Gravity and the Waltz of the Planets

> Geocentric Models
> Copernicus and Heliocentric Models
> Tycho Brahe's Observations
> Kepler
> Galileo
> Newton's Laws

http://physics.wm.edu/~hancock/171/
Greek Geocentric Models

The Greek model of the universe had the Earth fixed at the center and the Sun and Moon revolving around the Earth. The Moon was closer to the Earth since it revolved around the Earth in a month. The Sun was further away and require a year to return to the same place against the stars. The stars where fixed to the celestial sphere which also rotated around the Earth but in the opposite direction. Other ancient cultures such as China had similar models of the heavens.
There are five 'wandering stars' that moved with respect to the 'fixed' stars that rotated on the celestial sphere. In addition these 'wandering stars' had a strange 'retrograde' motion. They stopped, turned around, moved backwards, stopped again and then resumed their original motion. These five 'wandering stars' were the five known planets.
Ptolemy of Alexandria (200 CE)

- Ptolemy of Alexandria improved the geocentric model by assuming each planet moved on a small circle, which in turn had its center move on a much larger circle centered on the Earth.
- The small circles were called epicycles and were incorporated so as to explain retrograde motion.
Nicolaus Copernicus (1473-1543)

Could not reconcile centuries of data with Ptolemy’s geocentric model

- Reconsidered heliocentric model with the Sun at the center of the solar system and the planets moving in circular orbits

- Growing understanding of the distance to nearest stars, maybe parallax is too small to measure so it might not be the problem the Greeks had assumed

- Actually his model did a poor job of predicting the positions of the planets than the complex geocentric models with epicycles

- He assumed circular orbits, which let to inaccuracies in his model’s predictions
Heliocentric Model Explains the Retrograde motion

1. From point 1 to point 4, Mars appears to move eastward against the background of stars as seen from Earth (direct motion).

2. As Earth passes Mars in its orbit from point 4 to point 6, Mars appears to move westward against the background of stars (retrograde motion).

3. From point 6 to point 9, Mars again appears to move eastward against the background of stars as seen from Earth (direct motion).

Earth's faster orbital speed make Mars's motion appear to move backwards … it is not going backwards.
Copernicus and the Position of Planets

Mercury and Venus ('inferior' planets) are always seen near the sun at sunset or sunrise. They orbit inside the orbit of the Earth.

The 'superior' planets (Mars, Jupiter and Saturn) can be seen in 'opposition' ... high in the sky at midnight and opposite from the Sun. They orbit outside of the Earth's orbit.
Periods and radii of planets

In the Heliocentric model the periods (time for one orbit) and the distance from the Sun are related.

The synodic period is the time between the planet being in opposition … but the Earth moves too.

The sidereal period (time for one complete orbit around the Sun) must be calculated. Copernicus did the math. He only know the ratio (relative) size of the orbit radii

<table>
<thead>
<tr>
<th>Planet</th>
<th>Synodic period</th>
<th>Sidereal period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>116 days</td>
<td>88 days</td>
</tr>
<tr>
<td>Venus</td>
<td>584 days</td>
<td>225 days</td>
</tr>
<tr>
<td>Earth</td>
<td>—</td>
<td>1.0 year</td>
</tr>
<tr>
<td>Mars</td>
<td>780 days</td>
<td>1.9 years</td>
</tr>
<tr>
<td>Jupiter</td>
<td>399 days</td>
<td>11.9 years</td>
</tr>
<tr>
<td>Saturn</td>
<td>378 days</td>
<td>29.5 years</td>
</tr>
<tr>
<td>Uranus</td>
<td>370 days</td>
<td>84.1 years</td>
</tr>
<tr>
<td>Neptune</td>
<td>368 days</td>
<td>164.9 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Planet</th>
<th>Copernican value (AU*)</th>
<th>Modern value (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.22</td>
<td>5.20</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.07</td>
<td>9.55</td>
</tr>
<tr>
<td>Uranus</td>
<td>—</td>
<td>19.19</td>
</tr>
<tr>
<td>Neptune</td>
<td>—</td>
<td>30.07</td>
</tr>
</tbody>
</table>

*1 AU = 1 astronomical unit = average distance from Earth to the Sun.
The heliocentric model of Copernicus was certainly a major improvement over the geocentric model. But it did not work perfectly because Copernicus had assumed the planet orbits were circles. Long term observation of planet position were not significantly better than in the Ptolemaic model.

So it was time for better observations and data … you can't fool mother nature!
Tycho Brahe (1562-1601) was a Danish observational astronomer who built two state of the art observatory during the late 1500s with funds from the Danish king.

Tycho made the best measurements of the heavens before the invention of the telescope. His measurements were accurate to 1 arc-minute of angle!
Tycho Brahe's Observations

Tycho Brahe used parallax to show that a supernova in 1572 was very distant from the Earth. He also tried to measure the distance to a comet. In both cases he conclude both were far away.

Note his 'base' distance for his parallax measurement was the Earth's diameter (Not the the Earth's orbit)
Johannes Kepler (1571-1630) was a mathematician and assistant to Tycho who analyzed Tycho's data of planet motion.

Upon Tycho’s death, his data passed to Kepler, his young assistant.

Using the very precise Mars data, Kepler came to 3 major conclusions.
Kepler’s 1st Law

- Planets move in elliptical orbits with the Sun at one focus of the ellipse.

The geometry of an ellipse

Ellipses with different eccentricities
Kepler’s 2nd Law

- The orbital speed of a planet varies so that a line joining the Sun and the planet will sweep out equal areas in equal time intervals.
- The closer a planet is to the Sun during its orbit, the faster it moves.

Animation of the equal areas
Kepler’s 3rd Law

- The amount of time a planet takes to orbit the Sun (period) is related to its orbit’s size.
- This law implies that a planet with a larger distance from the Sun, will take longer to orbit the Sun.
- This hints at the nature of the force holding the planets in orbit.

\[ P^2 \text{ years} = a^3 \text{ AU} \]

- \( P = \) time to complete orbit
- \( a = \) semimajor axis
Kepler’s 3rd Law

The table show the data for the planets. (Uranus and Neptune were not discovered until later). The last two columns show amazing agreement of Kepler's 3rd law

Kepler's laws were a significant advancement and supported the Heliocentric model of the solar system.
Kepler's 1st Law
The orbit of each planet is an ellipse with the Sun at one focus

\[ a = \text{semimajor axis} = \text{average of perihelion & aphelion} = \text{average distance from Sun} \]

Kepler's 2nd Law
Equal areas are swept out in equal times

a planet moves faster when it is near the Sun than when it is far

Kepler's 3rd Law
More distant planets orbit the Sun at slower average speeds, obeying

\[ p^2 = a^3 \]

where \( p \): planet's orbital period in years

These Laws made the heliocentric model more accurate than the geocentric model.
Galileo Galilei (1564-1642)

- Contemporary of Kepler
- 1\textsuperscript{st} to use a telescope to observe the sky
  - Father of optical astronomy
- Fundamental contributions in many different sciences
Galileo’s contributions using his telescopes

- The Sun has spots $\Rightarrow$ The Sun is not perfect, changes its appearance, and rotates
- Jupiter has four objects orbiting it $\Rightarrow$ The objects are moons and they are not circling Earth
  - Jupiter is like a mini solar system
- Milky Way is populated by uncountable number of stars $\Rightarrow$ Earth-centered universe is too simple
- Phases of Venus found to disagree with the geocentric model $\Rightarrow$
Galileo’s contributions using his telescopes

Sunspots on the Sun that move across the face of the Sun with time.

Jupiter and the four Galilean satellites
Phases of Venus

Geocentric model:

- From earth we would never see more than half of Venus illuminated.
Phases of Venus

Galileo found all phases existed
  > Direct proof that the geocentric model was wrong
Galileo’s other contributions

- Credited with originating the experimental method for studying scientific problems
- Deduced the first correct physics “laws of motion”
  > Overturning the laws put forth by Aristotle, then generally accepted
- Was brought before the Inquisition and put under house arrest for the remainder of his life for advocating heliocentric solar system
Isaac Newton, father of astrophysics

- Isaac Newton (1642-1727) was born the year Galileo died
- He pioneered modern studies of motion, optics, and gravity
- co-discovered calculus
Newton's Laws of Motion

1st Law: An object remains at rest or moves in a straight line with constant speed unless acted upon by a net force. (Force is a push or a pull). This property is called inertia. Mass is a measure of inertia. Mass is not weight.

2nd Law: The net (outside) force on an object is equal to the mass of the object time the objects acceleration:

\[ \vec{F} = m \vec{a} \]

Acceleration is the time rate of change of the velocity. Both force and acceleration are vectors (they have direction and magnitude.

3rd Law: When one object exerts a force on another object the second object exerts a oppositely directed force of equal strength on the first object
Newton’s Law of gravity

- Every mass attracts every other mass
- The force of attraction is directly proportional to the product of the masses
- The force of attraction decreases with distance

\[ F_g = G \frac{M_1 M_2}{d^2} \]

\( G = \text{gravitational constant} = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2) \)
Newton’s thought experiment on gravity and orbits

“The knack of flying is learning how to throw yourself at the ground and miss”
-Hitchhikers Guide to the Galaxy
Consequences Newton's Laws

Earth as an example of an orbit

The centripetal force holding the earth in orbit is given by

\[ F_c = \frac{m_\oplus V^2}{d} \]

\[ F = G \frac{m_{\text{sun}} m_\oplus}{d^2} = \frac{m_\oplus V^2}{d} \]

\[ m_{\text{sun}} = \frac{V^2 d}{G} \]

\( V = 29.8 \text{ km/sec} \)

\( d = 1 \text{ AU} \)
Acceleration of gravity at the surface of the Earth

\[ F_g = \frac{GM_{\text{Earth}} M_{\text{rock}}}{d^2} = \frac{GM_{\text{Earth}} M_{\text{rock}}}{R_{\text{Earth}}^2} \]

The \( d \) value is from the surface to the center of the Earth, \( R_{\text{Earth}} \)

Newton 2nd Law: \( F=ma \)

\[ F_g = \frac{GM_{\text{Earth}} M_{\text{rock}}}{R_{\text{Earth}}^2} = M_{\text{rock}} a \]

\( M_{\text{rock}} \) cancels...
Acceleration of gravity at the surface of the Earth

\[ a_{\text{surface}} = \frac{GM}{R^2} = g \]

This evaluates to 9.8 m/s\(^2\)

The same acceleration for all masses

We call it “little \(g\)” or “surface gravity”
Newton's generalization of Kepler's 3rd law

\[ P^2 = \left[ \frac{4\pi^2}{G(m_1 + m_2)} \right] a^3 \quad \rightarrow \quad M_A + M_B = \frac{4\pi^2}{G} \times \frac{a^3}{P^2} \]

- It involves the **sum** of both objects’ masses
- If \( M_A \) (e.g. the Sun) is much greater than the mass of the \( M_B \) (e.g. The Earth) then this is approximately

\[ M_A = \frac{4\pi^2}{G} \times \frac{a^3}{P^2} \]

- Note: use M-K-S units, not AU & yr
Energy and Orbits

The idea of energy did not exist in Newton's time.

We are concerned with kinetic and potential energy. Kinetic energy is energy of motion given by $K.E. = \frac{1}{2}mv^2$. Potential energy is stored energy because of position.

Gravitational potential energy increases the greater the distance (e.g., semimajor axis).
Energy and Orbits

For a circular orbit, the kinetic and gravitational potential energy are both constant.

For an elliptical orbit, the kinetic and potential energy change. The sum is constant! When a satellite is nearer the Earth it moves faster (more K.E.) and it has less gravitational energy. Further away from Earth, the satellite move slower (less K.E.) but has more gravitational potential energy.
More types of orbits because of Newton's Law of gravity

Newton showed that an object can move around another object under the influence of gravity in other ways than an Ellipse or circle. Objects can also move around the Sun in a parabola or hyperbola. These 4 curves are called conic sections. They are all possible with a $1/r^2$ force law like gravity.
When an object has a certain velocity (kinetic energy), it can go into orbit. For a satellite around Earth in low Earth orbit, this is about 8 km/s. If it goes fast enough, it will eventually completely leave the Earth and never return. This speed is known as 'escape velocity' and is 11.2 km/s for the Earth. For the Sun, escape velocity is 618 km/s.

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$
Newton was the first person to understand why there are two high and low tides each day. It had been known from ancient times that the Moon (and Sun to a lesser degree) influenced the tides. The law of gravity explained why there are two tides each day.
Tides

The high tides are highest when the Sun and Moon 'work' together to deform the Earth (spring tides). Because the Moon is closer, it has the greatest effect.

When the Sun and Moon are at right angles, the tides are lower (neap tides).