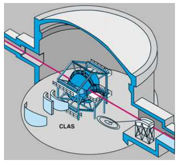

New experimental constraints on the polarizability corrections in the hydrogen hyperfine structure

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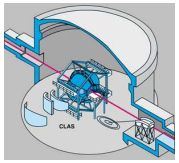
Outline



- Introduction to the world's smallest electron accelerator
- How I became amazed that the hyperfine splittings in hydrogen are not infinite
- Prerequisites:
  Q^2, $x = Q^2/2M\nu$
  Elastic form factors: $G_E(Q^2)$, $G_M(Q^2)$, $F_1(Q^2)$, $F_2(Q^2)$
  Inelastic structure functions: $F_1(x, Q^2)$, $F_2(x, Q^2)$, $g_1(x, Q^2)$, $g_2(x, Q^2)$

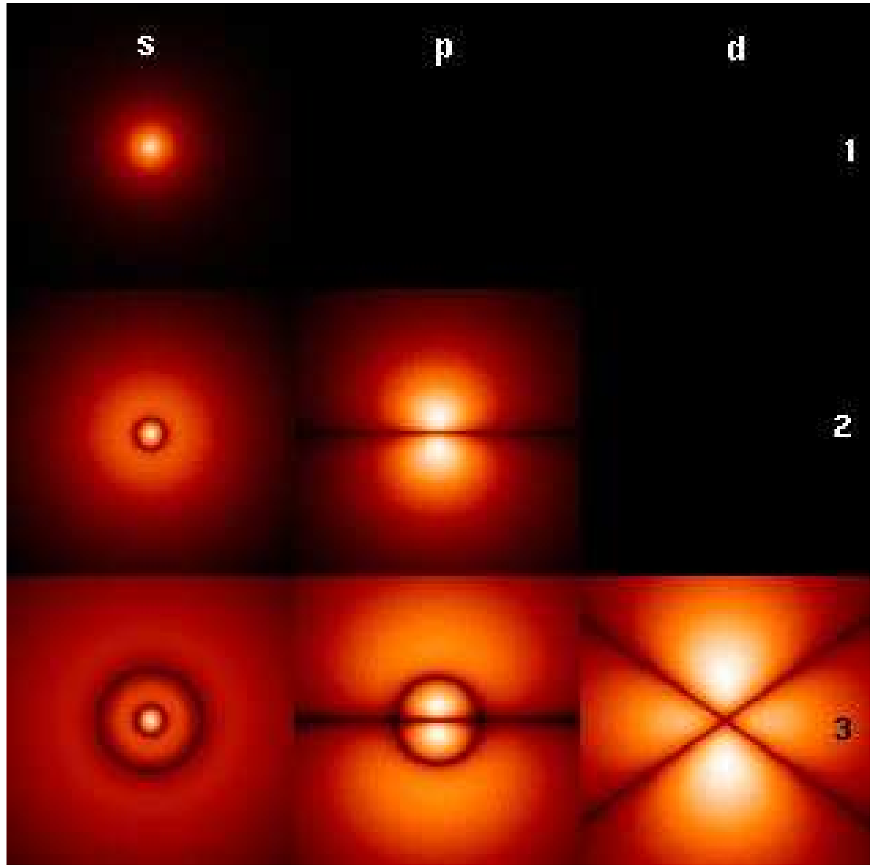
Electron Accelerators

- SLAC: 10^3 m
- JLab, Bonn, Mainz: 10^2 m
- Hydrogen atom: 10^{-10} m

Bohr radius $a = 52918$ fm
proton radius ~ 1 fm
electron momentum $p \sim 4$ keV

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

The electron feels the internal structure of the proton.
Hydrogen is the world's smallest electron accelerator.
Precise measurements of energy levels allows us to do nuclear physics with the atom.



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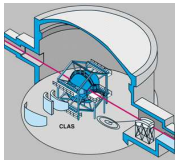
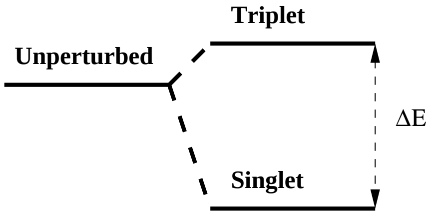
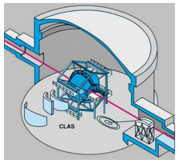
Hyperfine Splittings





- $\vec{\mu}_p = \frac{ge}{2M}\mathbf{S_p}$ and $\vec{\mu}_e = -\frac{e}{m}\mathbf{S_e}$
  $g = 5.59$, M (m) is proton (electron) mass
- $\mathbf{B} = \frac{\mu_0}{4\pi r^3}[3(\vec{\mu}\cdot\hat{r})\hat{r} - \vec{\mu}] + \frac{2\mu_0}{3}\vec{\mu}\delta^3(\mathbf{r})$ (dipole)
  $H = -\vec{\mu}_e \cdot \mathbf{B_p} = \frac{\mu_0 g e^2}{8\pi M m}\frac{[3(\mathbf{S_p}\cdot\hat{\mathbf{r}})(\mathbf{S_e}\cdot\hat{\mathbf{r}}) - \mathbf{S_p}\cdot\mathbf{S_e}]}{r^3} + \frac{\mu_0 g e^2}{3Mm}\mathbf{S_p}\cdot\mathbf{S_e}\delta^3(\mathbf{r})$
  Expectation value of first term vanishes
- $E_{\rm hf} = \frac{\mu_0 g e^2}{3\pi M m a^3}(\mathbf{S_p}\cdot\mathbf{S_e})$ using ψ_{100}
- $\mathbf{S} = \mathbf{S_e} + \mathbf{S_p}$ implies $\mathbf{S_p}\cdot\mathbf{S_e} = \frac{1}{2}(S^2 - S_e^2 - S_p^2) = -\frac{3}{4}(\frac{1}{4})$
  for $S = 0(1)$.
- $\Delta E \equiv E_F^p = \frac{4g\hbar^4}{3Mmc^2a^4} = 5.88\times 10^{-6}$ eV (Fermi energy)
- $\nu = \frac{\Delta E}{h} = 1.420$ GHz; $\lambda = \frac{c}{\nu} = 21$ cm.

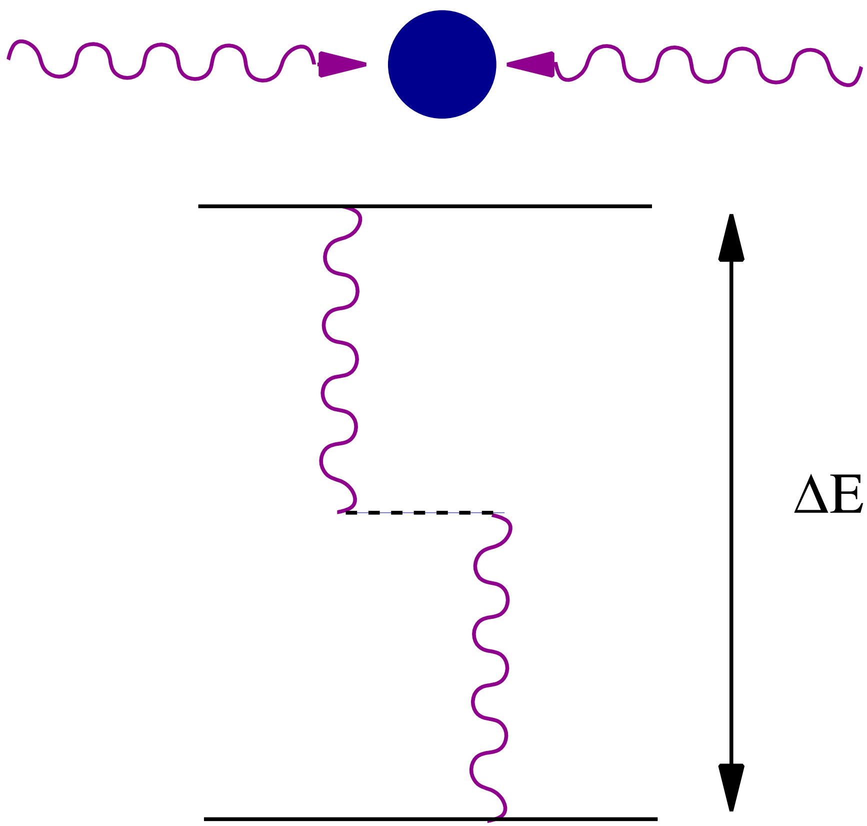
This 21 cm line is ubiquitous in the universe.



Double Photons

- Intense lasers make double photon transitions in atoms possible
- Doppler broadening is greatly reduced with two photons
- A moving atom sees a photon with energy $\gamma(1+\beta)k \approx (1+\beta)k$,but sees a combined two-photon energy of $\gamma 2k' \approx (1+\beta^2/2)2k'$ for $\beta << 1$.Scanned absorption spectrum is much narrower for 2 photons.

Double Photon Absorption

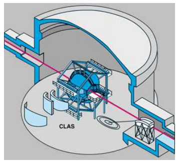


(see F. Biraben *et al.*, Springer LNP**570**(01)17.)

HFS Pieces

- see Brodsky, Carlson, Hiller and Hwang, PRL**94**(05)022001,169902(E).
- $$E_{\rm hfs}(e^- p) = E_F^p(1+\Delta_{\rm QED}+\Delta_R^p+\Delta_S+\Delta_{\rm hvp}^p+\Delta_{\mu\rm vp}^p+\Delta_{\rm weak}^p) = 1.420405751 7667(9)\,{\rm GHz}$$
- $\Delta_{\rm QED} = 1136.09(14)$ ppm ($\frac{\alpha}{2\pi} + \ldots$)
- $\Delta_R^p = 5.86(15)$ ppm (recoil)
- $\Delta_{\rm hvp}^p = 0.01$ ppm (hadronic vacuum polarization)
- $\Delta_{\mu\rm vp}^p = 0.07$ ppm (muonic vacuum polarization)
- $\Delta_{\rm weak}^p = 0.06$ ppm (weak interaction)
- $\Delta_S = -38.62(16)$ ppm (nucleon structure; deduced)
- Δ_S is the largest uncertainty in theoretical calculation of $E_{\rm hfs}(e^- p)$

Chronology for Δ_S



- Zemach, PR104(56)1771, calculates hfs contribution from proton form factors.
- Drell and Sullivan, PR154(67)1477, calculate the polarizability contribution to hydrogen hfs.
- De Rafael, PL37B(71)201, sets bounds on the polarizability.
- Faustov and Martynenko, EPJC24(02)281, estimate polarizability contribution to hydrogen hfs.
- Friar and Sick, PLB579(04)285, determine the Zemach radius from world form factor data.
- Brodsky, Carlson, Hiller and Hwang, PRL94(05) 022001, determine Zemach radius via Faustov.
- The inconsistencies call for an updated determination of the polarizability contribution.

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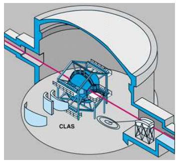
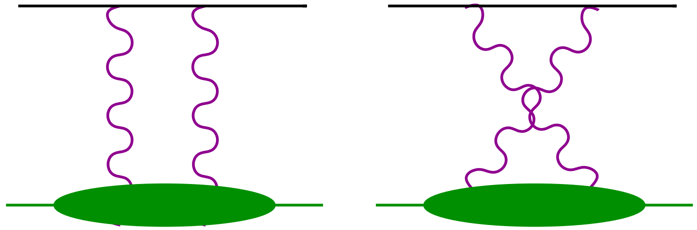
Hyperfine Splitting



- Feynman diagrams for proton polarizability term in the hydrogen hyperfine splitting



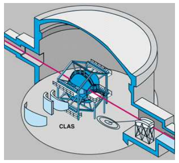
Ground-state hyperfine splittings have been measured to 13-digit accuracy. The largest theoretical uncertainty comes from Δ_S (proton structure).

$$E_{\mathrm{HFS}}(e^- p) = 1.420405751 7667(9)\mathrm{GHz} = (1{+}\Delta_{QED}{+}\Delta_R^p{+}\Delta_S)E_F^p$$

$$E_{\mathrm{HFS}}(e^- \mu^+) = 4.463302765(53)\mathrm{GHz} = (1 + \Delta_{QED} + \Delta_R^\mu)E_F^\mu$$

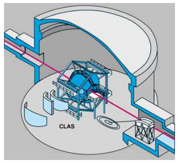
in which the Fermi energy $E_F^N = \frac{8}{3}\alpha^4 \mu_N \frac{m_e^2 m_N^2}{(m_N + m_e)^3}$

Δs



- Brodsky, Carlson, Hiller, Hwang use hydrogen and muonium to extract $\Delta_S = -38.62(16)$ ppm.
- $\Delta_S = \Delta_Z + \Delta_{\rm pol}$
- Zemach: $\Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\rm rad})$
- Karshenboim, PLA**225**(97)97: $\delta_Z^{\rm rad} = \frac{\alpha}{3\pi}[2 \ln \frac{\Lambda^2}{m^2} - \frac{4111}{420}]$
- $\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$
- $\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi(1+\kappa)M} (\Delta_1 + \Delta_2) = (0.2264798 \text{ ppm})(\Delta_1 + \Delta_2)$
- Friar and Sick: $\langle r \rangle_Z = 1.086 \pm 0.012$ fm from experiment. $\Delta_Z = -41.0(5)$ ppm.
- This all would imply that $\Delta_{\rm pol} = 2.38(58)$ ppm.
- Faustov and Martynenko obtain $\Delta_{\rm pol} = 1.4 \pm 0.6$ ppm from a model loosely constrained by SLAC E143 data.

Polarization Terms



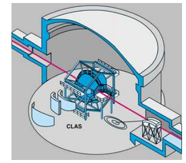
$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) - 4M\int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2}\beta_1(\tau)g_1(\nu,Q^2)\right\}$$

$$\Delta_2 = -12M\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2}\beta_2(\tau)g_2(\nu,Q^2)$$

in which

- $\nu_{\rm th} = m_\pi + \frac{m_\pi^2+Q^2}{2M}$
- $F_2(Q^2)$ is the Pauli form factor
- $\tau = \frac{\nu^2}{Q^2}$
- g_1 and g_2 are the polarized structure functions
- and $\beta_{1,2}$ are kinematic functions

x Integrals



$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8M^2}{Q^2}\int_0^{x_{\rm th}} dx\beta_1(\tau)g_1(x,Q^2)\right\}$$

$$\Delta_2 = -24M^2\int_0^\infty \frac{dQ^2}{Q^4}\int_0^{x_{\rm th}} dx\beta_2(\tau)g_2(x,Q^2)$$

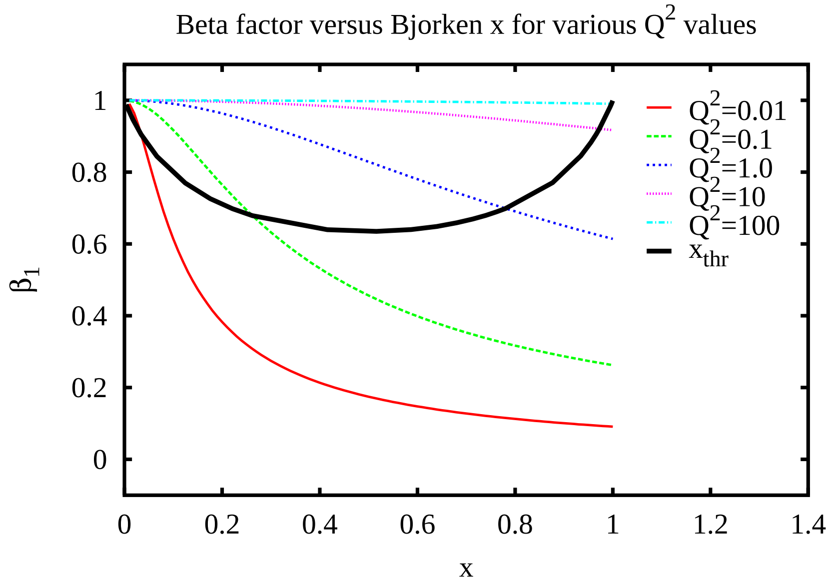
- $x_{\rm th} = \frac{Q^2}{Q^2+m_\pi^2+2Mm_\pi}$
- Advantage: experiments evaluate $\int f(x)g_{1,2}dx$, so error analysis is simplified.
- Disadvantage: large, canceling integrands as $Q^2 \to 0$.

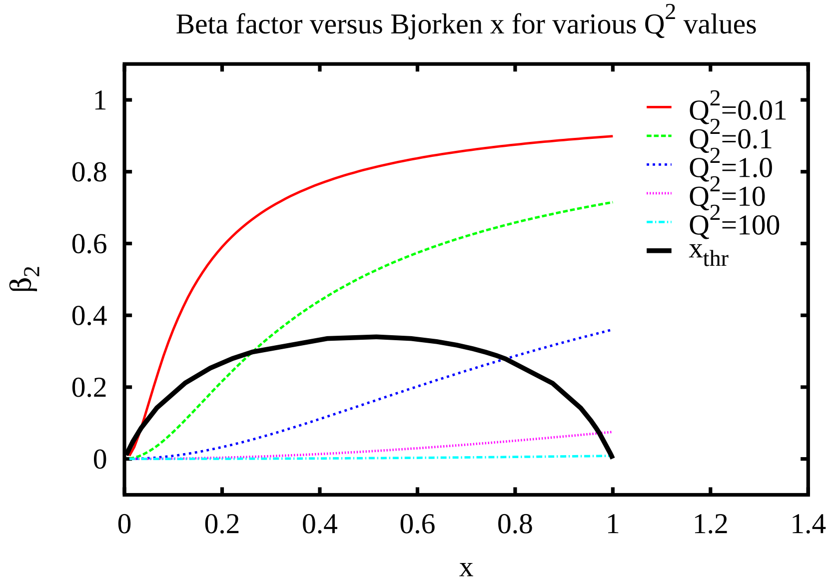
$\beta_1(\tau)$ **and** $\beta_2(\tau)$

- $\tau = \frac{\nu^2}{Q^2} = \frac{Q^2}{4M^2x^2}$
- $\beta_1(\tau) =$
$$\frac{4}{9}\left[-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}\right]$$
- $\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$

- $\beta_1(\tau) \to 0$ as $\tau \to 0$
- $\beta_1(\tau) \to 1$ as $\tau \to \infty$
- $\beta_2(\tau) \to 1$ as $\tau \to 0$
- $\beta_2(\tau) \to 1/4\tau$ as $\tau \to \infty$

- $\int \beta_1 g_1 dx \sim (0.8 - 1.0) \times \Gamma_1$
- $\int \beta_2 g_2 dx \sim (0.0 - 0.2) \times \Gamma_2$

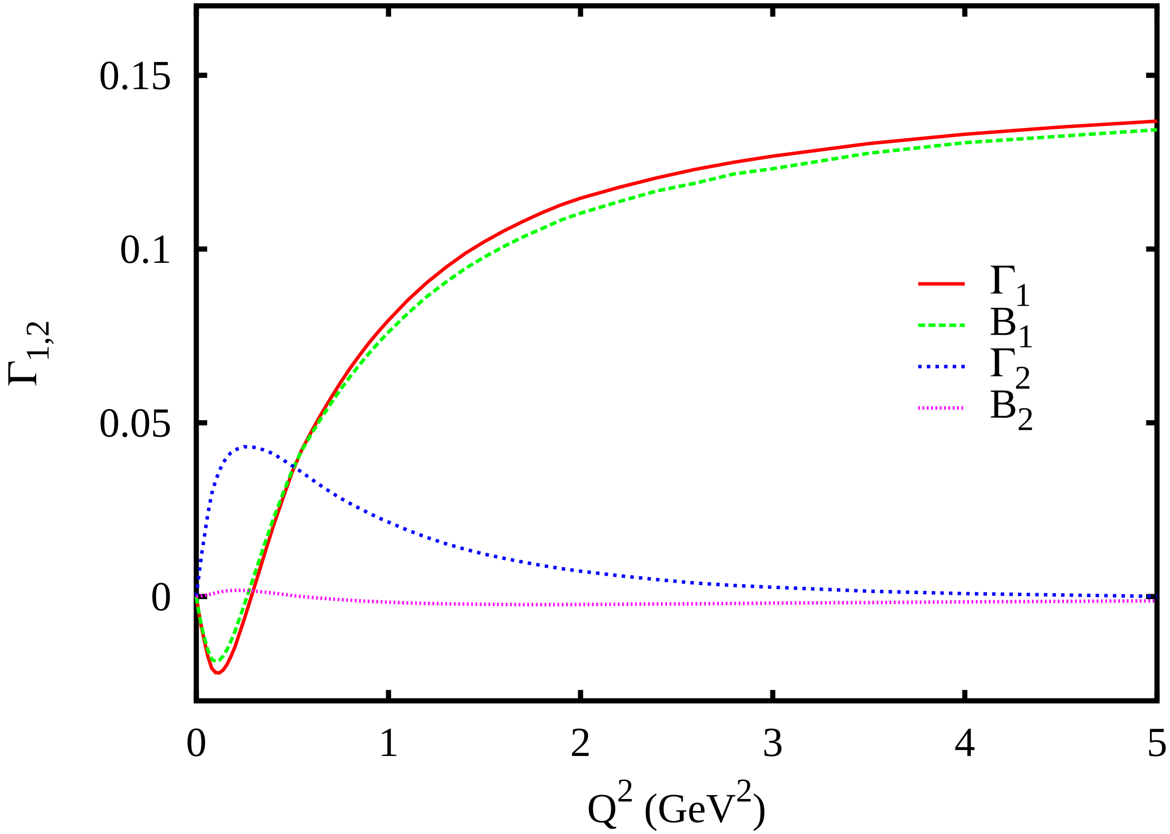




Integrals

Comparisons between $\Gamma_1 = \int g_1 dx$ and $B_1 = \int \beta_1 g_1 dx$ and between $\Gamma_2 = \int g_2 dx$ and $B_2 = \int \beta_2 g_2 dx$

- $B_1 \approx \Gamma_1$
- $B_2 \approx 0$
- Experimentally, errors on Γ_1 are understood; we exploit this fact.
- $\Gamma_2 = \int g_2 dx \neq 0$ at low Q^2.



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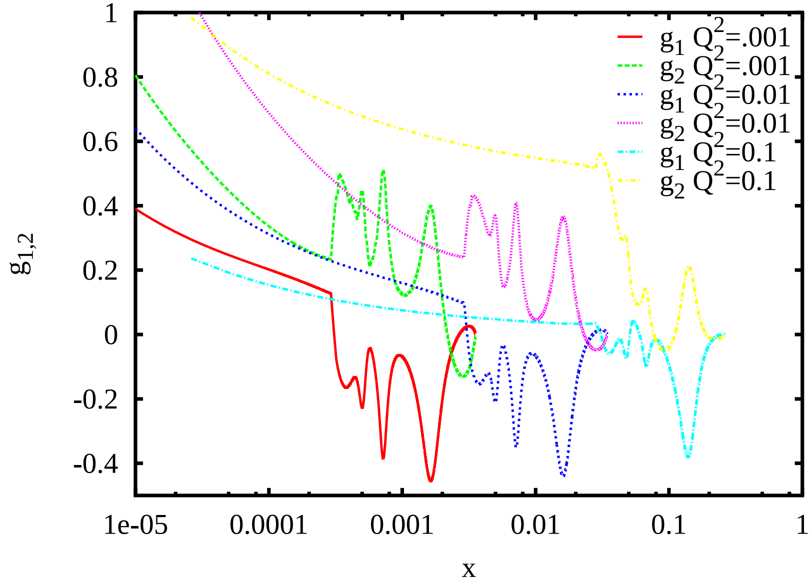
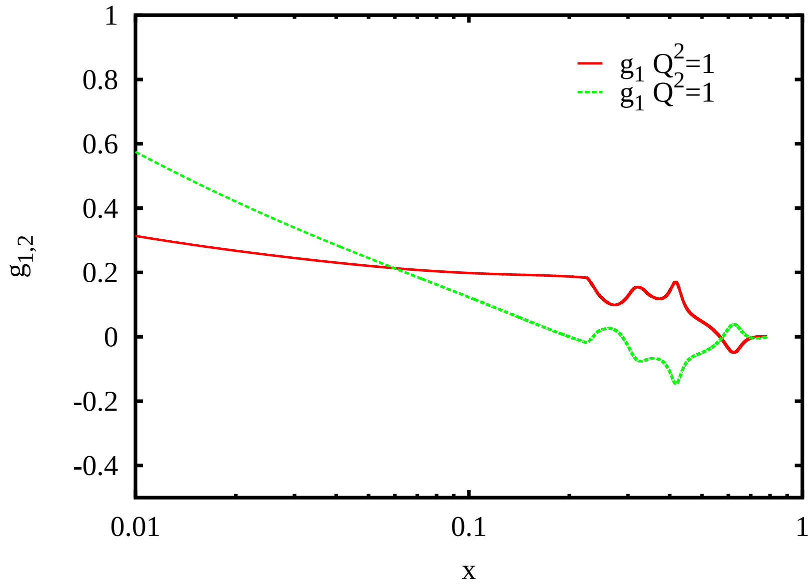
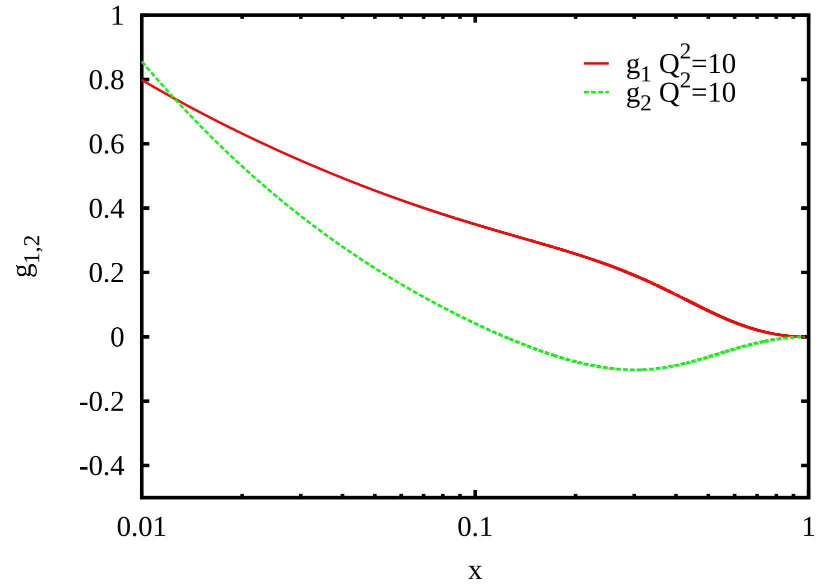
Model g_1 *and* g_2

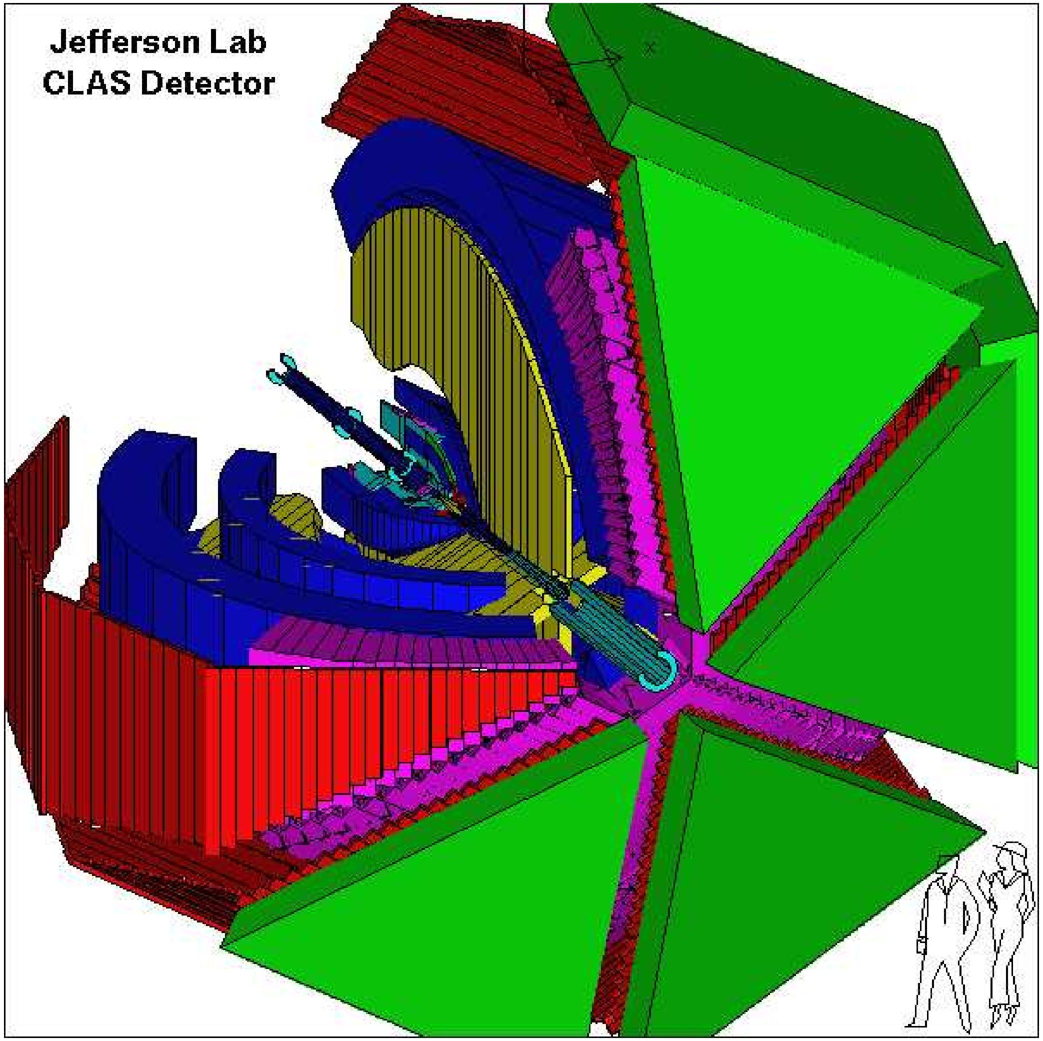


- MAID parameterization in resonance region
- E155 fit in DIS region
- g_2^{WW} in DIS region
- $Q^2 = 0.001, 0.01, 0.1, 1.0, 10.0$



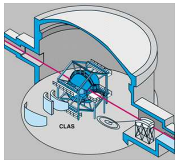


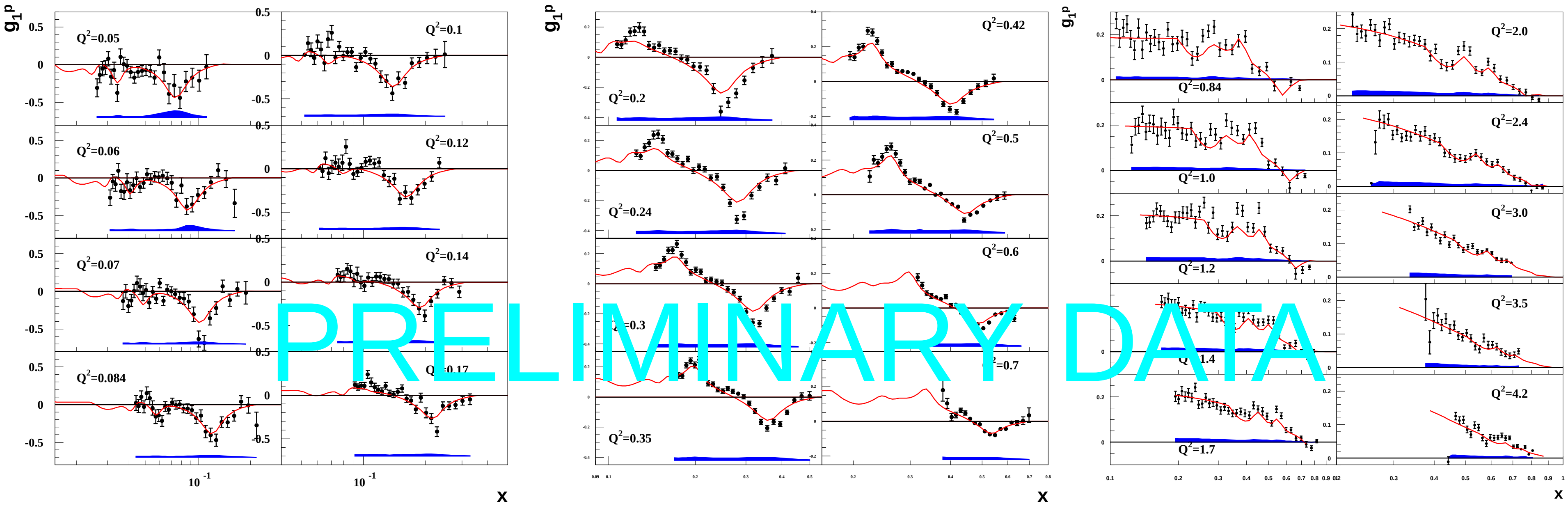
CLAS spectrometer



$E_e = $ 1.2–5.8 GeV
$^{15}NH_3$ and $^{15}ND_3$ targets
Luminosity: 10^{34}/cm^{2}s
green: EM calorimeter
magenta: Cherenkov
red: TOF scintillators
blue: drift chambers
yellow: SC magnet

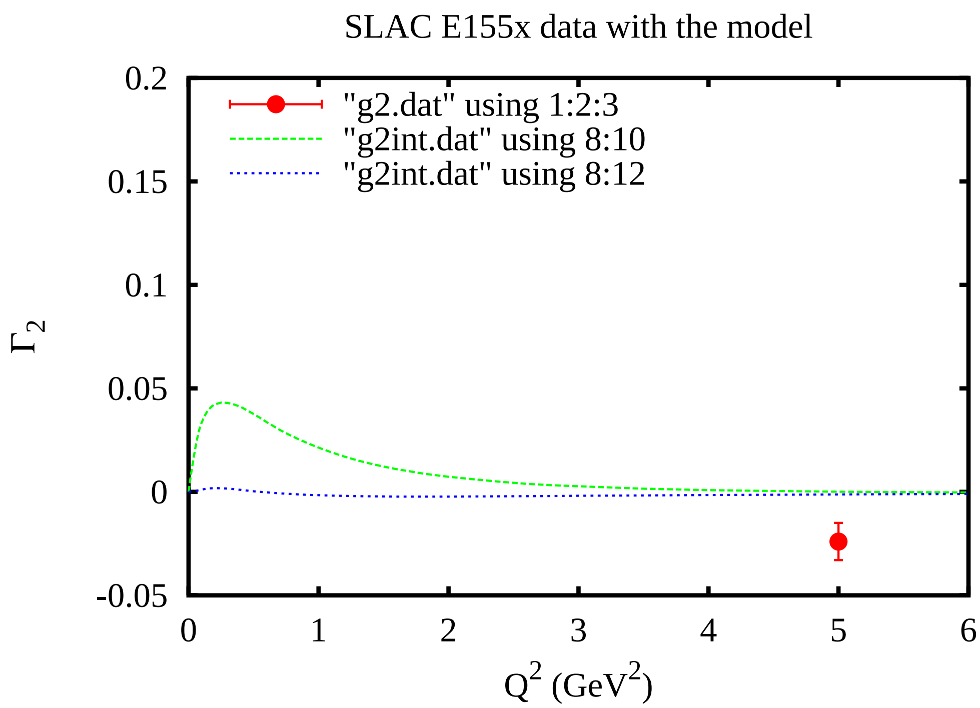
CLAS g_1 with Model

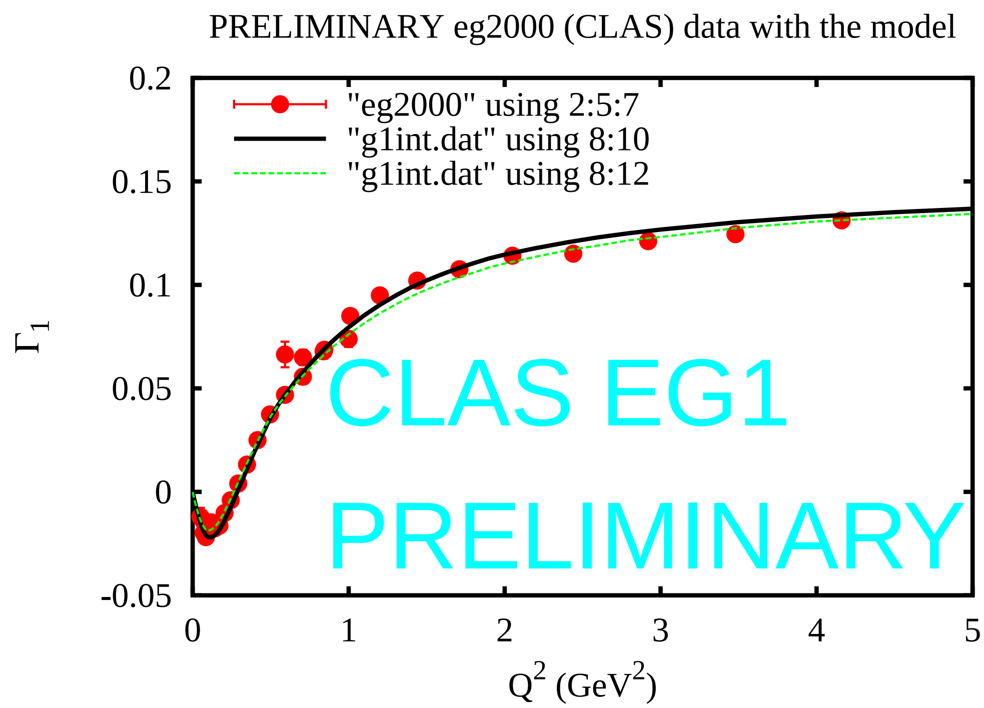




- Preliminary CLAS g_1 data
- $0.05 < Q^2 < 4.2$ GeV^2
- Red line: Model
- Model reproduces the data quite well over the full range kinematics.

$\Gamma_{1,2}$ Data

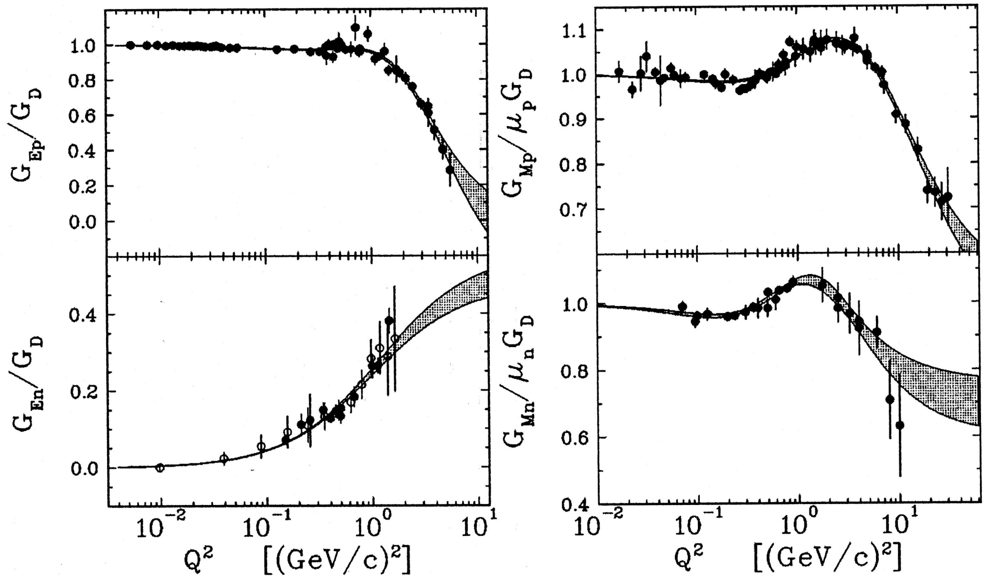




- Left plot: E155x data for $\Gamma_2 = \int g_2(x, Q^2)dx$ with model (green, upper curve) and $B_2 = \int \beta_2 g_2 dx$ (blue, lower curve)
- Right plot: CLAS data for $\Gamma_1 = \int g_1(x, Q^2)dx$ with model (green, upper curve) and $B_1 = \int \beta_1 g_1 dx$ (blue, lower curve)

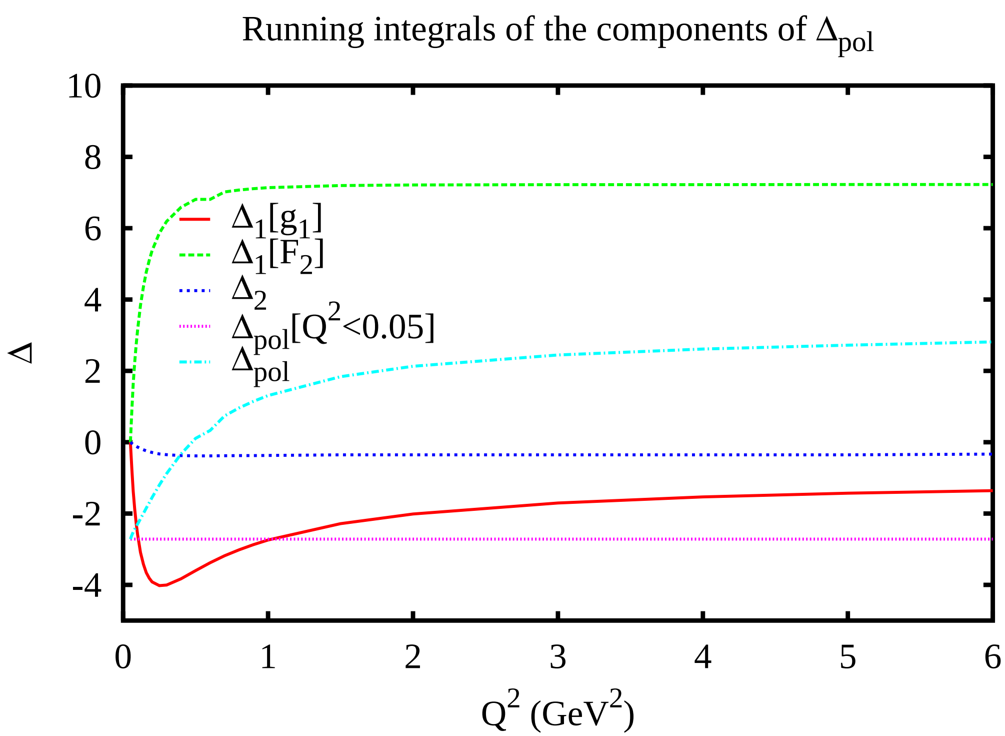
Best Form Factor Fits

- Kelly: PRC**70**(04)068202
  $G(Q^2) = \frac{1+a_1\tau}{1+b_1\tau+b_2\tau^2+b_3\tau_3}$
- Fit has good behavior both at low Q^2 and high Q^2.
- Δ_1 converges with this fit.



Contributions to $\Delta_{\rm pol}$

- Running integrals over Q^2
- Magenta: $\Delta_{\rm pol}$ up to $Q^2 = 0.05$ GeV^2
- Red: $\Delta_1^{g_1}$ for $[0.05, Q^2]$
- Blue: Δ_2 for $[0.05, Q^2]$
- Green: $\Delta_1^{F_2}$ for $[0.05, Q^2]$
- Cyan: $\Delta_{\rm pol} = \Delta_1^{g_1} + \Delta_2 + \Delta_1^{F_2}$



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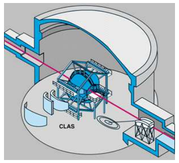
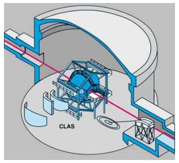
Δ_1 *at low* Q^2



- $G_E = F_1 - \frac{Q^2}{4M^2}F_2$ $\qquad$ $G_M = F_1 + F_2$
- $F_2(0) = \kappa \qquad F_1(0) = 1 \qquad G_E(0) = 1 \qquad G_M(0) = 1 + \kappa$
- $\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2}|_0 \qquad \langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{dG_M(Q^2)}{dQ^2}|_0$
- $\frac{dF_2}{dQ^2}|_0 = \frac{dG_M}{dQ^2}|_0 - \frac{dG_E}{dQ^2}|_0 - \frac{\kappa}{4M^2}$
- Friar and Sick:

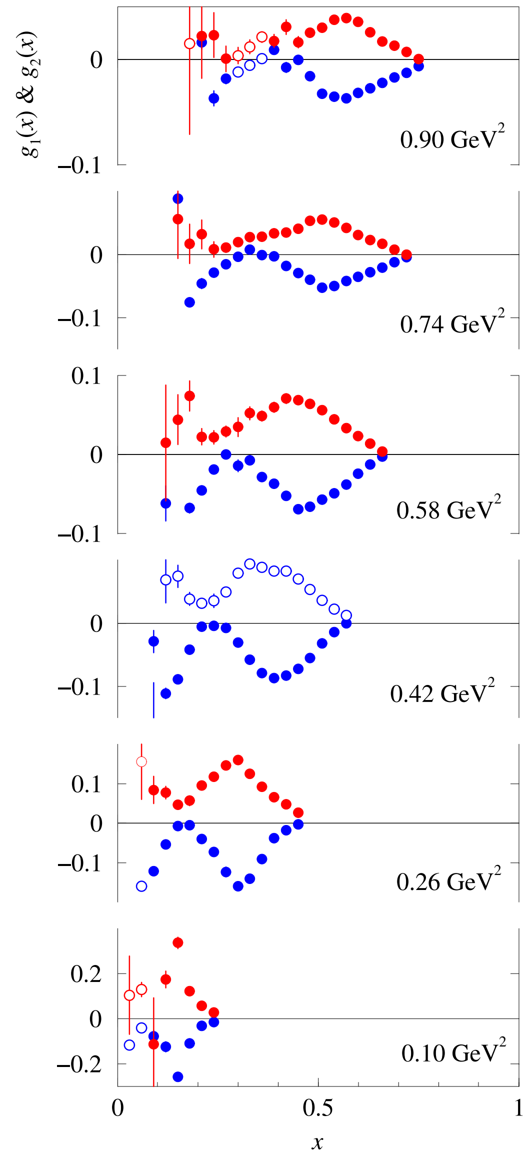
$\langle r_E^2 \rangle = (0.895 \pm 0.018 \text{ fm})^2 \qquad \langle r_M^2 \rangle = (0.855 \pm 0.035 \text{ fm})^2$

- GDH Sum Rule: $\frac{\Gamma_1}{Q^2} = -\frac{\kappa^2}{8M^2}$ as $Q^2 \to 0$
- $\Delta_1^{[0,0.05]} = \frac{9}{4}\int_0^{0.05} \frac{dQ^2}{Q^2}\left\{\kappa^2 + 2\kappa \frac{dF_2}{dQ^2}|_0 Q^2 - \kappa^2\right\}$
- $\kappa = 1.79284739(6) \qquad M = 0.938272029(80)$ GeV
- $\Delta_1^{[0,0.05]} = -2.35 \pm 0.30 \qquad (-2.07)$ in 2nd order
- Bosted form factor fit: $\Delta_1^{[0,0.05]} = -2.44301$



Δ_2 at low Q^2

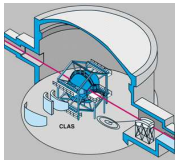
- Hall A ^{3}He data show $g_2 \approx -g_1$ for the neutron at low Q^2.
- $g_1 + g_2 \propto \sigma_{LT}$ which should go to zero as $Q^2 \to 0$.
- $\beta_2(\tau) \to \frac{1}{4\tau}$ as $\tau \to \infty$ with $\tau = \frac{Q^2}{4M^2x^2}$. Therefore, $\beta_2 = 0$ at $x = 0$ and $\beta_2 = \frac{M^2Q^2}{(Q^2+m^2)^2}$ at $x_{\rm th}$, with $m^2 = m_\pi^2 + 2Mm_\pi$
- Take average β_2 and $g_2 = -g_1$
- $\Delta_2^{[0,0.05]} = -24M^2 \int_0^{0.05} \frac{dQ^2}{Q^4} \frac{M^2Q^2}{2(Q^2+m^2)^2} \left(\frac{\kappa^2}{8M^2}Q^2\right)$ $= -2.276$ (numerically incorrect, but integral converges!)



Comments on $\langle r \rangle_Z$

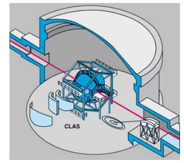
- $\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$
- Unless G_E and G_M go as $1 + \epsilon Q^2$, the Zemach radius diverges.
- Bosted fit, PRC51(95)409: $G_E = 1/(1 + 0.14Q + 3.01Q^2 + 0.02Q^3 + 1.20Q^4 + 0.32Q^5)$ and $G_M = (1 + \kappa)G_E$ fits all data well; yet the Zemach integral diverges.
- JLab fit, ARNPS54(04)217, $(1 + \kappa)G_E/G_M = 1 - 0.13(Q^2 - 0.29)$ yields a divergent $\langle r \rangle_Z$.
- Friar and Sick's analysis assumes a convergent Q^2 dependence (reasonable); however, data alone are consistent with $\langle r \rangle_Z = \infty$.

Results



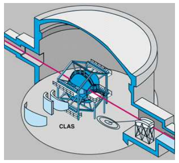
term	Q^2 (GeV^2)	value	component
Δ_1	$[0, 0.05]$	-2.44 ± 1.2	
	$[0.05, 20]$	7.22 ± 0.72	F_2
		-1.10 ± 0.55	g_1
	$[20, \infty]$	0.00 ± 0.01	F_2
		0.12 ± 0.01	g_1
total		3.80 ± 1.5	(3.55 ± 1.27)
Δ_2	$[0, 0.05]$	-0.28 ± 0.28	(Simula/Kelly)
	$[0.05, 20]$	-0.33 ± 0.33	(PRD**65**,034017)
	$[20, \infty]$	0.00 ± 0.01	
total		-0.61 ± 0.61	(-1.86 ± 0.36)
Δ_{pol}		0.72 ± 0.37 ppm	(0.38 ± 0.37)

Comments on Δ_{pol}



- Δ_{pol} is dominated by F_2 with a smaller (canceling) contribution from g_1, and a small contribution from g_2.
- Most of Δ_{pol} comes from $Q^2 < 1$ GeV^2.
- Unless $F_2 \to \kappa + \epsilon Q^2$ and $\Gamma_1 = -\kappa^2 Q^2/8M^2$ (generalized GDH Sum Rule) as $Q^2 \to 0$, Δ_1, Δ_Z diverge.
- If $\Gamma_2 \to \kappa^2 Q^2/8M^2$ ($g_2 = -g_1$ and GDH) as $Q^2 \to 0$, Δ_2 converges.
- $\Delta_{\text{pol}} = 0.7 \pm 0.4$ ppm is small compared to $\Delta_{\text{pol}} = 2.4 \pm 0.6$ ppm from the HFS+Zemach analysis.
- Discrepancy most likely lies in the low-Q^2 dependencies of g_1, g_2, G_E and G_M.

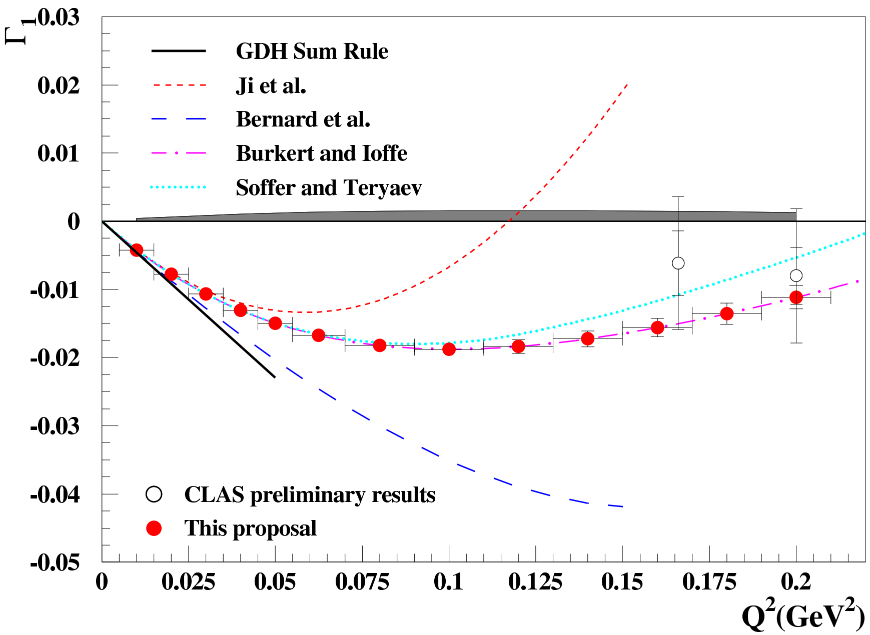
Generalized Sums



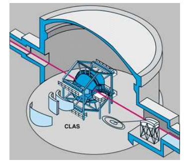
- $\sigma_{1/2} = \frac{4\pi^2\alpha}{KM}(F_1 + g_1 - g_2/\tau)$;$\sigma_{3/2} = \frac{4\pi^2\alpha}{KM}(F_1 - g_1 + g_2/\tau)$
  $\sigma_{LT} = \frac{4\pi^2\alpha}{KM}\frac{Q}{\nu}(g_1 + g_2)$; $\quad W^2 = M^2 + 2MK$; $\quad 1 - x \approx 1$
- $\frac{-\kappa^2}{8M^2} = \frac{\alpha}{16\pi^2}\int_{\nu_{\mathrm{th}}}^{\infty} d\nu \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} = \frac{1}{Q^2}\int_0^{x_{\mathrm{th}}} dx (g_1 - g_2/\tau)$
- Polarizability: $\gamma_0 = -0.94 \pm 0.15 \times 10^{-4}$ fm^4 $=$
  $-\frac{1}{4\pi^2}\int_{\nu_{\mathrm{th}}}^{\infty}\frac{d\nu}{\nu^3}(\sigma_{1/2} - \sigma_{3/2}) = -\frac{16\alpha M^2}{Q^6}\int_0^{x_{\mathrm{th}}} dx x^2 (g_1 - g_2/\tau)$
- $\delta_{LT} = \frac{1}{2\pi^2}\int_{\nu_{\mathrm{th}}}^{\infty}\frac{d\nu}{Q\nu^2}\sigma_{LT} = \frac{16\alpha M^2}{Q^6}\int_0^{x_{\mathrm{th}}} dx x^2 (g_1 + g_2)$
- $\Gamma_1 = -\frac{\kappa^2}{8M^2}Q^2 + \frac{M\delta_{LT}}{4\alpha}Q^4 + \frac{\gamma_0 M}{4\alpha}Q^6 + \ldots =$
  $-0.456Q^2 + 32.144\delta_{LT}Q^4 - 1.993Q^6$
- Fit to data: $\delta_{LT} \approx 1.3 \times 10^{-4}$ fm^4.
- Can the generalized sum rules be extended to $Q^2 = 0.05 - 0.1$? More data will tell.

New Measurements

- CLAS E03-006: The GDH Sum Rule with Nearly Real Photons and the Proton g_1 Structure Function at Low Momentum Transfer
- No measurements of g_2 with transverse targets are planned at CLAS.

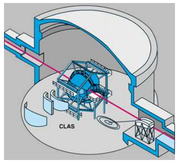


Conclusions



- Determination of $\Delta_{\rm pol}$ can be improved only by precision data for g_1, g_2 and F_2 with $Q^2 < 1$ GeV^2
- The behavior of g_1, g_2, and F_2 for $Q^2 < 0.05$ is crucial, since a large part of $\Delta_{\rm pol}$ comes from this region.
- Although beautiful g_1 data exist from CLAS at JLab over a large kinematic region, the errors on this part are dominated by the lowest Q^2 data.
- Finite hyperfine splittings imply: $\Gamma_1 \to -\kappa^2 Q^2/8M^2$ $g_2 \to -g_1$, $F_2 \to \kappa - \epsilon Q^2$, $G_E \to 1 - \epsilon_E Q^2$, and $G_M/(1+\kappa) \to 1 - \epsilon_M Q^2$ as $Q^2 \to 0$.
- Higher orders (Q^4, Q^6, etc.) are crucial at low Q^2 for an accurate determination of $\Delta_{\rm pol}$.

Questions



- Page 4: Verify that the expectation value of the first term in H is zero.
- Page 8: Show that E_F^N is the same as E_F^p on Page 4.
- Page 11: Derive Δ_1 and Δ_2 from the forms given on Page 10.
- Page 11: Derive the expression for $x_{\rm th}$.
- Page 12: Expand β_1 and β_2 to two terms in τ as $\tau \to 0$ and $\tau \to \infty$.
- Page 25: Derive Γ_1 from the formulas above.
- Extra Credit: Find the mistakes in my formulae.