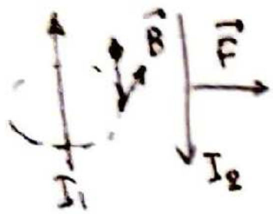
Homework 7

Problem 1

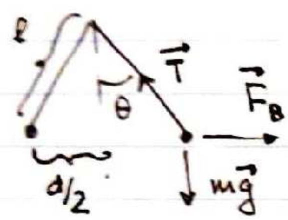
a) The magnetic field created by the first current acts on the second (and vice versa). If the two currents are counter propagating, this force is directed away from the first current



$$B = \frac{\mu_0 I_1}{2\pi d} \qquad F_B = L I_2 B = \frac{\mu_0 I_1 I_2}{2\pi d} L$$

$$F_B/L = \frac{\mu_0 I^2}{2\pi d} \qquad I_1 = I_2$$

b)



Equilibrium: $F_B - T\sin\theta = 0$

$mg - T\cos\theta = 0$

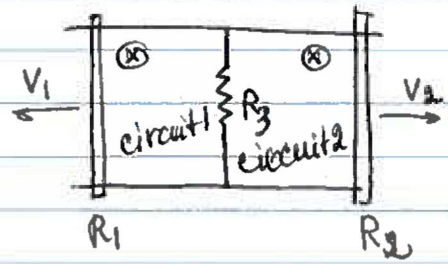
$$\underbrace{F_B = mg\tan\theta}_{} = \frac{\mu_0 I^2}{2\pi d} L$$

$\theta = 8^\circ$ $\quad d/2 = l\sin\theta$ $\quad \boxed{d = 2l\sin\theta = 1.67\text{ cm}}$

$$\frac{m}{L} g\tan\theta = \frac{\mu_0 I^2}{2\pi d}$$

$$I = \sqrt{2\pi d g\tan\theta \cdot \frac{m}{L}\frac{1}{\mu_0}} = 67.8A$$

Q 2.



Circuit 1

$$\Delta\Phi_1 = B\cdot d\cdot v_1 \Delta t$$

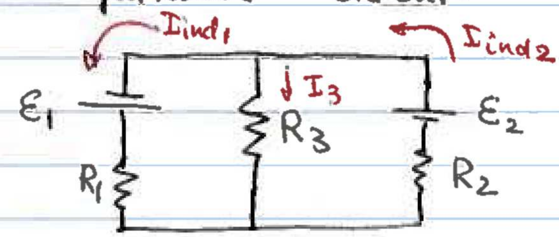
$$\mathcal{E}_1 = \left| -\frac{\Delta\Phi_1}{\Delta t} \right| = Bdv_1$$

Circuit 2

$$\Delta\Phi_2 = B\, d\, v_2 \Delta t$$

$$\mathcal{E}_2 = \left| -\frac{\Delta\Phi_2}{\Delta t} \right| = Bdv_2$$

Equivalent circuit



Kirchhoff's rules

$$\begin{cases} I_2 = I_1 + I_3 & (1) \\ \mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 & (2) \\ \mathcal{E}_1 - R_1I_1 + I_3R_3 = 0 & (3) \end{cases}$$

$$I_1 = \frac{\mathcal{E}_1}{R_1} + I_3\frac{R_3}{R_1} \quad \text{from (3)}$$

$$I_2 = \frac{\mathcal{E}_2}{R_2} - I_3\frac{R_3}{R_1} \quad \text{from (2)}$$

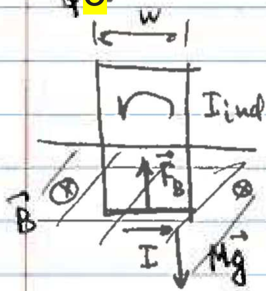
$\mathcal{E}_{1,2}$ direction → from Lenz rule to reduce the flux

from (1) $$\frac{\mathcal{E}_2}{R_2} - I_3\frac{R_3}{R_2} = \frac{\mathcal{E}_1}{R_1} + I_3\frac{R_3}{R_1} + I_3$$

$$\frac{\mathcal{E}_2}{R_2} - \frac{\mathcal{E}_1}{R_1} = I_3\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)$$

$$I_3 = \frac{\mathcal{E}_2/R_2 - \mathcal{E}_1/R_1}{1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}} = B\cdot d\,\frac{v_2/R_2 - v_1/R_1}{1 + R_3/R_1 + R_3/R_2}$$

Q3.



$$\frac{d\Phi_B}{dt} = Bw \cdot v = |\mathcal{E}_{ind}|$$

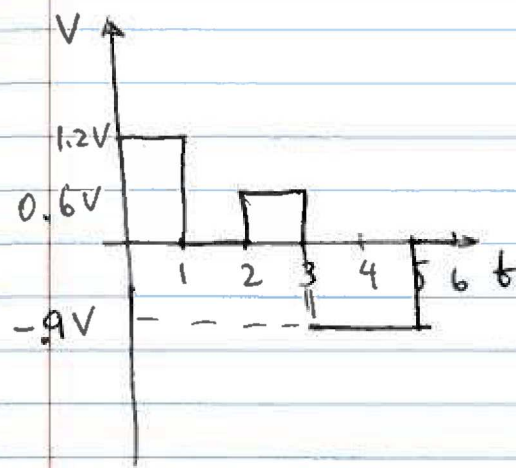
$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{Bwv}{R}$$

$$F_B = w \cdot B I_{ind} = \frac{B^2 w^2 v}{R}$$

In the beginning v is small, so the loop accelerates down $F_B - Mg = ma$ $a < 0$ (down)
However, as v increases, F_B approaches Mg, and the velocity increase becomes smaller
For the terminal velocity

$$Mg = F_B = \frac{B^2 w^2 v_t}{R} \Rightarrow v_t = \frac{MgR}{B^2 w^2}$$

Q 4



0 - 1s $\frac{dI}{dt} = -6 A/s$ $V = -L\frac{dI}{dt} =$

$= 1.2V$

1-2s $\frac{dI}{dt} = 0$ $V = 0$

2-3s $\frac{dI}{dt} = -3 A/s$ $V = 0.6V$

3-5s $\frac{dI}{dt} = 4.5 A/s$ $V = -0.9V$

5-6s $\frac{dI}{dt} = 0$ $V = 0$

Q5

Four possible capacitor configurations

$C_1 = 6\mu F$

$C_2 = 3\mu F$

in series $C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = 2\mu F$

parallell $C_p = C_1 + C_2 = 9\mu F$

$\omega_1 = 1/\sqrt{LC_1} = 4082\ rad/s = 2\pi \times 650\ Hz$

$\omega_2 = 1/\sqrt{LC_2} = 5773\ rad/s = 2\pi \times 919\ Hz$

$\omega_s = 1/\sqrt{LC_s} = 7071\ rad/s = 2\pi \times 1125\ Hz$

$\omega_p = 1/\sqrt{LC_p} = 3333\ rad/s = 2\pi \times 531\ Hz$