Physics 102H Midterm test #1
February 14 2025

Name (please print): solutions

This test is administered under the rules and regulations of the honor system of the College of William & Mary.

Signature: ______________________________

Final score: ____________

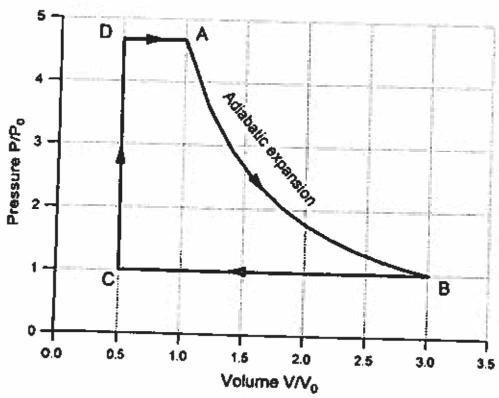
Some useful constants

$k_B = 1.38 \times 10^{-23}$ J/K $N_A = 6.022 \times 10^{23}$ $R = k_B N_A = 8.315$ J/mol · K 0°C=273K

one atmosphere = 760 mm Hg = 10^5 Pa 1 cal = 4.186 J 1amu = 1.66×10^{-27}kg

Show all work to receive full credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 1 (50 points)



In 1859, Scottish engineer William John Macquorn Rankine proposed a cycle that became the foundation for modern heat engine and steam engine theory. The diagram shows a simplified Rankine cycle, in which one mole of an ideal diatomic gas ($\gamma = C_P/C_V = 7/5$) is taken though the cycle, consisting of an adiabatic expansion ($V_A = V_0 \rightarrow V_B = 3V_0$), constant-pressure compression ($V_B = 3V_0 \rightarrow V_C = 0.5V_0$, $P = P_0$) and expansion ($V_D = 0.5V_0 \rightarrow V_A = V_0$) and one constant-volume heating C→D.

a) Find the value of pressure during D→A expansion.

$P_A V_A^\gamma = P_B V_B^\gamma$ $\quad \boxed{P_A = P_B \left(\frac{V_B}{V_A}\right)^\gamma = 3^{7/5} P_0 = 4.66 P_0}$

b) Calculate the change in the gas internal energy ΔE_{int}, the work done by the gas W and the heat Q added to the system during each process (in the units of P_0V_0):

	AB	BC	CD	DA
Internal energy change ΔE_{int}	$-W = -4.14 P_0V_0$	$-\frac{5}{2}P_0 \cdot \frac{5}{2}V_0 = -6.25 P_0V_0$	$\frac{5}{2} \cdot \frac{V_0}{2} \cdot 3.66 P_0 = 4.57 P_0V_0$	$\frac{5}{2} \cdot \frac{V_0}{2} \cdot 4.66 P_0 = 5.82 P_0V_0$
Work W	$\frac{P_AV_A - P_BV_B}{\gamma - 1} = \frac{3^\gamma - 3}{\gamma - 1} P_0V_0$	$-2.5 P_0V_0$	0	$\frac{V_0}{2} \cdot 4.66 P_0 = 2.33 P_0V_0$
Added heat Q	0	$-\frac{7}{2}P_0 \cdot \frac{5}{2}V_0 = -8.75 P_0V_0$	$4.57 P_0V_0$	$\frac{7}{2} \cdot \frac{V_0}{2} \cdot 4.66 P_0 = 8.15 P_0V_0$

c) Using data from b), calculate the efficiency of this engine.

Total work: $W = W_{BC} + W_{CD} + W_{DA} = 4.14 P_0V_0 - 2.5 P_0V_0 + 2.33 P_0V_0 = 3.97 P_0V_0$

Added heat: $Q_H = Q_{CD} + Q_{DA} = 4.57 P_0V_0 + 8.15 P_0V_0 = 12.72 P_0V_0$ $\quad \boxed{e = \frac{W}{Q_H} = 31.2\%}$

d) Find the entropy change during each part of the cycle. Explain why we expect total entropy change to be zero?

$\Delta S_{AB} = 0$

$\Delta S_{BC} = C_P \ln \frac{T_C}{T_B} = C_P \ln \frac{V_C}{V_B} = \frac{7}{2} R \ln \frac{1}{6} = -6.27R$

$\Delta S_{CD} = C_V \ln \frac{T_D}{T_C} = C_V \ln \frac{P_D}{P_C} = \frac{5}{2} R \ln 4.66 = 3.85R$

$\Delta S_{DA} = C_P \ln \frac{T_A}{T_D} = C_P \ln \frac{V_A}{V_D} = \frac{7}{2} R \ln 2 = 2.45R$

Entropy is a state variable, so its change over closed cycle is zero.

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Problem 2 (25 points)

Someone *accidentally* dropped a water bottle (m_{water}=0.3kg) at room temperature 20°C into a large vessel of liquid nitrogen at 77 K (nitrogen boiling temperature). Ignore any heat exchange with the outside world, and assume the evaporated nitrogen is instantly removed and does not absorb any additional energy.

a) How many kilograms of nitrogen will boil away by the time the water bottle freezes and reaches 77 K?

b) All the evaporated nitrogen gas is captured and put inside a round red air balloon. What is its radius at room temperature and normal atmospheric pressure?

Some possibly useful properties: Latent heat of evaporation for nitrogen L_{N2}= 5.56×10^3 J/kg, specific heat capacity of ice C_{ice}=2100 J/kg·°C, latent heat of fusion of ice L_{ice} = 3.34×10^{5} J/kg, specific heat capacity of water C_{water}= 4186 J/kg·°C. Mass of a single N_2 molecule m_{N2} = 4.65×10^{-26}kg. *The volume of the sphere is $4\pi/3R^3$.*

a) Heat removed from the water = Heat absorbed by LN

$$m_w C_w \cdot 20°C + m_w L_{ice} + m_w C_{ice} \cdot 196°C = M_{N_2} \cdot L_{N_2}$$

$$0.3\,kg \left[4186 \frac{J}{kg\cdot°C} \cdot 20°C + 3.34\cdot10^5 \frac{J}{kg} + 2100 \frac{J}{kg\cdot°C} \cdot 196°C \right] = M_{LN} \cdot 5.56\cdot10^3 \frac{J}{kg}$$

$$\boxed{M_{LN} = 44.7\,kg}$$

b) $$N_{N_2} = \frac{M_{LN}}{m_{N_2}} = \frac{44.7\,kg}{4.65\cdot10^{-26}\,kg} = 9.6\cdot10^{26} \text{ molecules}$$

$$PV = N k_B T$$

$$V = \frac{N\cdot k_B T}{P} = \frac{9.6\cdot10^{26}\cdot 1.38\cdot10^{-23}\, J/K \cdot 293K}{10^5\,Pa} = 38.9\, m^3$$

$$V = \frac{4\pi}{3}R^3 \Rightarrow \boxed{R = \sqrt[3]{\frac{3V}{4\pi}} = 2.1\, m}$$

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Problem 3 (25 points)

a) Majority of cryogenic experiments use liquid helium, since it liquifies at 4K. Calculate the average rms velocity of a helium atom in a gas for at this temperature.

b) If a heavier atom had the same the same rms velocity at 300K, what would be its mass?

c) One attempts to cool down a fixed amount of He from 300K to 4K by adiabatically expanding the gas. If the original volume was V_0, what is the final volume?

He is a monoatomic gas with $\gamma=5/3$, and its atomic mass is $m_{He}=6.65\times10^{-27}$ kg.

a) $\boxed{V_{RMS} = \sqrt{\dfrac{3k_B T_{He}}{m_{He}}} = 158\ \text{m/s}}$ $\left(\frac{1}{2} m V_{RMS}^2 = \frac{3}{2} k_B T\right)$

b) $\sqrt{\dfrac{3k_B T_{He}}{m_{He}}} = \sqrt{\dfrac{3k_B T_{room}}{m_x}}$ $\boxed{m_x = m_{He}\dfrac{T_x}{T_{He}} = 5\cdot10^{-25}\ \text{kg}}$

c) Adiabatic expansion $P_0 V_0^{\gamma} = P_f V_f^{\gamma}$ or $T_0 V_0^{\gamma-1} = T_f V_f^{\gamma-1}$

$V_f^{\gamma-1} = V_0^{\gamma-1}\dfrac{T_0}{T_f}$ $V_f = V_0\left(\dfrac{T_0}{T_f}\right)^{1/\gamma-1}$

It may be easier to substitute $\gamma = 5/3$

$V_f^{2/3} = V_0^{2/3}\dfrac{T_0}{T_f}$ $V_f = V_0\left(\dfrac{T_0}{T_f}\right)^{3/2} = V_0\left(\dfrac{300K}{4K}\right)^{3/2}$

$\boxed{V_f = 650 V_0}$

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