General Physics II Honors (PHYS 102H)

Problem set # 3 *(due February 23)*

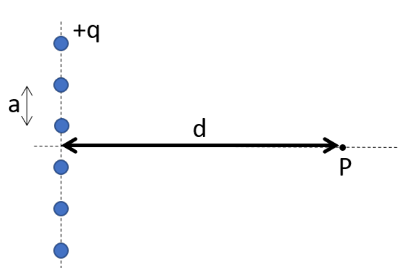
All problems are mandatory and each problem is 10 points, unless marked otherwise.

Q1 You are invited to the Coulomb game show! Several charges are scattered in the room (their charges and locations are known), and your goal is to place one more charge (let's call it the play charge) to make sure the electric field in the center of the room is zero. This is a tough task, but you can get some lifelines:
for 5 coins you can choose the value of the play charge, but it will be placed in the random (known) location;
for 10 coins you can choose the location of the play charge, but it will have a preset random (known) charge;
for 20 coins you can select both the value and the location of the play charge.
What is the minimum amount you have to pay to ensure the win if you play in 1D, 2D or 3D universe?

Q2 (15 points) Let's investigate how charges move under the Coulomb force. Let's assume that a positive charge q_1 is pinned in the origin $x = 0$, while the second positive charge q_2 with mass m_2 is initially placed at rest at $x_0 = 0.5$ m away, and released at $t = 0$. For the calculations we will assume that $kq_1q_2/m_2 = 0.03\ \mathrm{m}^3/\mathrm{s}^2$.
(a) Before doing any calculations, qualitatively describe how this charge is going to move, sketching its position and velocity as functions of time.
(b) Since the Coulomb force is space-dependent, there is no easy analytical solution, so we have to make these graphs numerically. You can use any software for accomplish that, as long as you know how to produce digital final graphs for position and velocity for the time interval between 0 and 6s.

While there are several approaches, here is the most straightforward one: let's break the time interval into small pieces Δt, and assume that during each small interval the charge moves with the constant acceleration given by the value of the Coulomb force at its beginning. Namely, if at time t_i the particle is in the position x_i and has velocity v_i, then during the next interval Δt its acceleration is assumed to be constant $a_i = kq_1q_2/(m_2x_i^2)$, and you can calculate the charge's new position and velocity x_{i+1} and v_{i+1}.

Q3 (15 points) Consider the charge configuration shown in the figure. Identical charges $+q$ as set along the line at the same interval a. The number of charges can change: the minimum number is $n = 2$, and then pairs can be added, so there is always even number.



Our goal is to determine the electric field in point P, which is positioned at distant d from the charge line and passes through its center (i.e., it is the symmetry line for any charge configuration).
(a) Write the expression of the electric field for two charges, indicate its direction.
(b) From now on let's assume $d = 10a$. Using that calculate the value of the electric field of the two charges in the point P in the units of $E_0 = kq/d^2$.
(c) Now let's do a little bit of computational physics. Using any software or hardware of your choice, calculate the electric field in point P for the sets of 4, 6 and 8 charges.
(d) In the near future we will calculate the electric field of a uniformly charged wire to be $E_{wire} = 2kq/ad$, if we assume the charge density to be q/a. Using the computational tool you developed in part (c), estimate how many charges one needs to place in line for their total electric field to be within 5% of the theoretical value E_{wire}.