Balanced homodyne detection

We want to take advantage of high quantum efficiency of the conventional detectors by cleverly "amplifying" our weak optical signal.

What we measure: \( \hat{\Delta n} = \hat{\Delta n}_c - \hat{\Delta n}_d = \hat{\Delta a}_s^+ a_c - \hat{\Delta a}_d^+ d_d \)

For an ideal 50/50 beam-splitters:

\[ \hat{a}_c = \frac{1}{\sqrt{2}} (\hat{a}_s + i\hat{a}_{10}) \]

\[ \hat{a}_d = \frac{1}{\sqrt{2}} (\hat{a}_{10} - i\hat{a}_s) = \frac{1}{\sqrt{2}} (\hat{a}_s - i\hat{a}_{10}) \]

\[ \hat{n}_c = \hat{\Delta a}_s^+ \hat{a}_c = \frac{1}{2} (\hat{a}_s^+ - i\hat{a}_{10}^+)(\hat{a}_s + i\hat{a}_{10}) = \]

\[ = \frac{1}{2} (\hat{a}_{10}^+ \hat{a}_{10}^+ + i\hat{a}_s^+ \hat{a}_s + i \hat{a}_{10}^+ \hat{a}_s + \hat{a}_s^+ \hat{a}_{10}^+ ) \]

_largest contribution, puts the signal above the dark noise_

\[ \hat{n}_d = \hat{\Delta a}_d^+ \hat{a}_d = \frac{1}{2} (\hat{a}_s^+ + i\hat{a}_{10}^+)(\hat{a}_s^+ - i\hat{a}_{10}) = \]

\[ = \frac{1}{2} (\hat{a}_{10}^+ \hat{a}_{10}^+ - i\hat{a}_s^+ \hat{a}_s + i \hat{a}_{10}^+ \hat{a}_s - \hat{a}_s^+ \hat{a}_{10}^+ ) \]

\[ \Delta \hat{n} = i (\hat{a}_s^+ \hat{a}_{10}^+ - \hat{a}_{10}^+ \hat{a}_s) \]

Detected current \( \overline{\Delta n} \Rightarrow \langle \Delta n \rangle = i \langle \hat{a}_s^+ \hat{a}_{10} - \hat{a}_{10}^+ \hat{a}_s \rangle \)
Again, here we explicitly assume that the local oscillator and the quantum signal are in the identical spatial and temporal modes, since we consider that after the beam-splitter we cannot distinguish which channel the photons came from.

Normally, the local oscillator is a strong coherent state \( |d_0\rangle \), and \( d_0 = |d_0\rangle \cdot e^{i\pi} \)

\[
\langle a_{d0}^\dagger \rangle = d_{0}^* \quad \langle a_{d0} \rangle = d_{0} \\
\langle \hat{\Delta} \rangle = i \langle a_{d0}^\dagger a_{d0} - a_{d0} a_{d0}^\dagger \rangle = \langle i |d_0\rangle |e^{i\pi} a_{d0}^\dagger - i |d_0\rangle e^{-i\pi} a_{d0} \rangle \\
= |d_0| \langle \hat{a}_{d} e^{-i\theta} + \hat{a}_{d}^* e^{i\theta} \rangle \quad \text{if } \theta = x + \pi/2 \\
\langle \hat{\Delta} \rangle = 2 |d_0| \langle \hat{X}_\theta \rangle \quad \text{quadrature operator} \\
\text{Fluctuations of the differential photon flux} \\
\langle \hat{\Delta}^2 \rangle = 4 |d_0|^2 \langle \hat{X}^2 \rangle \\
\Delta \langle \hat{\Delta} \rangle = 4 |d_0|^2 \langle \Delta \hat{X}_\theta \rangle
Squeezed vacuum:

\[ \langle \hat{X}_1 \rangle = \langle \hat{X}_2 \rangle = 0 \]

For \( \theta = 0 \)

\[ \langle \delta \hat{X}_1 \rangle = \frac{1}{N} e^{-2r} \]

\[ \langle \delta \hat{X}_2 \rangle = \frac{1}{N} e^{2r} \]

When analyzed using a balanced photodetector

\[ \Delta (\Delta n) = 4 |d_{\theta}|^2 \langle \delta \hat{X}_0^2 \rangle \]

\[ \min: 1|d_{\theta}|^2 e^{-2r} \]

\[ \max: 1|d_{\theta}|^2 e^{2r} \]