Written assignment #2
Solutions

Q1. $PV^{\gamma} = C$ (constant) $\Rightarrow P = \frac{C}{V^{\gamma}}$

a) $W_{on\,gas} = -\int_{V_i}^{V_f} \frac{C}{V^{\gamma}} dV = \frac{1}{\gamma-1} \frac{C}{V^{\gamma-1}}\Big|_{V_i}^{V_f} =$

$= \frac{1}{\gamma-1}(P \cdot V)\Big|_{V_i}^{V_f} = \frac{1}{\gamma-1}(P_f V_f - P_i V_i)$

b) Adiabatic process $\Delta Q = 0$ $W_{on\,gas} = \Delta E_{int}$

$W_{on\,gas} = n C_V (T_f - T_i)$

Consistency $\frac{1}{\gamma-1} = \frac{1}{\frac{C_P}{C_V} - 1} = \frac{1}{\frac{C_V + R}{C_V} - 1} = \frac{C_V}{R}$

$\frac{1}{\gamma-1}(P_f V_f - P_i V_i) = \frac{C_V}{R}(nRT_f - nRT_i) = n C_V (T_f - T_i)$

Q.2. Two lead objects collide:
one ~~moving~~ $m_1 = 12g$ moves with $V_1 = 300\ m/s$,
the other one $m_2 = 8g$ moves with $V_2 = 400\ m/s$.
After the collision they stick together and continue moving with $V_f = 20\ m/s$, while they partially melt.
Calculate the mass ~~that~~ of final liquid fraction.

This is one boring version!

Q3: Using Q1, the work done by the gas

is $W_{by} = \frac{1}{\gamma-1}(P_i V_i - P_f V_f)$

This work is used to accelerate the ~~bullet~~ pellet,

$$W_{by} = \frac{1}{2} m v^2 = \frac{1}{\gamma-1}(P_i V_i - P_f V_f)$$

$$P_i V_o - P_f V_f = \frac{\gamma-1}{2} m v^2 \qquad V_i = V_o$$

$$V_f = V_o + A \cdot L$$

$$P_f = P_i \left(\frac{V_i}{V_f}\right)^\gamma = P_i \left(\frac{V_o}{V_o + A L}\right)^\gamma$$

$$P_i V_o - P_i \left(\frac{V_o}{V_o + A \cdot L}\right)^\gamma \cdot (V_o + AL) = \frac{\gamma-1}{2} m v^2$$

$$P_i = \frac{\gamma-1}{2} m v^2 \frac{1}{V_o\left(1 - \left(V_o/V_o+AL\right)^{\gamma-1}\right)} = 6.33 \cdot 10^7 \, Pa$$

$$\approx 630 \text{ atm}$$

Q4 Escape velocity: $K + U_{gr} = 0$

$$\frac{m v_{esc}^2}{2} - G\frac{m M_E}{R_E} = 0 \qquad v_{esc} = \sqrt{2G\frac{M_E}{R_E}} \approx 1.1 \cdot 10^4 \, m/s$$

In Maxwell distribution

a) $$v_{ave} = \sqrt{\frac{8 k_B T}{\pi m_{He}}} \qquad T = \frac{\pi}{8} \frac{m_{He} v_{esc}^2}{k_B} = 22900 k$$

b) $$P_v = 4\pi \left(\frac{m_{He}}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_{He} v^2 / 2 k_B T} \Delta v$$

$m_{He} = 6.65 \cdot 10^{-27} \, kg$ $\qquad \Delta v = 2 \, m/s$

$P_v = 2.46 \cdot 10^{-44}$ not a significant loss of the atmosphere