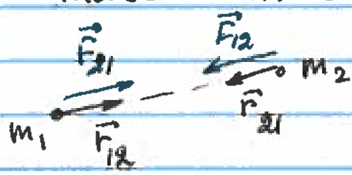
Electric charge and electric force

Gravity

mass $m > 0$



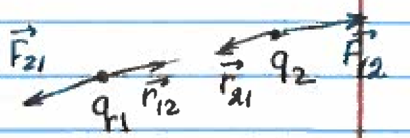
$$\vec{F}_{12} = \frac{G m_1 m_2}{r_{12}^2}\hat{r}_{21} = -\frac{G m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

always attractive

Electricity

charge q (can be positive or negative) or zero

units - Coulumb



$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

if $q_1 q_2 > 0$ — repulsive
$q_1 q_2 < 0$ — attractive

k- Coulumb constant $k = 9 \cdot 10^9 \, \frac{N \cdot m^2}{C^2}$

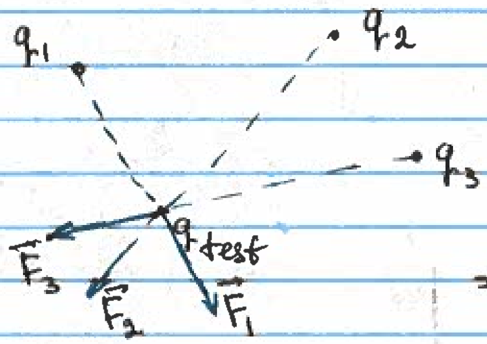
alternative : $k = \frac{1}{4\pi\varepsilon_0}$ $\quad \varepsilon_0$ – permittivity of vacuum

$\varepsilon_0 = 8.85 \cdot 10^{-12} \, \frac{C^2}{N \cdot m^2}$

we need those because of "random" definition of SI units

(For the same reason 1 Coulumb is a huge charge

Electric forces are vectors!



$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 =$$

$$= \frac{k q_1 q_t}{r_1^2}\hat{r}_1 + \frac{k q_2 q_t}{r_2^2}\hat{r}_2 + \frac{k q_3 q_t}{r_3^2}\hat{r}_3$$

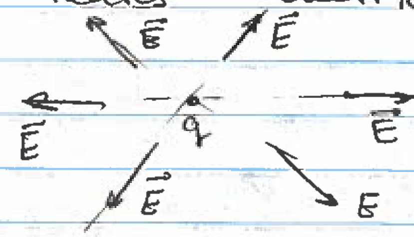
$$= q_t \underbrace{\left[\frac{k q_1}{r_1^2}\hat{r}_1 + \frac{k q_2}{r_2^2}\hat{r}_2 + \frac{k q_3}{r_3^2}\hat{r}_3\right]}_{\text{electric field } \vec{E}_{total}}$$

Electric field describes the "landscape" due to all surrounding charges, so that

$$\vec{F}_{test} = q_{test} \cdot \vec{E}$$

A single charge creates electric field

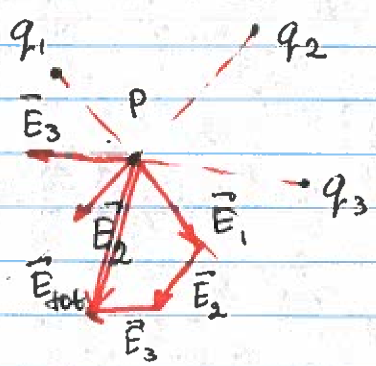
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$



Electric force lines help visualize E-field direction
At any point $\vec{E}$ is tangential to the field line.

Similarly, the electric fields produced by different sources add up as vectorse

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$



That means that

$$E_{tot_x} = E_{1x} + E_{2x} + E_{3x} + \ldots$$

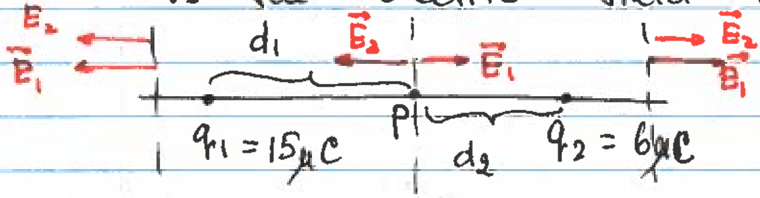
$$E_{tot_y} = E_{1y} + E_{2y} + E_{3y} + \ldots$$

Magnitude

$$|\vec{E}_{tot}| = E_{tot} = \sqrt{E_{tot\,x}^2 + E_{tot\,y}^2}$$

Example 1 : 1D world

Two charges are on a line. Where is the electric field is zero?



q_1, q_2 both positive! the distance b/w them is $L = 2m$

This point must be b/w two charges

$$E_1 = \frac{kq_1}{d_1^2} \qquad E_2 = \frac{kq_2}{d_2^2}$$

$$\text{if } E_1 = E_2 \quad \frac{q_1}{d_1^2} = \frac{q_2}{d_2^2} \Rightarrow \frac{d_2}{d_1} = \sqrt{\frac{q_2}{q_1}}$$

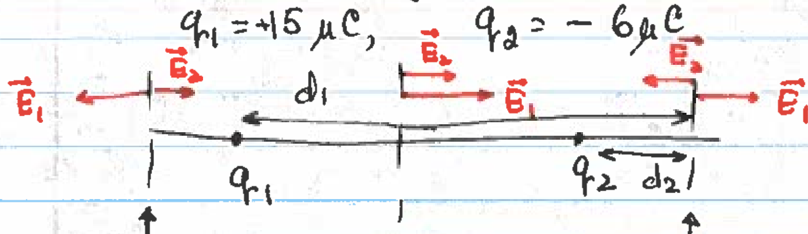
$$d_1 + d_2 = L \Rightarrow d_2 = L - d_1 \Rightarrow \frac{L}{d_1} - 1 = \sqrt{q_2/q_1}$$

$$d_1 = \frac{L}{1+\sqrt{q_2/q_1}}$$

$$d_2 = \frac{\sqrt{q_2/q_1}}{1+\sqrt{q_2/q_1}} L$$

If the charges ~~of~~ are opposite:

$q_1 = +15\mu C, \quad q_2 = -6\mu C$



suspect

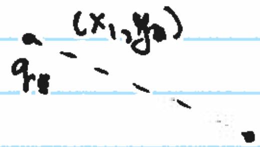
but! $|q_2|$ is smaller and it is farther so $|E_2| < |E_1|$ at any location to the left of q_1

suspect → may work

$E_1 = E_2$

$$\frac{kq_1}{d_1^2} = \frac{kq_2}{d_2^2} \qquad d_2 = 2 + d_1$$

2D electric fields



$\vec{E}(x_2, y_2) = ?$

$\vec{r} = [(x_2 - x_1), (y_2 - y_1)]$

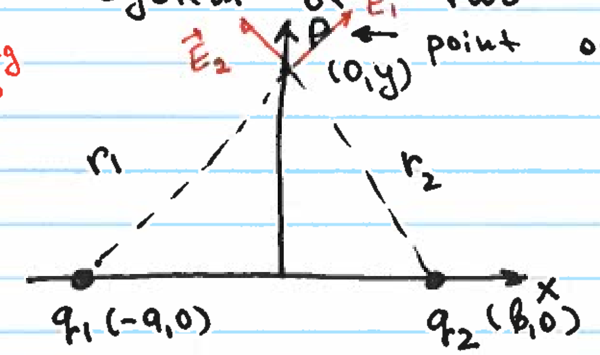
$|\vec{r}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$$\hat{r} = \left[\frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}, \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}\right]$$

$$\vec{E} = \frac{kq}{r^2}\hat{r} = \left[\frac{kq(x_2-x_1)}{[(x_2-x_1)^2+(y_2-y_1)^2]^{3/2}}; \frac{kq(y_2-y_1)}{[(x_2-x_1)^2+(y_2-y_1)^2]^{3/2}}\right]$$

System of two charges

assuming $q_1, q_2 > 0$



$r_1 = \sqrt{a^2 + y^2}$

$r_2 = \sqrt{b^2 + y^2}$

$$\hat{r}_1 = \left\{\frac{+a}{\sqrt{a^2+y^2}}, \frac{y}{\sqrt{a^2+y^2}}\right\}$$

$$\hat{r}_2 = \left\{\frac{-b}{\sqrt{b^2+y^2}}, \frac{y}{\sqrt{b^2+y^2}}\right\}$$

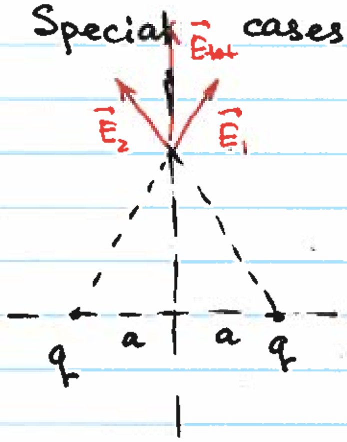
$$\vec{E}_1 = \left[\frac{kq_1 a}{\sqrt{(a^2+y^2)}^{3/2}}; \frac{kq_1 y}{\sqrt{(a^2+y^2)}^{3/2}}\right]$$

$$\vec{E}_2 = \left[\frac{-kq_2 b}{\sqrt{(b^2+y^2)}^{3/2}}; \frac{kq_2 y}{\sqrt{(b^2+y^2)}^{3/2}}\right]$$

$$\vec{E}_{tot} = [E_{1x} + E_{2x}, E_{1y} + E_{2y}] =$$

$$= \left[\frac{kq_1 a}{r_1^3} - \frac{kq_2 b}{r_2^3}; \frac{kq_1 y}{r_1^3} + \frac{kq_2 y}{r_2^3}\right]$$

Special cases $a = b$ $q_1 = q_2 > 0 = q$



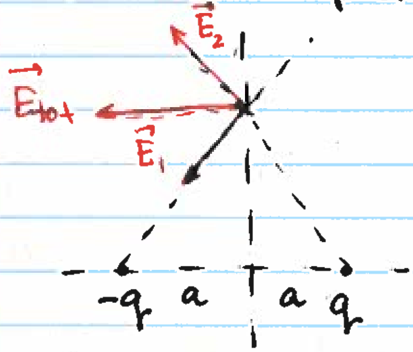
From the symmetry of the problem and/or graphic vector addition we expect $\vec{E}_{tot} = [0, \vec{E}_y]$

indeed since now $r_1 = r_2 = \sqrt{a^2+y^2}$

$$E_{tot\,x} = \frac{kqa - kqa}{r^3} = 0$$

$$E_{tot\,y} = \frac{2kqy}{r^3} = \frac{2kqy}{(a^2+y^2)^{3/2}}$$

Another special case $a = b$ $q_1 = -q_2$



In this case $\vec{E}_{tot} = [E_x, 0]$

$$E_{tot\,x} = -\frac{2kqa}{r^3} = -\frac{2kqa}{(a^2+y^2)^{3/2}}$$

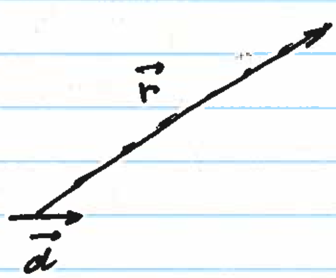
$$E_{tot\,y} = 0$$

electric dipole

dipole moment $\vec{p} = q \cdot \vec{d}$ $\vec{d}$ goes from negative to positive charge

in our case $\vec{d} = [2a, 0]$
$\vec{p} = [2qa, 0]$

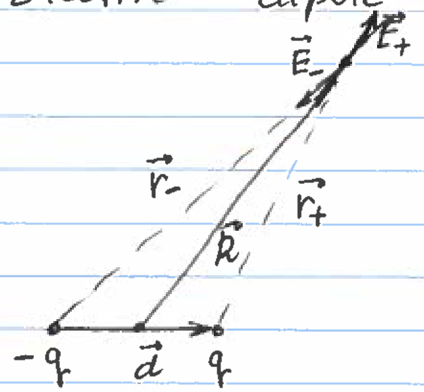
So $\vec{E}_{dipole} = -\dfrac{k\vec{p}}{(a+y)^3}$ if $y >> a$ $\vec{E}_{dipole} \approx -\dfrac{k\vec{p}}{y^3}$



In general

$$\vec{E}_{dipole} = -\frac{k\vec{p}}{r^3}$$

Electric dipole – most common charge configuration



$$\vec{r}_- = \vec{R} + \frac{\vec{d}}{2} \qquad \vec{r}_+ = \vec{R} - \frac{\vec{d}}{2}$$

$$|\vec{r}_\pm| = \sqrt{\left(\vec{R} + \frac{\vec{d}}{2}\right)\left(\vec{R} + \frac{\vec{d}}{2}\right)} =$$

$$= \sqrt{R^2 + \vec{R}\cdot\vec{d} + d^2/4}$$

We can simplify the calculation if we assume that the observation point is far from the dipole $d \ll R$

Then $R^2 \gg \vec{R}\cdot\vec{d} \gg d^2$

$$\sqrt{R^2 + \vec{R}\cdot\vec{d} + d^2} \approx \sqrt{R^2 + \vec{R}\cdot\vec{d}} = R\sqrt{1 + \underbrace{\frac{\vec{R}\cdot\vec{d}}{R^2}}_{\text{small}}}$$

Your favourite Taylor expansion

$$(1+x)^d \approx 1 + dx$$

$$\vec{E}_- = \frac{k(-q)}{r_-^3}\vec{r}_- \qquad \vec{E}_+ = \frac{kq}{r_+^3}\vec{r}_+$$

$$\frac{1}{r_-^3} = \frac{1}{R^3}\left(1 + \frac{\vec{R}\cdot\vec{d}}{R^2}\right)^{-3/2} \approx \frac{1}{R^3}\left(1 - \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}\right)$$

$$\frac{1}{r_+^3} \approx \frac{1}{R^3}\left(1 + \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}\right)$$

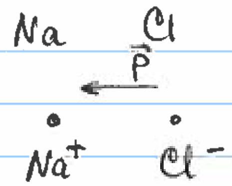
$$\vec{E}_{tot} = \vec{E}_- + \vec{E}_+ = \left(\frac{1}{R^3} - \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^5}\right)\left(\vec{R} + \frac{\vec{d}}{2}\right)(-kq) +$$

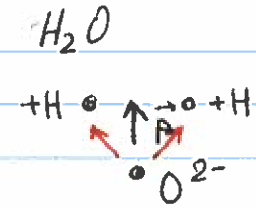
$$+ \left(\frac{1}{R^3} + \frac{3}{2}\frac{\vec{R}\cdot\vec{d}}{R^5}\right)\left(\vec{R} - \frac{\vec{d}}{2}\right)(kq) =$$

① and ③ cancel out!

$$\vec{E}_{tot} = -\frac{kq\vec{d}}{R^3} = -\frac{k\vec{p}}{R^3} \qquad \vec{p} = q\cdot\vec{d} \text{ dipole moment}$$

Polar molecules





$$\vec{p} = \sum_{\text{charges}} q_i \vec{r}_i$$

Special unit for a molecular dipole moment

1 Debay $= 10^{-20}$ C·m

Does an electron has a dipole moment?
We consider electron to be a point-like particle → no internal ~~stru~~ charge structure.
Many physicists are trying to measure EDM → we know that it is

Dipole's electrostatic potential

$$V = V_+ + V_- = \frac{kq}{r_+} - \frac{kq}{r_-} =$$

$$= \frac{kq}{R\sqrt{1+\frac{\vec{R}\vec{d}}{R^2}}} - \frac{kq}{R\sqrt{1+\frac{\vec{R}\vec{d}}{R^2}}} \approx \frac{kq}{R}\left(1+\frac{\vec{R}\vec{d}}{2R^2}\right) - \frac{kq}{R}\left(1-\frac{\vec{R}\cdot\vec{d}}{2R^2}\right)$$

$$= \frac{kq \cdot \vec{R}\cdot\vec{d}}{R^3} = \frac{k\vec{p}\cdot\vec{R}}{R^3}$$