

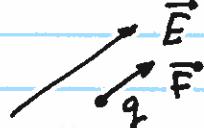
## Magnetic force and magnetic field

Magnetic force acts only on moving charges (free particles or electric currents)

Electric field → any charged object  
easy (moving or stationary)

Electric force is straight forward

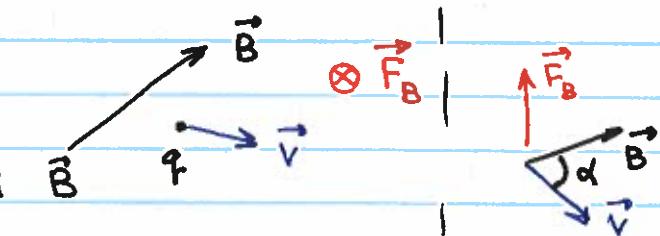
$$\vec{F}_E = q \cdot \vec{E}$$



Magnetic field → only moving object  
Magnetic force is round about

$$\vec{F}_B = q \cdot \vec{v} \times \vec{B}$$

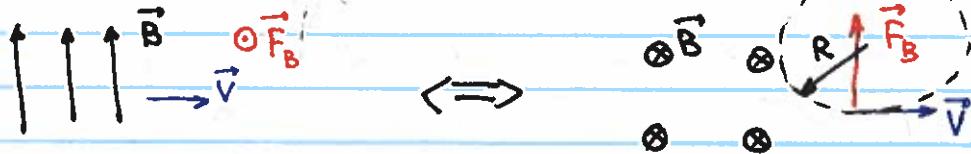
The magnetic field is perpendicular to both  $\vec{v}$  and  $\vec{B}$



$$|\vec{F}_B| = q \cdot v \cdot B \cdot \sin \alpha$$

Maximum force when  $\vec{v} \perp \vec{B}$   $|\vec{F}_B| = q \cdot v \cdot B$

Minimum (zero) force when  $\vec{v} \parallel \vec{B}$  ( $\alpha=0$ )  $|\vec{F}_B| = 0$



Acceleration due to magnetic force is perpendicular to the velocity → it only changes its direction, not magnitude!

$$\vec{a}_B = \frac{\vec{F}_B}{m} - \text{centrifugal acceleration}$$

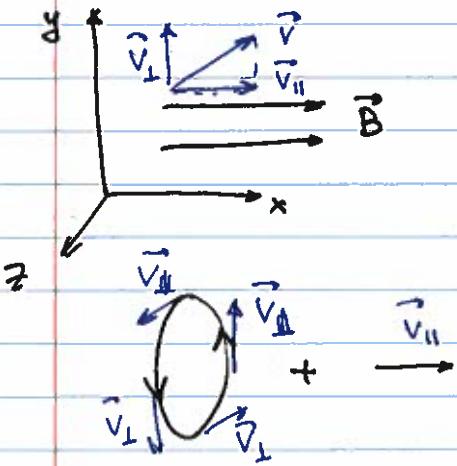
$$\frac{v^2}{R} = a_B = \frac{q \cdot v \cdot B}{m} \Rightarrow R = \frac{mv}{qB}$$

If a particle circulates in a constant orthogonal magnetic field, its period

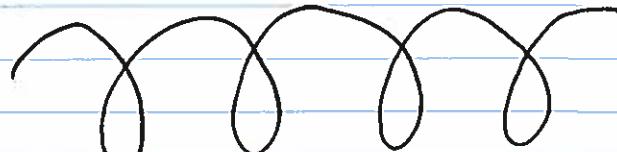
$$T = \frac{2\pi R}{v} = 2\pi \frac{m}{qB}$$
, it depends on its mass, but not velocity

$$\omega = \frac{2\pi}{T} = \frac{qB}{m}$$
 cyclotron frequency

What if velocity is not perpendicular to  $\vec{B}$



no acceleration in x direction  
 → moving with constant speed  
 centripetal acceleration in  
 $y^z$  direction plane → circular motion

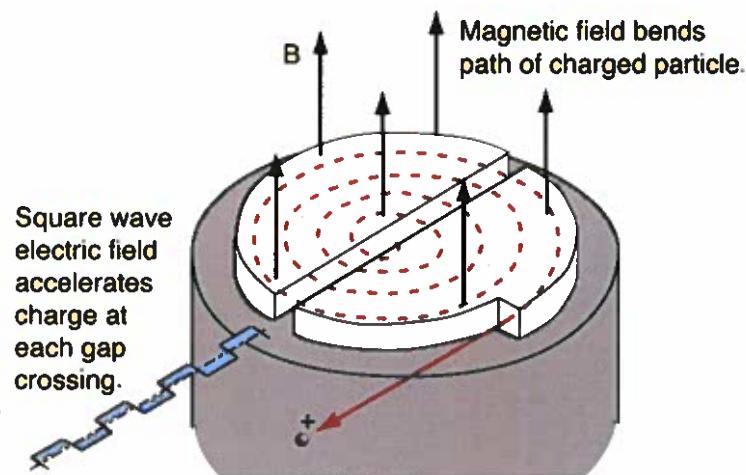


spiralizing

What if both magnetic and electric fields are present?

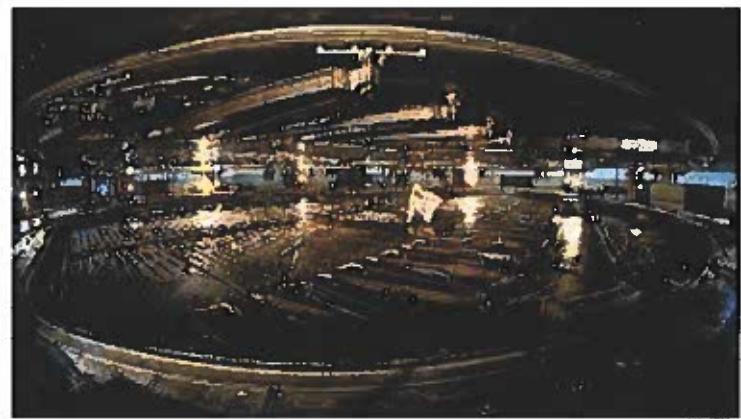
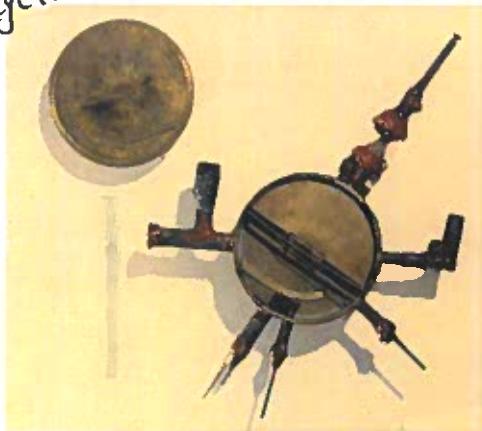
Lorentz force  $\vec{F}_L = q \cdot \vec{E} + q \vec{v} \times \vec{B}$

Applications: mass spectrometers  
 isotope separation  
 cyclotron accelerators  
 Hall effect



It always takes the same time for a particle to return to the gap, so it is always gaining energy

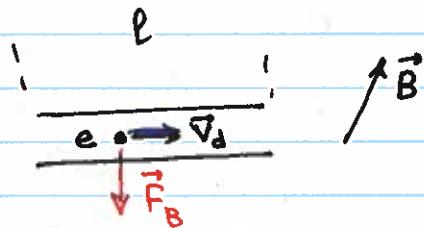
Flips  
of the cyclotron  
frequency



## Magnetic force on a current-carrying conductor

electric

Since current is a stream of moving charges, and magnetic field exerts a force on each, there will be a total force on the ~~some~~ whole wire



$$\vec{F}_{\text{total}} = \mu_0 N_{\text{charges}} e \vec{v}_d \times \vec{B}$$

$$N_{\text{charges}} = n \cdot A \cdot \ell$$

↑ length of the wire  
 ↑ area of the wire  
 charge density

$$\vec{F} = n A \ell \cdot e \vec{v}_d \times \vec{B} = I \vec{\ell} \times \vec{B}$$

$\vec{\ell}$  - vector length of the current-carrying wire, direction is along the current

