}{dt} = \beta \cdot L$

However, current through an inductance cannot change as a step $\longrightarrow$ infinite EMF

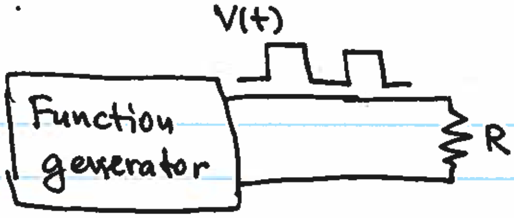


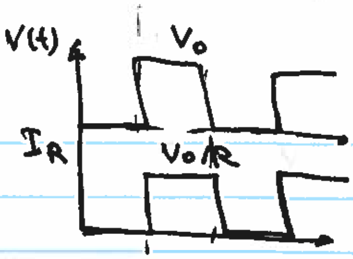
unphysical!

Thus, if something changes in a circuit, at the first moment the current through the induction does not change but develops an EMF to support the constant current.

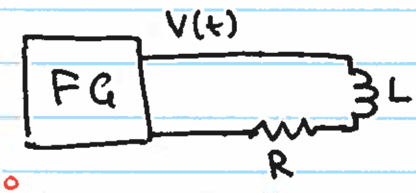
Unlike a resistor, ~~the~~ whose voltage / current can change instantaneously.

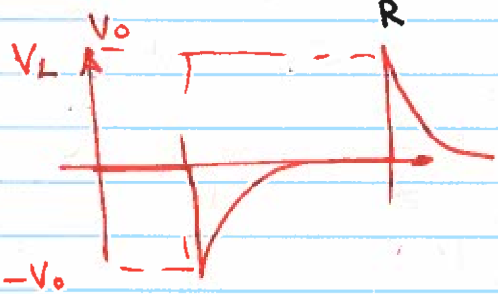
Resistor:

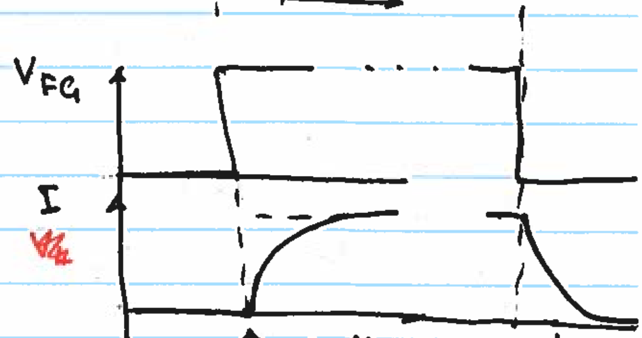




Inductor

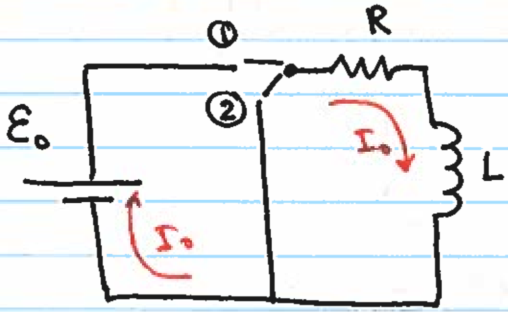




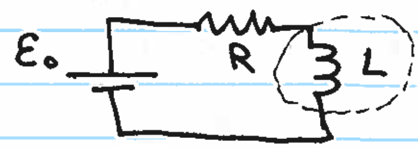


↑ at the moment of the switch, no current, induced EMF opposese V_{FG}. Then current starts changing slowly

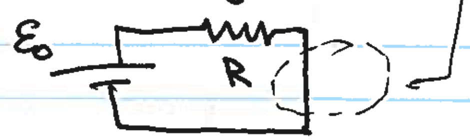
Example:



Switch in position 1



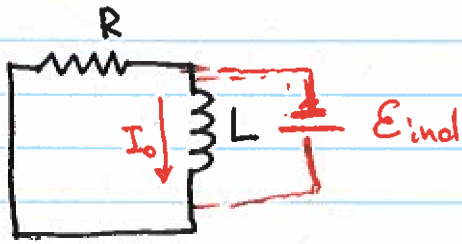
in steady state



no role in steady state

$I_0 = \mathcal{E}_0/R$

Switch in position 2, $t = 0_+$



Current through L (and R) stays the same, I_0

$\mathcal{E}_{ind} = I_0 \cdot R$ (Kirchhoff rules

As time goes by $\mathcal{E}_{ind}(t) - I(t)\cdot R = 0$

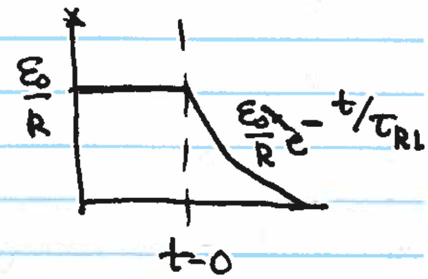
$$-L\frac{dI}{dt} - I\cdot R = 0$$

$$\frac{dI}{dt} = -\frac{R}{L} I$$

Exponential solution $I(t) = I_0 e^{-t/\tau_{RL}}$

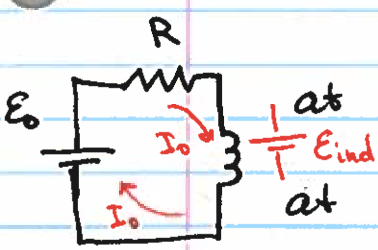
$$\frac{dI}{dt} = -\frac{1}{\tau_{RL}} \cancel{I_0 e^{-t/\tau_{RL}}} = -\frac{R}{L} \cancel{I_0 e^{-t/\tau_{RL}}}$$

$$\tau_{RL} = \frac{L}{R}$$



Current $I(t) = \frac{\mathcal{E}_0}{R} e^{-R/L\cdot t}$

Switching from ~~2 to~~ position 2 to position 1

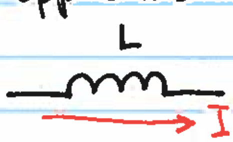


at $t=0$ $I=0$, at $t\to\infty$ $I = I_0 = \mathcal{E}_0/R$

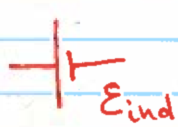
at any time $\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$

$$I(t) = I_0 - I_0 e^{-t/\tau_{RL}} \qquad \tau_{RL} = L/R$$

The direction of the induced EMF always opposes the change of the current



Current increases



value $\mathcal{E}_{ind} = \left|-L\frac{dI}{dt}\right|$

Current decreases