Exclusive $\pi^+\pi^-$ Electroproduction
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Abstract

I have studied the effect of the HERMES acceptance on the determination of Legendre moments for exclusive $\pi^+\pi^-$ electroproduction.

Exclusive $\pi^+\pi^-$ production

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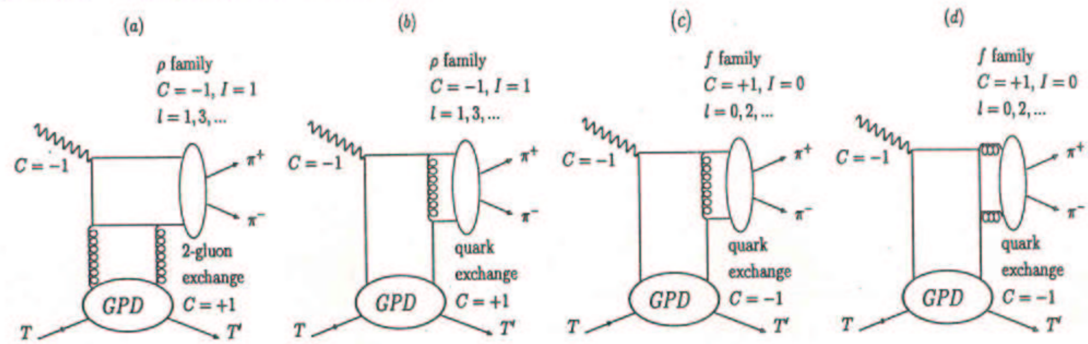


Figure 1. Leading twist diagrams for the hard exclusive reaction $e^+T \to e^+T' \pi^+\pi^-$. Gluon exchange (a) gives rise to pions in the isovector state only, while the quark exchange mechanism (b,c,d) gives rise to pions in both the isoscalar and the isovector state.

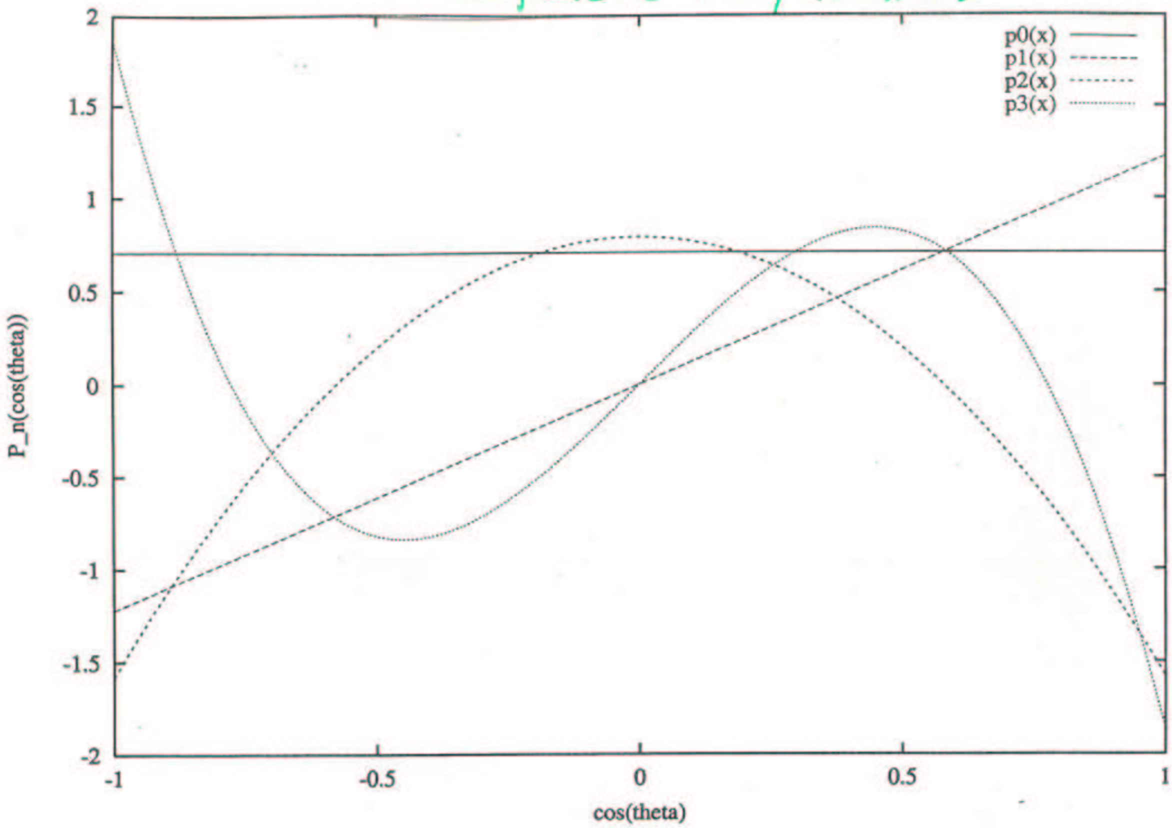
- Either 2 gluon or 2 quark exchange
- ρ-like or f-like quantum numbers
  - ρ-like: odd ℓ
  - f-like: even ℓ
- Amplitude $\propto \sum_\ell a_\ell P_\ell(\cos\theta)$
- $\dfrac{d\sigma^{\pi^+\pi^-}}{d\cos\theta} \propto \sum_{\ell\ell'} a_\ell a_{\ell'} P_\ell(\cos\theta) P_{\ell'}(\cos\theta)$
- Look at interference terms $\ell = 0$ $\ell' = 1, 3$

  $\dfrac{d\sigma}{d\cos\theta} \sim P_{\ell'}(\cos\theta)$ since $P_0(\cos\theta) = 1$
- P_ℓ's are orthogonal $\Rightarrow$

  $$\langle P_n \rangle = \frac{\int \frac{d\sigma}{d\cos\theta} P_n(\cos\theta)\, d\cos\theta}{\int \frac{d\sigma}{d\cos\theta}\, d\cos\theta}$$

  projects out the interference term

Legendre Polynomials



$$\frac{d\sigma^{\pi^+\pi^-}}{d\cos\theta} = \sum_{\ell\ell'} a_{\ell\ell'} P_\ell(\cos\theta) P_{\ell'}(\cos\theta)$$

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

Riccardo Fabri's paper (now circulating in HERMES) I=0 ℓ=2

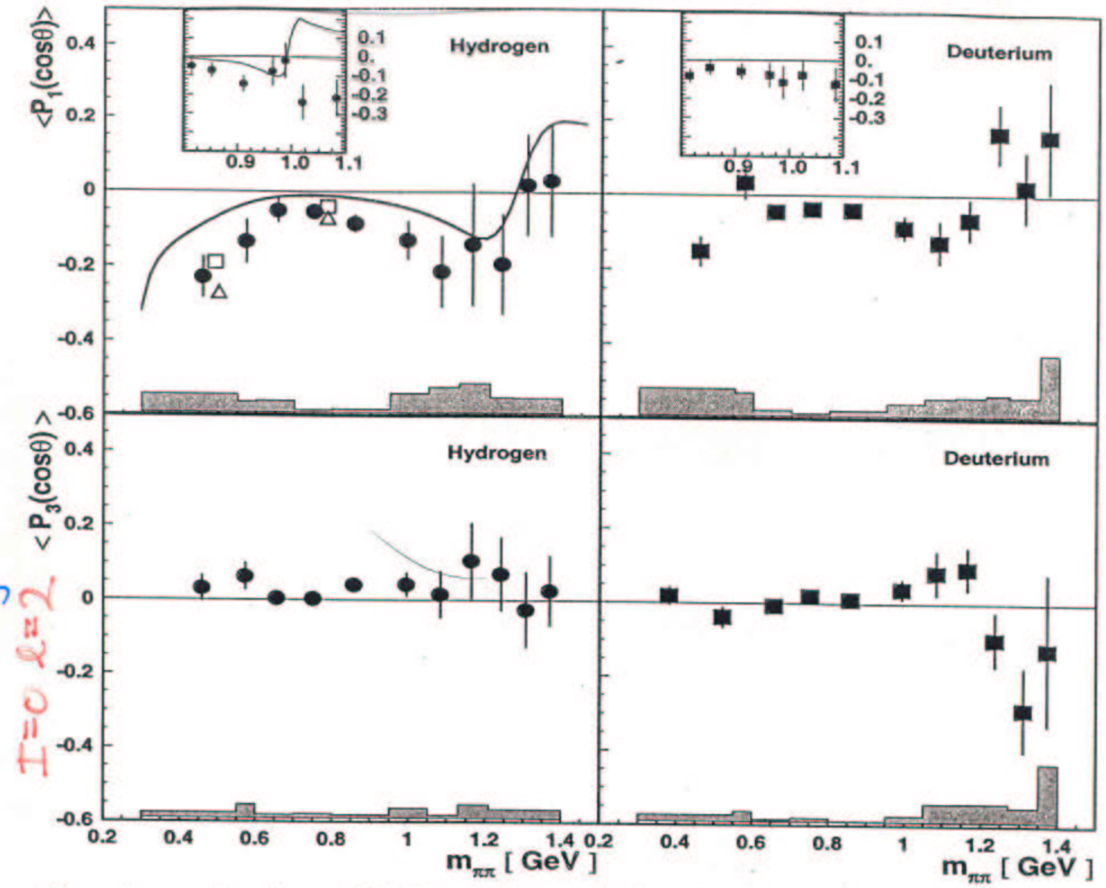


Figure 4. $m_{\pi\pi}$-dependence of the *intensity densities* $\langle P_1(\cos\theta)\rangle$, upper panels, and $\langle P_3(\cos\theta)\rangle$, bottom panels, for both hydrogen and deuterium, left and right panels respectively. In the upper panels, the region $0.8 < m_{\pi\pi} < 1.1$ GeV rebinned in finer channels to better investigate possible contributions from the narrow $f_0(980)$ meson resonance. Also shown are leading twist predictions for the hydrogen target including the two-gluon exchange mechanism contribution, LSPG [4,5] (solid curve at $x = 0.16$). A calculation without the gluon exchange contribution is showed for limited $m_{\pi\pi}$ values, LPPSG [6] (open squares at $x = 0.1$, open triangles at $x = 0.2$). Fig. 1-a. In the above predictions, the contribution from f_0 meson decay was not considered. Instead, in the zoomed panel for the hydrogen target, the prediction from [18], which includes the f_0 meson contribution, is shown. All experimental data have $< x >= 0.16$ and $< Q^2 > = 3$ GeV^2. The systematic uncertainty is represented by error band.

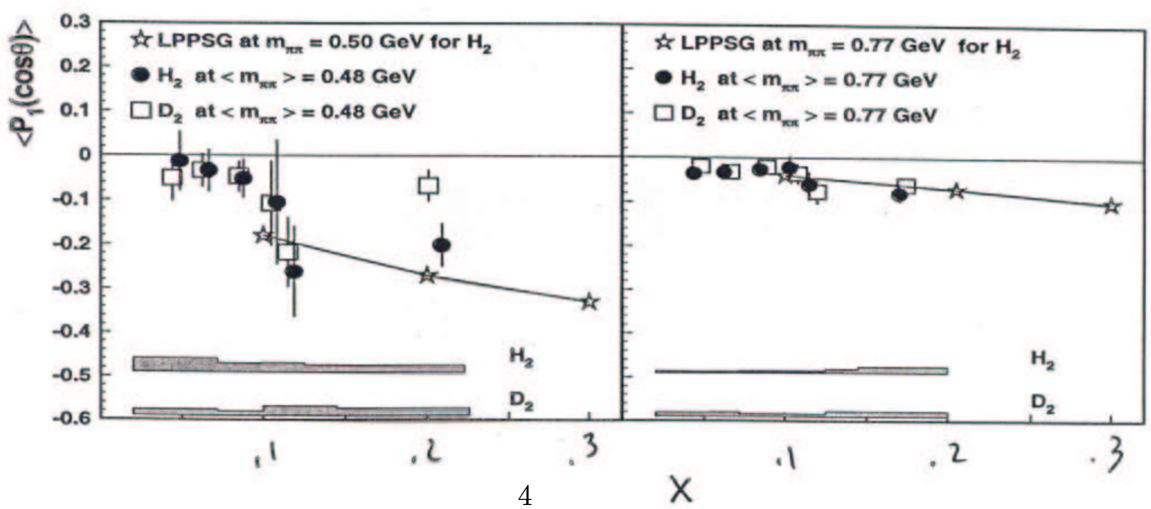


Figure 6. The x-dependence of the *intensity densities* $\langle P_1(\cos\theta)\rangle$ for both targets separately, in the regions $0.30 < m_{\pi\pi} < 0.60$ GeV (left panels) and $0.60 < m_{\pi\pi} < 0.95$ GeV (right panels). Theoretical predictions from LPPSG [6] (stars) for hydrogen are compared with the data. In these computations, the two-gluon exchange mechanism contribution to the process is neglected. The systematic uncertainty is given by the error band.

- Cross Check of analysis done by Sasha Borissov

- Monte Carlo simulation showed with large statistical errors that acceptance corrections to $\langle P_1 \rangle$ and $\langle P_3 \rangle$ were probably not important.

- $\langle P_n \rangle = \frac{1}{N} \sum_{i=1}^{N} P_n(\cos\theta_i)$ ← measured values

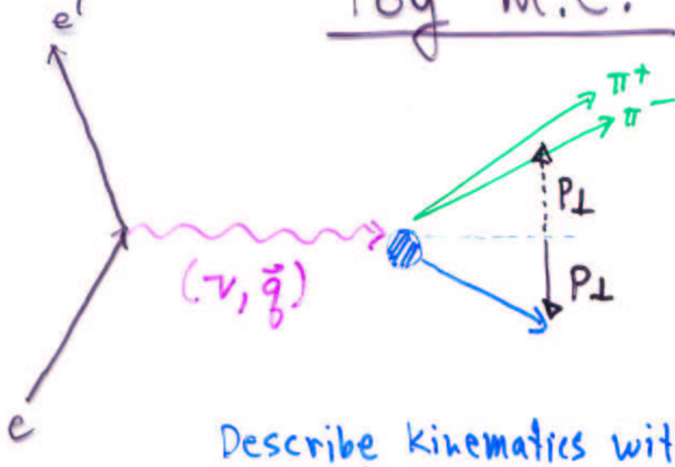
⇒ cannot make acceptance corrections to a spectrum and then fit

⇒ How wrong might $\langle P_n \rangle$ be due to holes in 4π acceptance?

This is a general problem also applicable to SSA $\langle \sin\phi \rangle$ moments : $\frac{1}{N} \sum_i \sin\phi_i$

- Create a toy Monte Carlo to study the effects of acceptance on moments $\langle P_n \rangle$

Toy M.C.



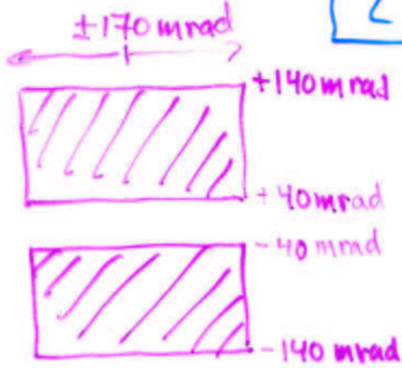
$ep \to ep(\pi^+\pi^-)$
$ed \to ed(\pi^+\pi^-)$
Exclusive

Describe kinematics with 4 variables:

$$\{x, Q^2, m_{\pi\pi}, P_\perp\}$$

⇓

for each quadruplet we can ask what is the $\pi^+\pi^-$ detection efficiency and measured $\langle P_1\rangle$ $\langle P_3\rangle$ moments



HERMES Geometrical acceptance
+ momentum > 1 GeV
(takes field into account)

CHOOSE at random

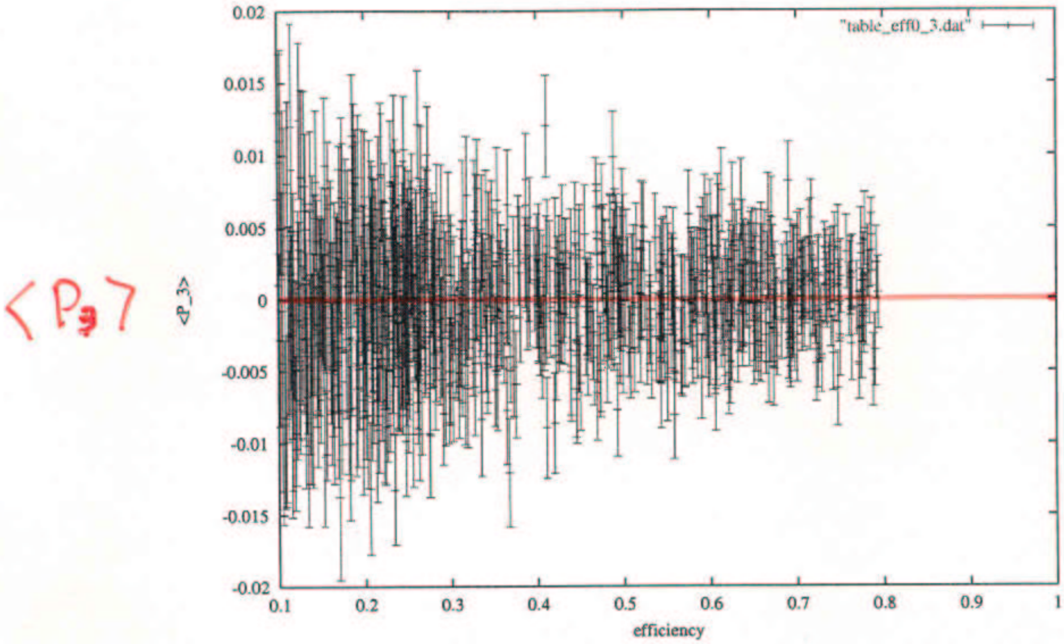
- ★ ϕ_q azimuthal angle of $\hat{q}$
- ★ $\phi_\perp$ azimuthal angle of $\vec{P}_\perp$ around $\hat{q}$
- ★ ϕ_{cm} azimuthal angle of π^+ emission around direction of $\pi^+\pi^-$ momentum.

CHOOSE θ_{cm} direction of π^+ in $\pi^+\pi^-$ CM frame with $-z$ the recoil direction of N.

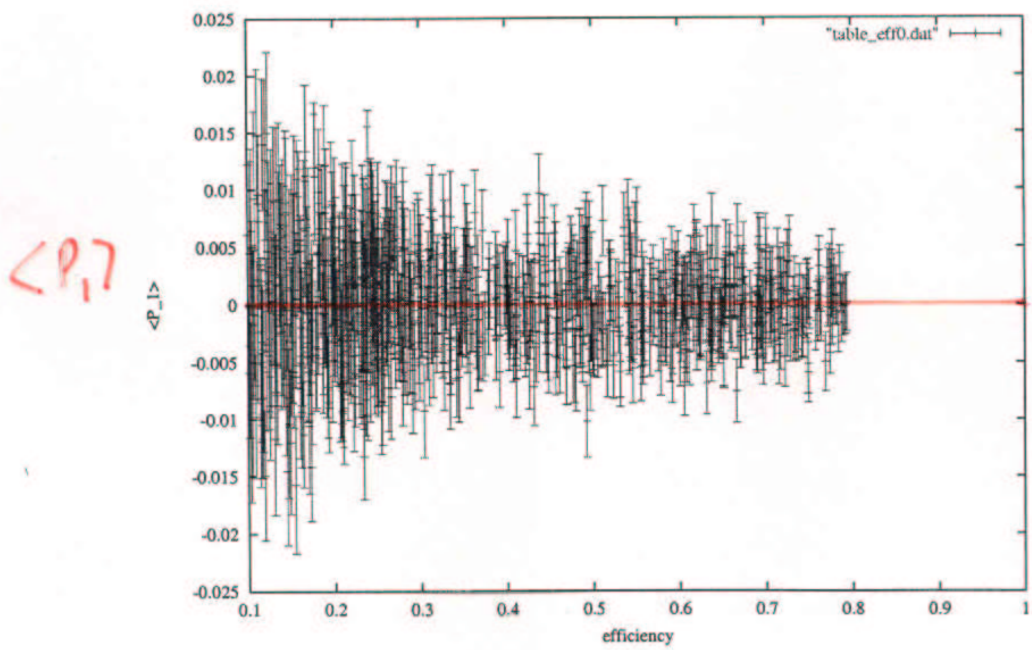
Distributions [in $\cos\theta_{cm}$]

- flat $f(x) = \frac{1}{2}$ on $[-1, 1]$
- triangular $f(x) = \frac{1}{2}(x+1)$
- quadratic $f(x) = \frac{3}{2}x^2$
- cubic $f(x) = \frac{1}{2}(1+x^3) = \alpha_0 P_0 + \alpha_1 P_1 + \alpha_3 P_3$

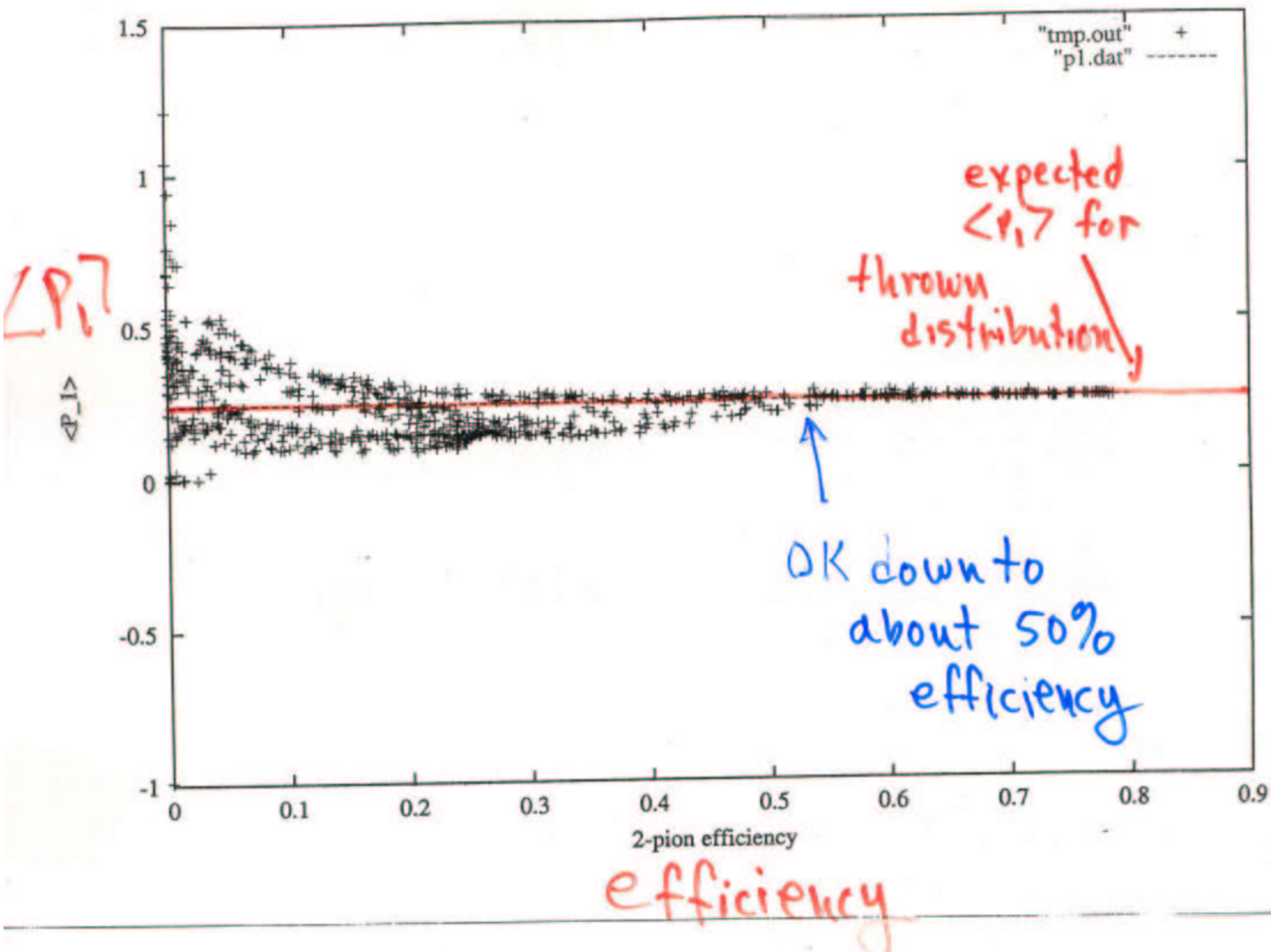
Do isotropic CM distributions generate non-zero $\langle P_1 \rangle$ and $\langle P_3 \rangle$ due to holes in acceptance?

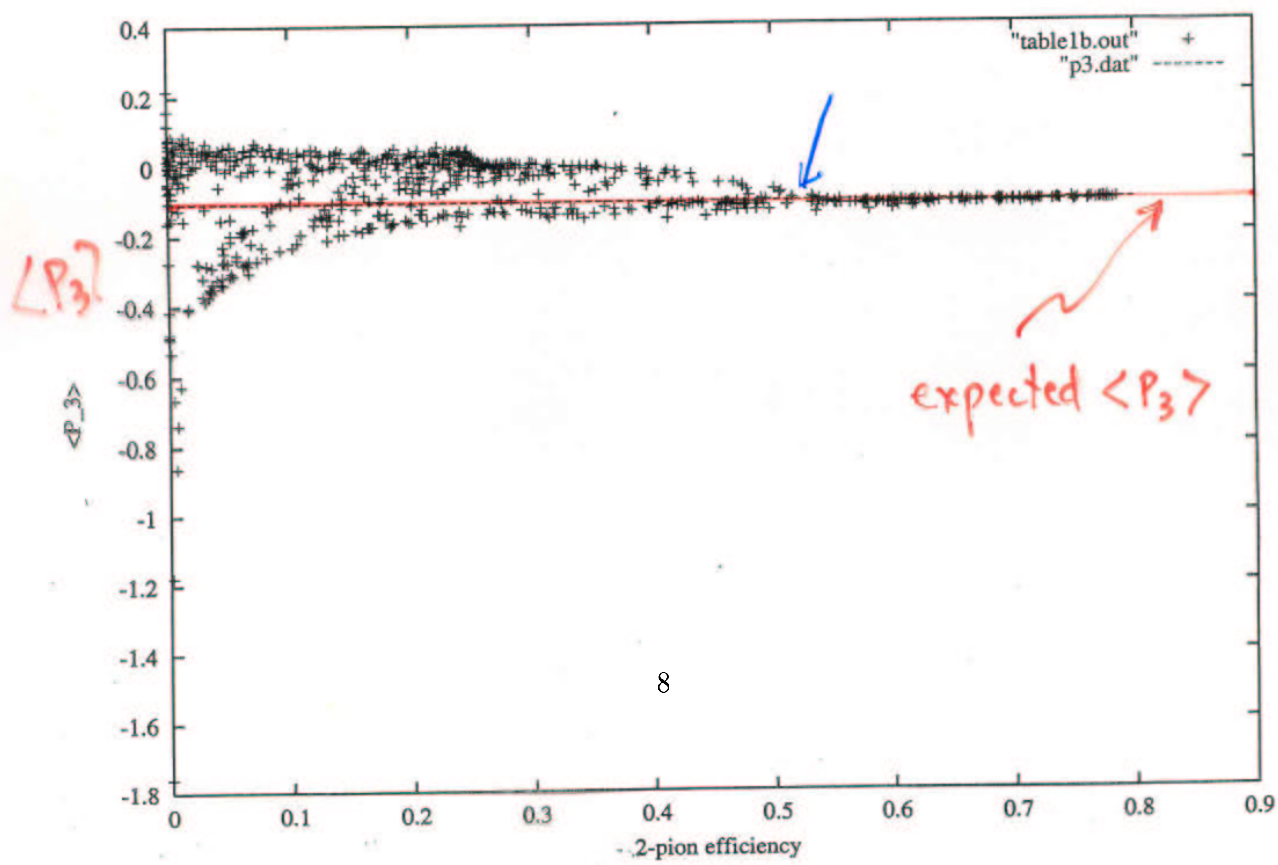


NO!

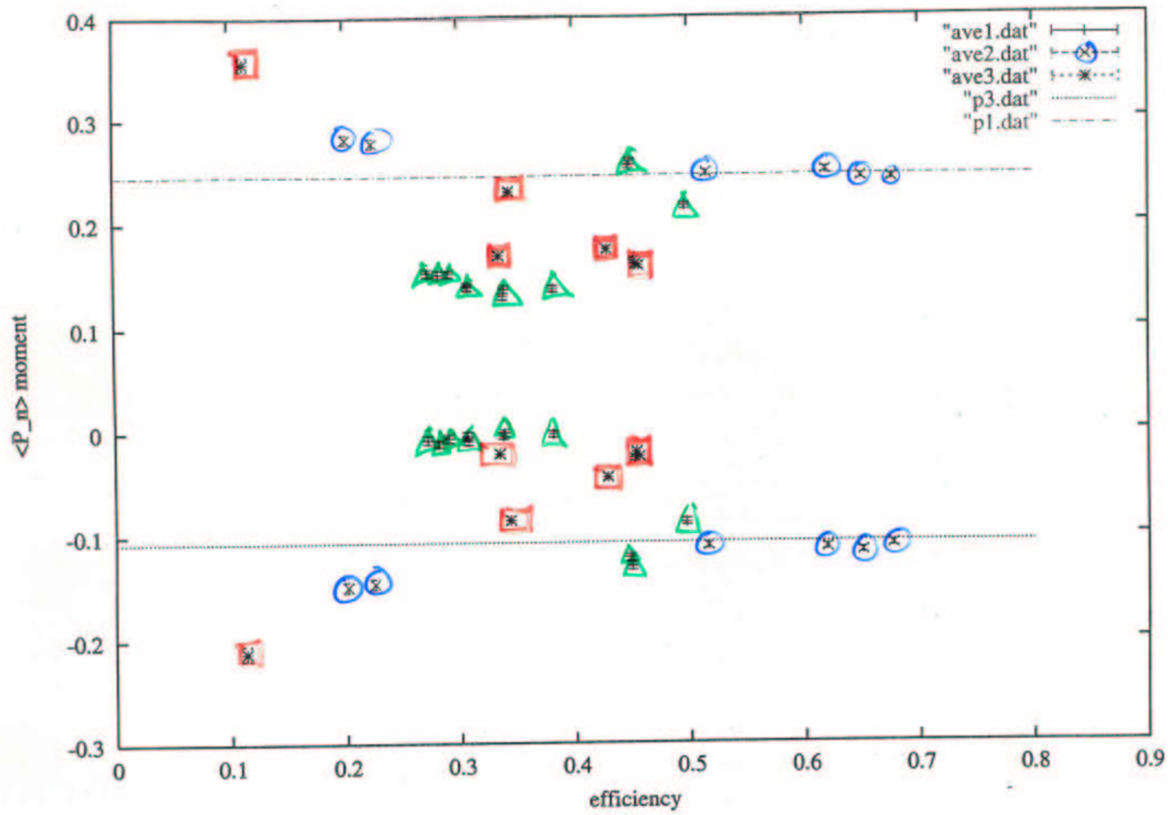


efficiency for $\pi^+\pi^-$ detection in HERMES


"tmp.out" +
"p1.dat"
<P_1>
2-pion efficiency
expected <P1> for thrown distribution
OK down to about 50% efficiency
<P1>
efficiency


"table1b.out" +
"p3.dat"
<P_3>
2-pion efficiency
<P3>
expected <P3>

MC



○ 6 bins in x for $0.3 < m_{\pi\pi} < 0.6$ GeV

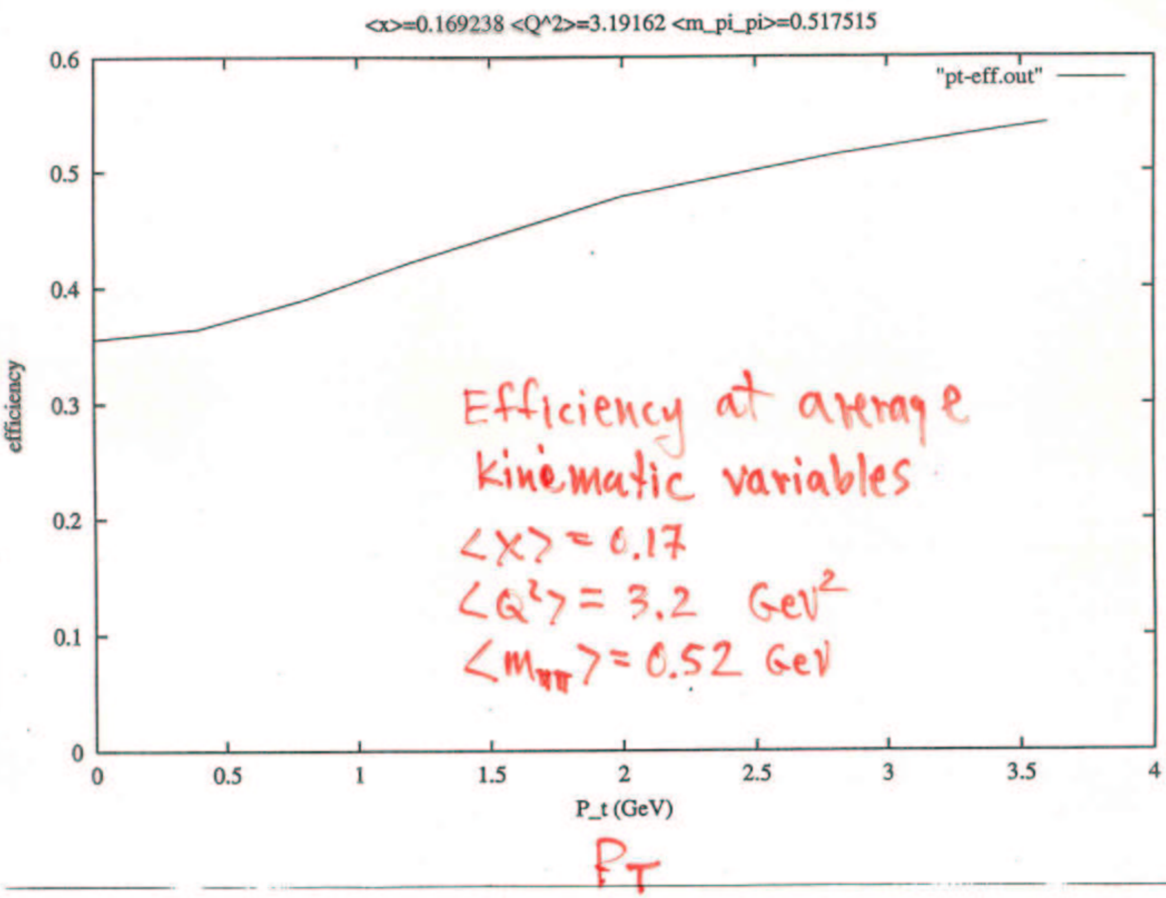
□ 6 bins in x for $0.6 < m_{\pi\pi} < 0.95$ GeV

△ 11 bins in $m_{\pi\pi}$ for $x > .1$

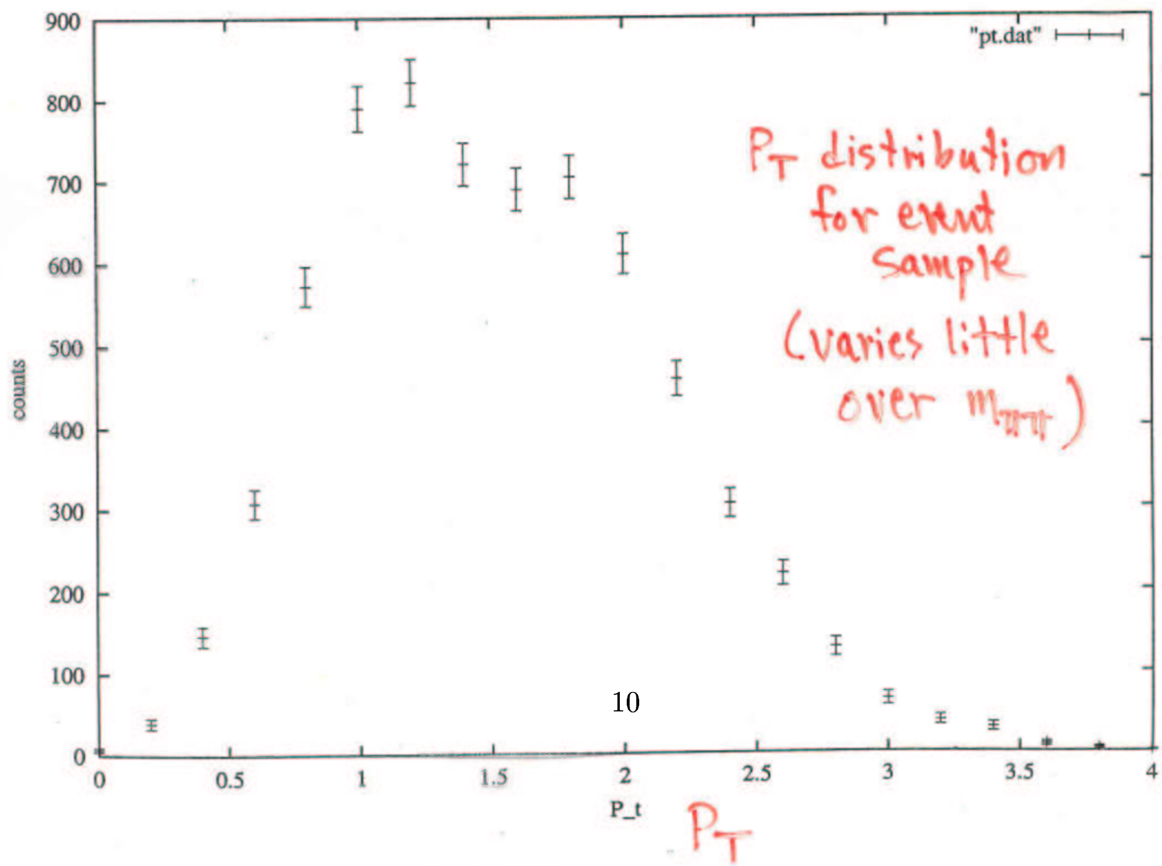
INPUT $\langle x \rangle$ $\langle Q^2 \rangle$ $\langle m_{\pi\pi} \rangle$ for each bin
$\langle P_T \rangle$

In general, extracted $\langle P_n \rangle$ is far from expected value

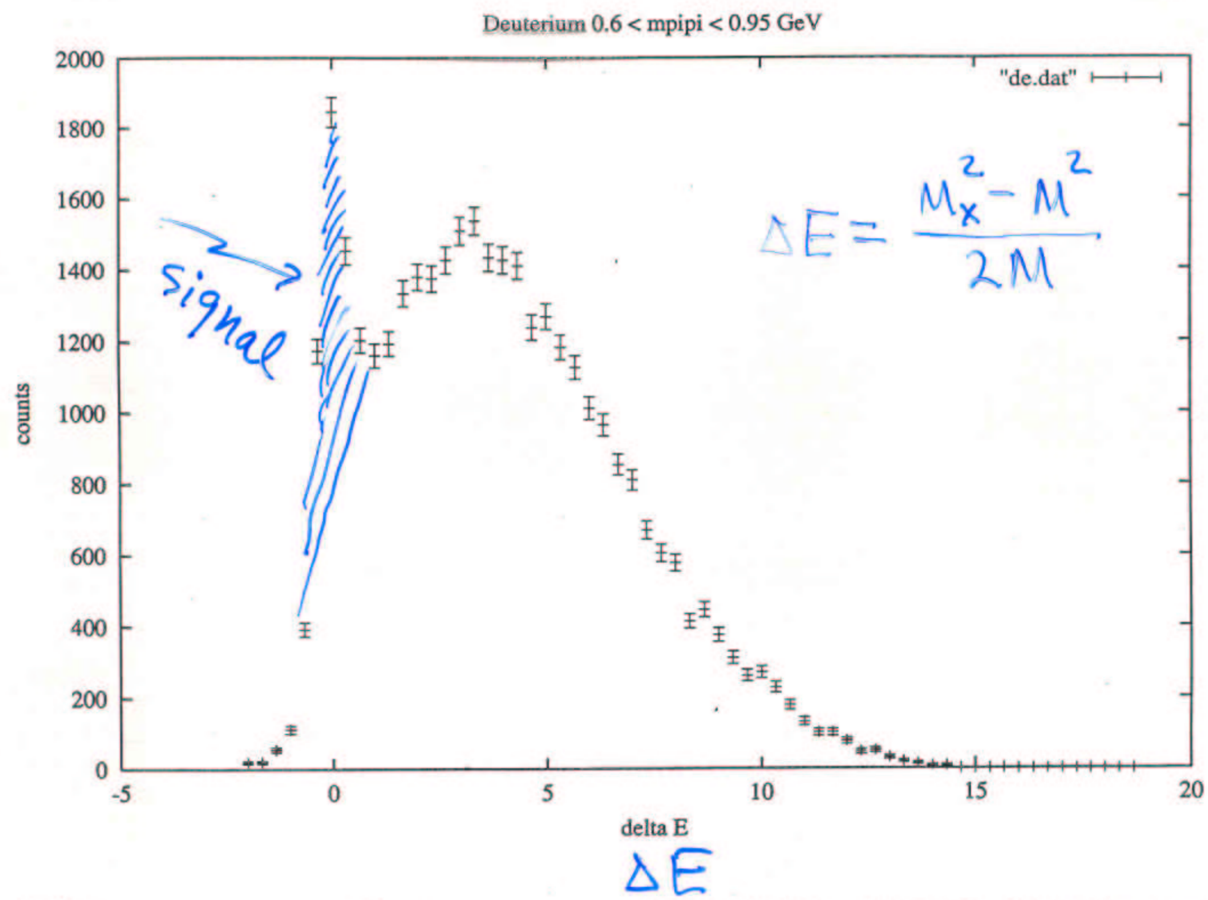
Full averages over a bin will likely come closer to ideal value... but not completely.

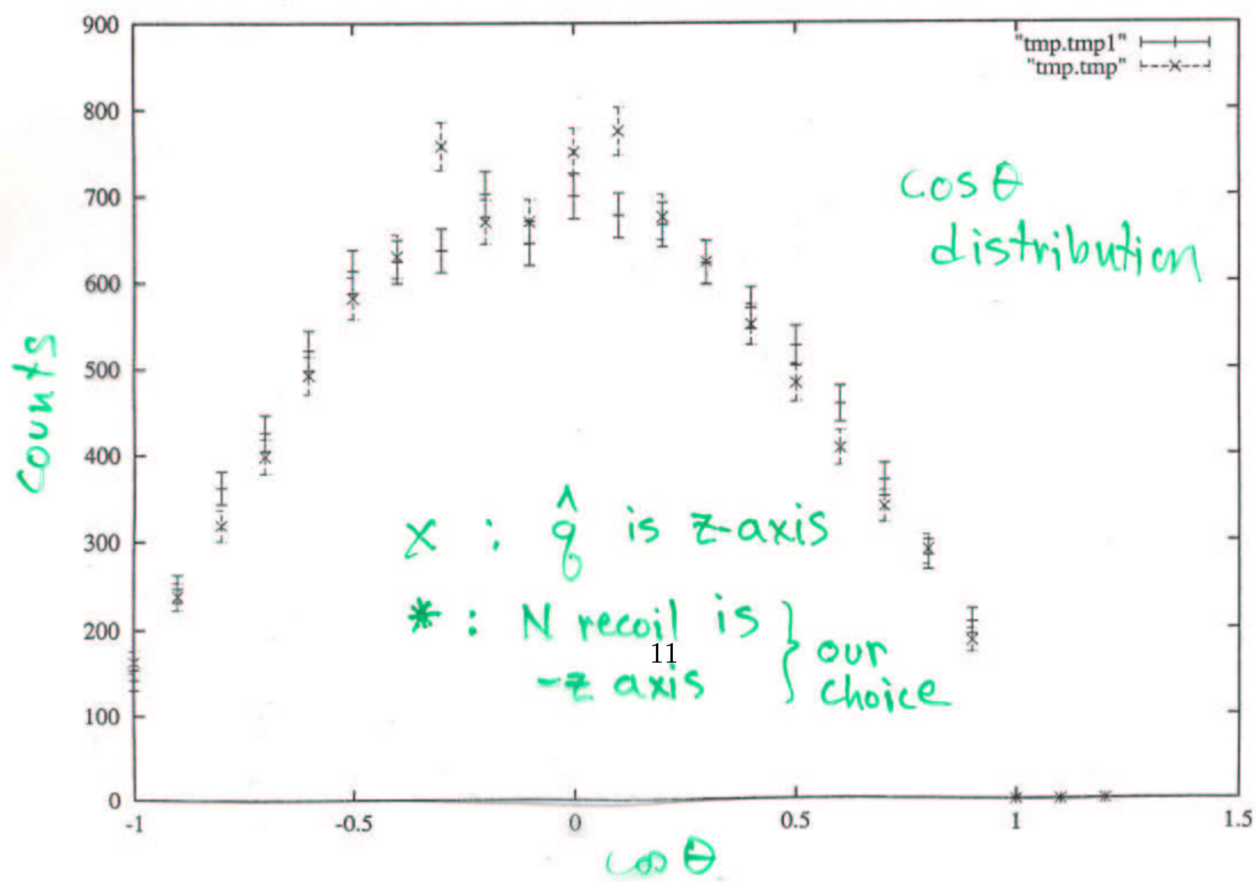


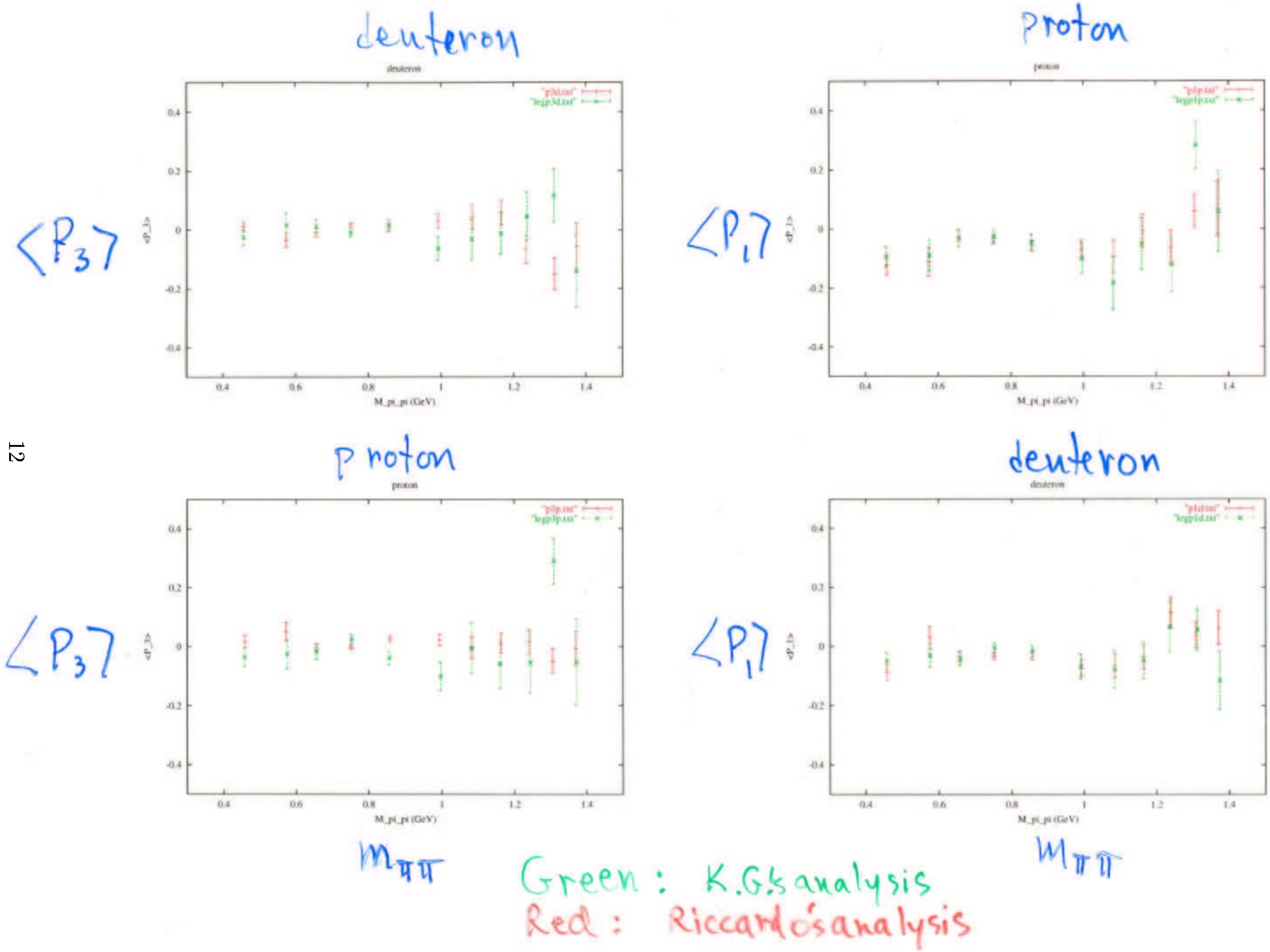
<x>=0.169238 <Q^2>=3.19162 <m_pi_pi>=0.517515
"pt-eff.out"
efficiency
P_t (GeV)
Efficiency at average kinematic variables
<x> = 0.17
<Q²> = 3.2 GeV²
<m_ππ> = 0.52 GeV
P_T

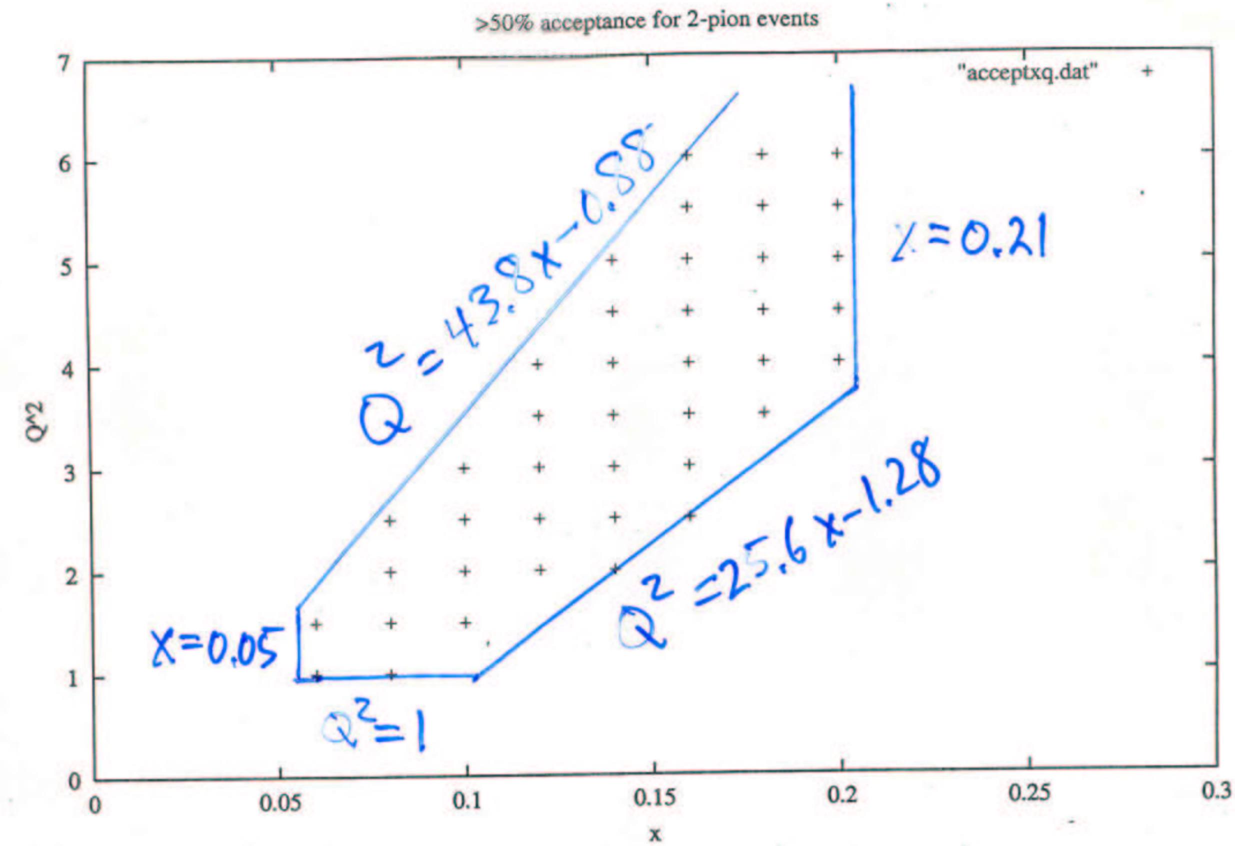


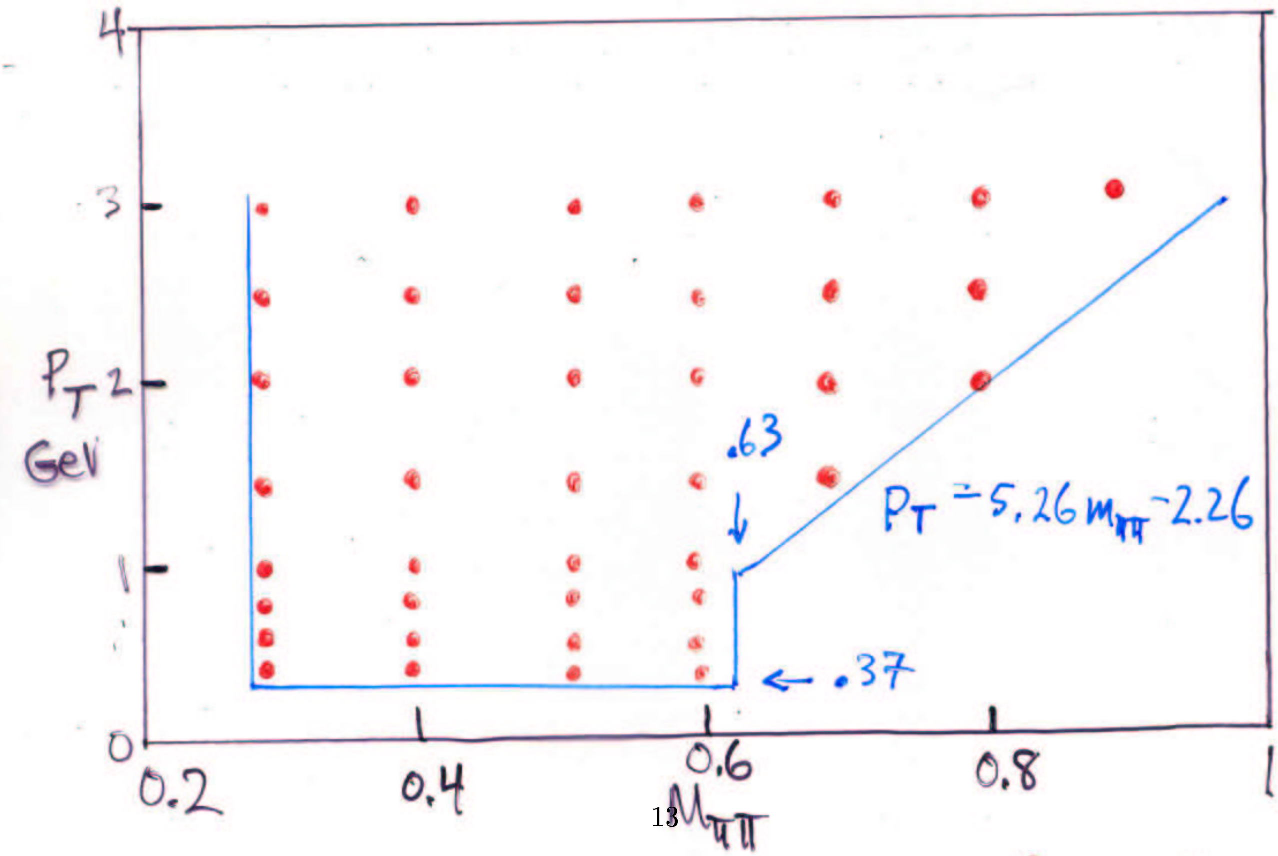
"pt.dat"
counts
P_t
P_T distribution for event sample (varies little over m_ππ)
P_T


Deuterium 0.6 < mpipi < 0.95 GeV
"de.dat"
counts
delta E
signal
ΔE = (M_x^2 − M^2) / 2M
ΔE

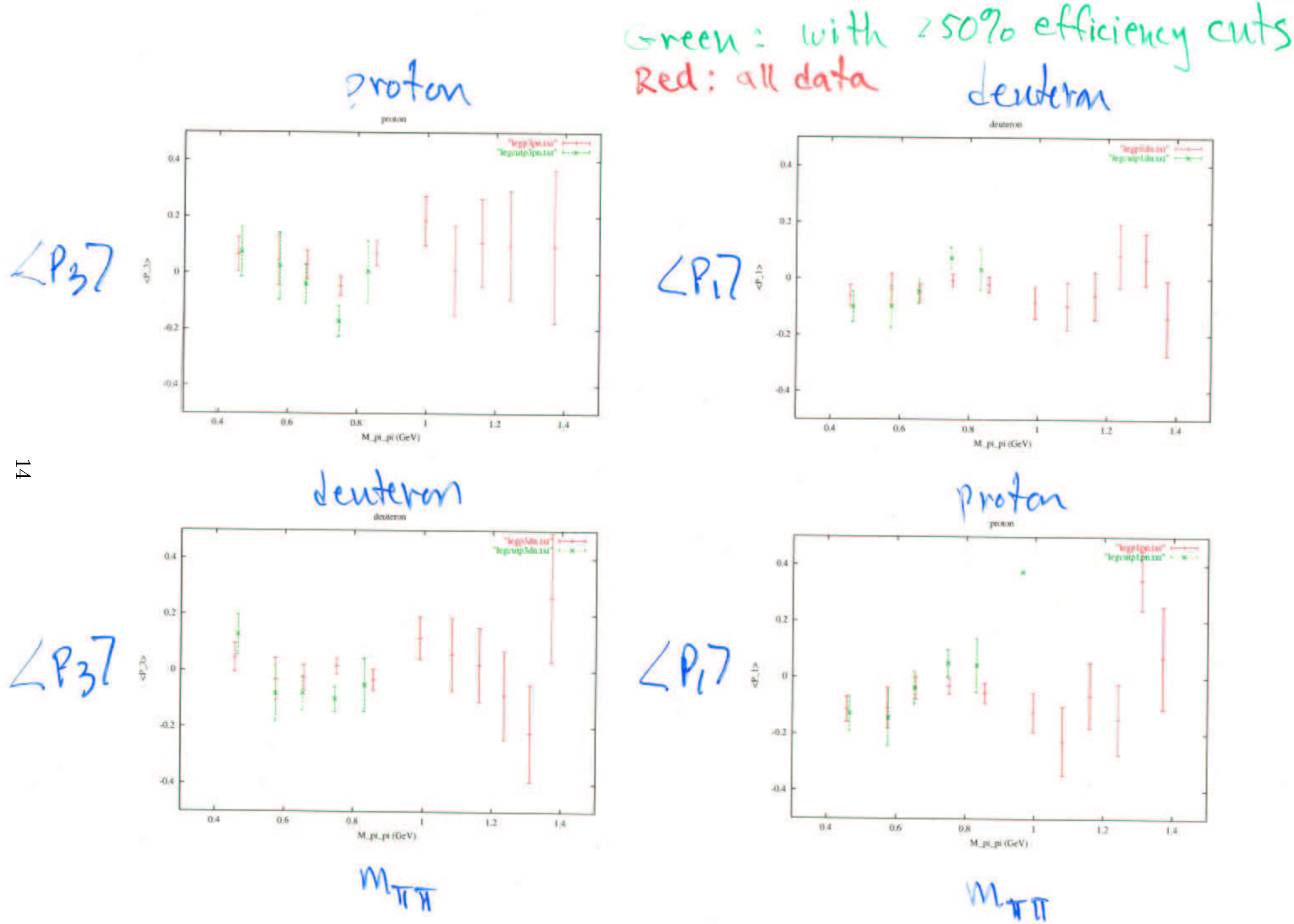

"tmp.tmp1"
"tmp.tmp"
cos θ distribution
Counts
x : q̂ is z-axis
* : N recoil is -z axis } our choice
cos θ

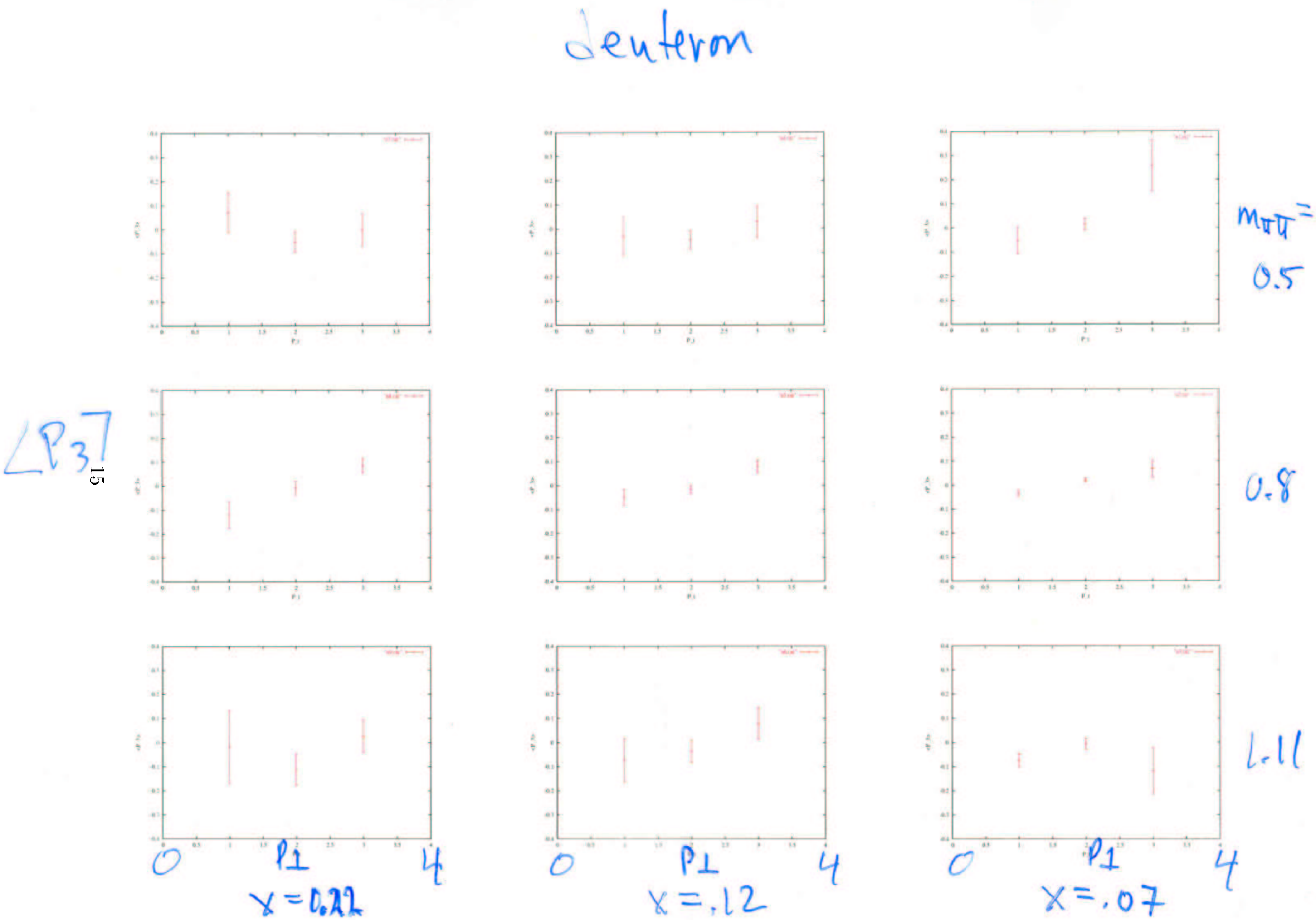

deuteron
proton
proton
deuteron
$\langle P_3 \rangle$
$\langle P_1 \rangle$
$\langle P_3 \rangle$
$\langle P_1 \rangle$
M_pi_pi (GeV)
$m_{\pi\pi}$
$M_{\pi\pi}$
Green : K.G.'s analysis
Red : Riccardo's analysis

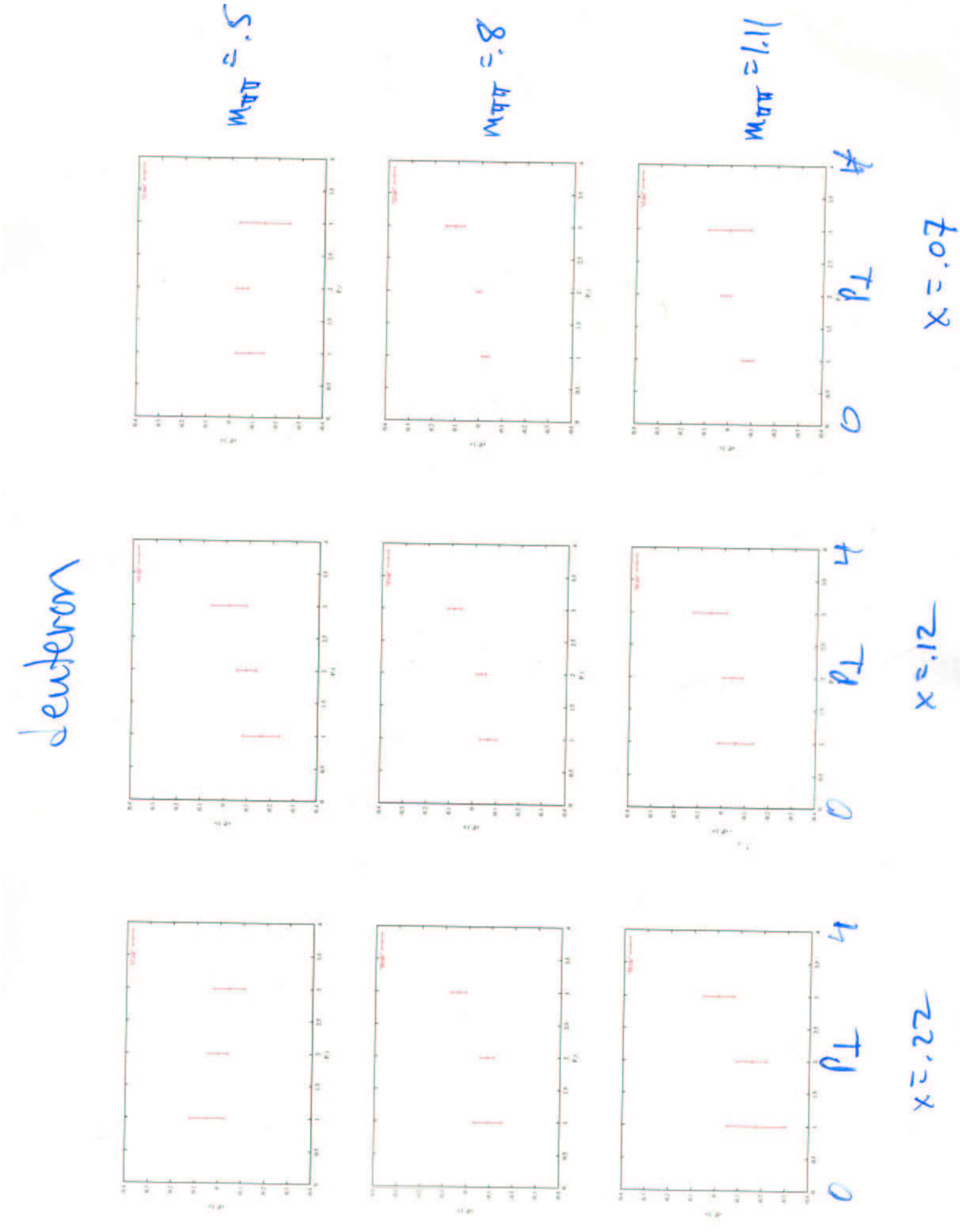




• population in bottom is the same for each point in the x-Q^2 plot


Green: with ≥50% efficiency cuts
Red: all data
proton
"legp3pn.txt"
"legcutp3pn.txt"
<P_3>
M_pi_pi (GeV)
<P3>
deuteron
"legp1dn.txt"
"legcutp1dn.txt"
<P_1>
M_pi_pi (GeV)
<P1>
deuteron
"legp3dn.txt"
"legcutp3dn.txt"
<P_3>
M_pi_pi (GeV)
<P3>
$M_{\pi\pi}$
proton
"legp1pn.txt"
"legcutp1pn.txt"
<P_1>
M_pi_pi (GeV)
<P1>
$M_{\pi\pi}$


deuteron
mππ =
0.5
0.8
1.11
⟨P3⟩
0
P⊥
4
x = 0.22
x = .12
x = .07



deuteron
x=.07
x=.12
x=.22
Td

Conclusions

- Toy Monte Carlo is a nice way to get an understanding of $\langle P_n \rangle$ within HERMES acceptance

- Present estimations of errors in $\langle P_n \rangle$ due to acceptance are small compared to the statistical error bars.

$$\Downarrow$$

- Exclusive $\pi^+\pi^-$ analysis and paper are OK as they presently stand.

- Any future analysis with improved statistics will need to reckon with acceptance corrections to $\langle P_n \rangle$