Physics 102H Midterm test #2
March 21 2024

Name (please print): solutions

This test is administered under the rules and regulations of the honor system of the College of William & Mary.

Signature: __________________

Final score: __________

Some useful constants

$k = 1.38 \times 10^{-23}$ J/K $N_A = 6.022 \times 10^{23}$ $R = kN_A = 8.315$ J/mol · K $0°C = 273K$

one atmosphere = 760 mm Hg = 10^5 Pa 1 cal = 4.186 J 1amu = 1.66×10^{-27}kg

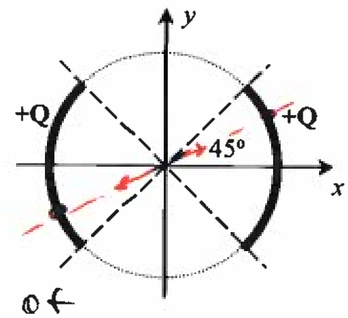
$e=1.6\cdot10^{-19}$ C $\varepsilon_0=8.84\cdot10^{-12}$ C^2/Nm^2 $k=9\cdot10^9$ Nm^2/C^2 $k = \dfrac{1}{4\pi\varepsilon_0}$

$V_{sphere}=4\pi R^3/3$ $V_{cylinder}=\pi R^2 L$

Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 1 (30 points)

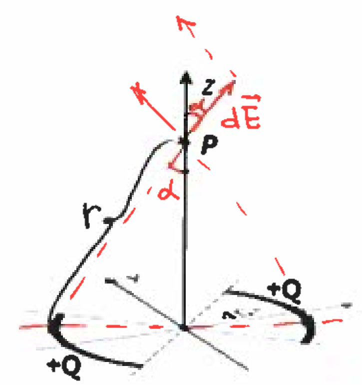
a) Two arcs of radius **R** carry each positive electric charge +Q that is uniformly distributed between 45° and -45°, and 135° and -135°, as shown. Find both *x* and *y* components of the electric field, as well as electric potential V_{center} at the center of the circle.



In the center $\vec{E}=0$, since each section of the charge arc ~~prodi~~ has an opposite symmetric section, so that their contributions to the electric field compensate each other.

$$V_{center} = \frac{2kQ}{R}$$

b) Point P(0,0,z) that is elevated above the center of the circle, as shown. Find *x, y* and *z* components of the electric field at this point.



Due to symmetry no x & y components

Only z-component of the electric field contributions from the different charges "survive"

same for all sections of the arc

$$dE_z = \frac{kdq}{r^2}\cdot\cos\alpha = \frac{k\,dq\cdot z}{r^3}$$

$$\text{Total } E_z = \frac{2kQ\cdot z}{r^3} = \frac{2kQ\cdot z}{(z^2+R^2)^{3/2}}$$

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c) Calculate the electric potential V_P at the point P and verify the relation between electric field and potential $E_z = -\frac{\partial V}{\partial z}$.

$$V_P = \frac{2kQ}{r} = \frac{2kQ}{\sqrt{R^2+z^2}}$$

$$\frac{\partial V_P}{\partial z} = 2kQ\left(-\frac{1}{\cancel{2}}\right)\frac{\cancel{2}z}{\sqrt{(R^2+z^2)^{3/2}}} = -2kQ\frac{z}{(R^2+z^2)^{3/2}} = -E_z \quad @\ b)$$

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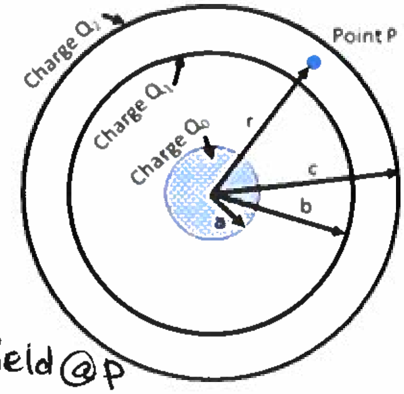
Problem 2 (35 points)

A charged sphere of radius a is surrounded by two conducting shells of radii b and c, as shown.



a) If the total charge of the inner sphere is Q_0, and the charge of the external (largest) shell is $Q_2 = -2Q_0$, what must be the charge of the intermediate shell Q_1 so that the electric field in point P is zero?

Using Gauss's law: ~~for~~ the electric field @ P to be zero, total enclosed charge in the ~~spt~~ Gaussian surface ~~of~~ passing through P must be zero

So $Q_1 = -Q_0$

b) If the inner sphere is made of a conductor, find the value of the electric field everywhere. *Make sure to define it in all four regions: r<a, a<r<b, b<a<c, and r>c.*

$r<a$ $E=0$ (no electric field inside a conductor)

$a<r<b$ $\vec{E} = \frac{kQ_0}{r^2}\hat{r}$

$b<r<c$ $E=0$ (in general $\vec{E} = \frac{k(Q_0+Q_1)}{r^2}\hat{r}$)

$r>c$ $E = -\frac{2kQ_0}{r^2}\hat{r}$ (in general $\vec{E} = \frac{k(Q_0+Q_1+Q_2)}{r^2}\hat{r}$)

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c) Now assume that the inner sphere is non-conductive, and its volume charge distribution is defined as $\rho(r) = \rho_0 {}^{r}/_{a}$. In which of the four regions will the electric field be different from b)?

Only in $\underline{r<a}$ region the electric field is different

d) Calculate the new electric field value in this region. *Reminder*: you can calculate the amount of charge inside a sphere of radius *R* by integrating the volume density as $\int_0^R \rho(r)4\pi r^2 dr$

The Gauss's law

$$4\pi r^2 \cdot E(r) = \frac{1}{\varepsilon_0} q_{enc}$$

$$q_{enc} = \int_0^r \rho_0 \frac{r}{a} \cdot 4\pi r^2 dr = 4\pi \rho_0 \frac{1}{a} \int_0^r r^3 dr = \pi \rho_0 \frac{1}{a} r^4$$

$$E(r) = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} q_{enc}$$

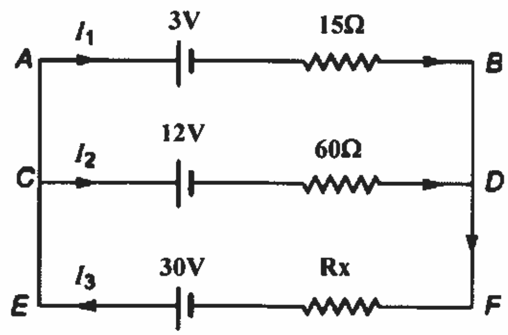
$$Q_0 = \int_0^a \rho_0 \frac{r}{a} \cdot 4\pi r^2 dr = \pi \rho_0 a^3$$

$$q_{enc}(r) = Q_0 \frac{r^4}{a^4}$$

$$E(r) = \frac{1}{4\pi\varepsilon_0} Q_0 \frac{r^2}{a^4} = \frac{kQ_0 r^2}{a^4}$$

Problem 3 (35 points)

a) What value of the resistor Rx should one choose so that there is no current flowing through the CD branch (I_2=0).



$I_2 = 0 \quad I_3 = I_1 + \cancel{I_2} = I_1 \qquad I_1 = I_3 = I$

Same current in AB & EF

ABCD loop: $-3V - 15\Omega \cdot I + 12V = 0$

$I = 3/5 A$

FECD loop: $-R_x \cdot I + 30V - 12V = 0$

$R_x = 18V/I = \frac{5}{3} \cdot 18\Omega = 30\Omega$

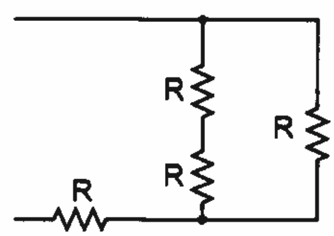
b) Under this condition, what are the values of currents I_1 and I_3?

$I_1 = I_3 = 0.6A$

c) While putting the circuit in part a) together, students discover they are out of 15Ω resistors. What should the value of R be to make sure that the equivalent resistance of the circuit below is 15Ω?

$R_{eq} = R + \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{5}{3} R = 15\Omega$

$R = 9\Omega$



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