# Physics 102H Midterm test #3
# April 11 2025

Name (please print): solutions

*This test is administered under the rules and regulations of the honor system of the College of William & Mary.*

Signature: ______________________

Final score: __________

**Some useful constants**

$k = 1.38 \times 10^{-23}$ J/K $N_A = 6.022 \times 10^{23}$ $R = kN_A = 8.315$ J/mol · K 0°C=273K

one atmosphere = 760 mm Hg = $10^5$ Pa 1 cal = 4.186 J 1amu = $1.66\times10^{-27}$kg

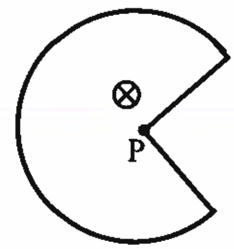
$e=1.6\cdot10^{-19}$ C $\varepsilon_0=8.84\cdot10^{-12}$ $C^2/Nm^2$ $k=9\cdot10^9$ $Nm^2/C^2$ $k = \frac{1}{4\pi\varepsilon_0}$

$\mu_0=4\pi\cdot10^{-7}$ H/m

*Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.*

.Problem 1 (25 points)

a) Find the magnitude of magnetic field at point P if the loop shown below carrying the electric current $I$. The angle between two straight segments is 90°, and the radius of the ark is $R$.

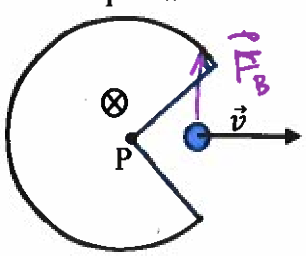



b) If the magnetic field direction at point P is into the page, does the current in the loop flow clockwise or counterclockwise?

$$dB = \frac{\mu_0}{4\pi} \cdot I \frac{dl}{R^2} \rightarrow \frac{3}{4} \text{ circumference } = \frac{3\pi}{2} \cdot R$$

$$B = \frac{\mu_0}{4\pi} I \frac{1}{R^2} \cdot \frac{3\pi}{2} R = \frac{3\mu_0}{8} \frac{I}{R}$$

c) A proton (positive electric charge $+e$) is dropped at point P with known velocity $\vec{v}$ directed as shown. Calculate the magnitude and the direction of the force acting on the proton at this point.




$$F_B = q \cdot v \cdot B = e \cdot v \cdot \frac{3\mu_0}{8} \frac{I}{R}$$

vertical up

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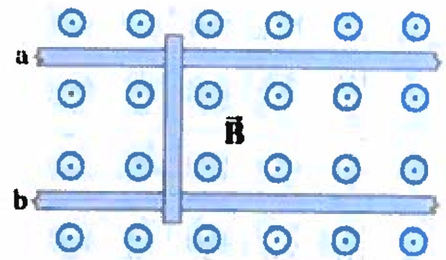
Problem 2 (40 points)

A conducting rod (mass $m$, resistance $R$) rests on two frictionless and resistanceless parallel rails a distance $\ell$ apart in a uniform magnetic field $B$ perpendicular to the rails and the rod, as shown. At $t$=0, the rod is at rest and a source of emf is connected to the points $a$ and $b$.




Determine the speed of the rod as a function of time

a) If the source puts out a constant current $I_0$.

b) If the source puts out a constant emf $\mathcal{E}_0$.

c) Does the rod reach a terminal speed in either case? If so, what is it?

*Hint*: a solution of the differential equation $dy/dt = A - \beta y$ is $y(t) = A/\beta\,(1 - e^{-\beta t})$, if $y(t=0)=0$.

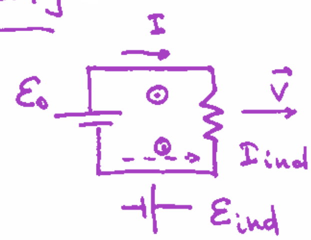
a) Force on the moving wire

$F_B = I\ell \cdot B$ $\quad I = \text{const} \rightarrow$ motion with constant acceleration

$a = \frac{F_B}{m} = \frac{I\ell B}{m}$ $\quad v(t) = a \cdot t = \frac{I\ell B}{m} \cdot t$

no terminal velocity

b) $F_B = I\ell \cdot B$ $\quad I = \frac{\mathcal{E}_0 - \mathcal{E}_{ind}}{R}$




$\mathcal{E}_{ind} = \frac{d}{dt}\Phi_B = B\ell \cdot v$

$I = \frac{\mathcal{E}_0 - B\ell v}{R}$ $\quad F_B = B\ell\left(\frac{\mathcal{E}_0}{R} - \frac{B\ell}{R}v\right)$

$a = \frac{dv}{dt} = \frac{F_B}{m} = \underbrace{\frac{B\ell\mathcal{E}_0}{mR}}_{A} - \underbrace{\frac{B^2\ell^2}{mR}}_{\beta} \cdot v$ $\quad \frac{A}{\beta} = \frac{B\ell\mathcal{E}_0}{mR} \cdot \frac{mR}{B^2\ell^2}$

$v(t) = \frac{\mathcal{E}_0}{B\ell}\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right)$

c) Terminal velocity in case B is $v_t = \frac{\mathcal{E}_0}{B\ell}$

when $\mathcal{E}_0 = \mathcal{E}_{ind} = B\ell v_t$

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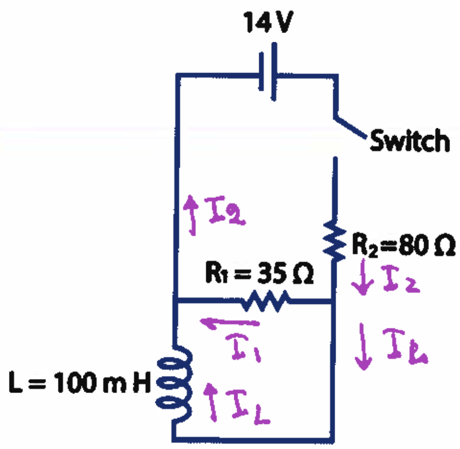
Problem 3 (35 points)

In the circuit shown here determine the current in each resistor at the moment …





a) immediately after the switch is closed.

Current through L is still zero

$I_{R_1} = I_{R_2} = \frac{\mathcal{E}}{R_1+R_2} = \frac{14\,V}{115\,\Omega} = 0.12\,A$

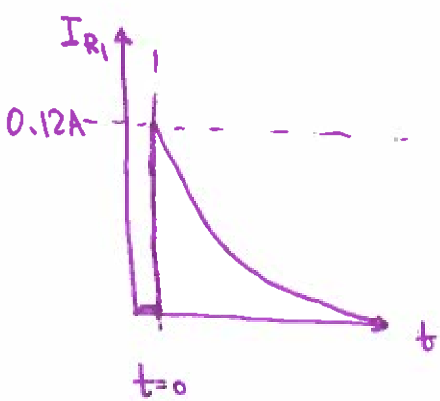
b) a long time after the switch is closed.

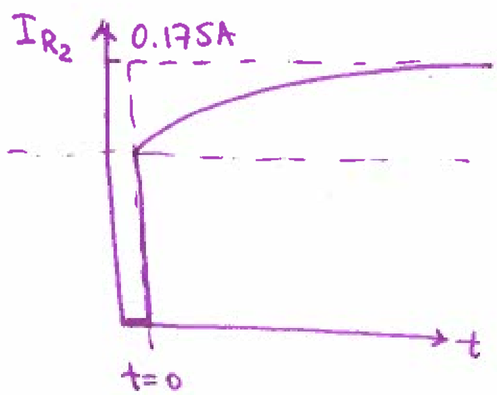
No potential difference across L → no current through $R_1$

$I_{R_1} = 0$

$I_{R_2} = \frac{\mathcal{E}}{R_2} = \frac{14\,V}{80\,\Omega} = 0.175\,A$

c) Sketch the graph of current through the resistors $R_1$ and $R_2$ as a function of time.




d) Write the set of Kirchhoff's equations for calculations of currents as function of time, when the switch is closed (you do not need to solve them).

$$\begin{cases} I_2 = I_1 + I_L \\ \mathcal{E} - I_2 R_2 - I_1 R_1 = 0 \\ I_1 R_1 - L\frac{dI_L}{dt} = 0 \end{cases}$$

or

$$\mathcal{E} - I_2 R_2 - L\frac{dI_L}{dt} = 0$$

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