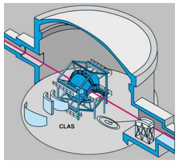

New experimental constraints on the polarizability corrections in the hydrogen hyperfine structure

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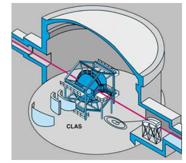
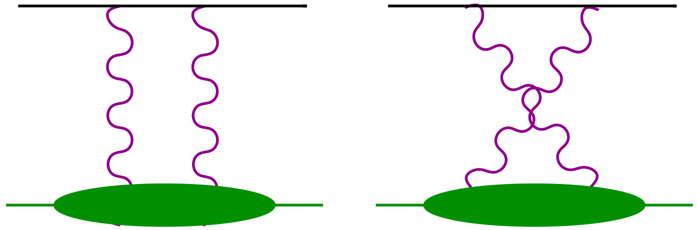
Introduction



- It has long been known that nuclear structure influences hyperfine splittings in atoms.
- Zemach, PR104(56)1771, calculates hfs contribution from proton form factors.
- Drell and Sullivan, PR154(67)1477, calculate the polarizability contribution to hydrogen hfs.
- Faustov and Martynenko, EPJC24(02)281, estimate polarizability contribution to hydrogen hfs.
- Friar and Sick, PLB579(04)285, determine the Zemach radius from world form factor data.
- Brodsky, Carlson, Hiller and Hwang, PRL94(05) 022001, determine Zemach radius via Faustov.
- The inconsistencies call for an updated determination of the polarizability contribution.

Hyperfine Splitting

- Feynman diagrams for proton polarizability term in the hydrogen hyperfine splitting



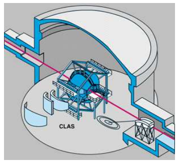
Ground-state hyperfine splittings have been measured to 13-digit accuracy. The largest theoretical uncertainty comes from Δ_S (proton structure).

$$E_{\mathrm{HFS}}(e^- p) = 1.420405751 7667(9)\,\mathrm{GHz} = (1 + \Delta_{QED} + \Delta_R^p + \Delta_S) E_F^p$$

$$E_{\mathrm{HFS}}(e^- \mu^+) = 4.463302765(53)\,\mathrm{GHz} = (1 + \Delta_{QED} + \Delta_R^\mu) E_F^\mu$$

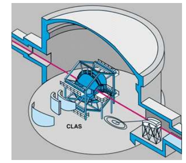
in which the Fermi energy $E_F^N = \frac{8}{3}\alpha^4 \mu_N \frac{m_e^2 m_N^2}{(m_N + m_e)^3}$

Δs



- Brodsky, Carlson, Hiller, Hwang use hydrogen and muonium to extract an experimental $\Delta_S = -37.66(16)$ ppm.
- $\Delta_S = \Delta_Z + \Delta_{\rm pol}$
- Zemach: $\Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\rm rad})$
- $\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$
- $\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi(1+\kappa)M} (\Delta_1 + \Delta_2) = (0.2264798 \text{ ppm})(\Delta_1 + \Delta_2)$
- Friar and Sick: $\langle r \rangle_Z = 1.086 \pm 0.012$ fm from experiment. $\Delta_Z = -41.0(5)$ ppm.
- This all would imply that $\Delta_{\rm pol} = 3.34(58)$ ppm.
- Faustov and Martynenko obtain $\Delta_{\rm pol} = 1.4 \pm 0.6$ ppm from a model loosely constrained by SLAC E143 data.

Polarization Terms



$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2}\left\{\frac{9}{4}F_2^2(Q^2) - 4M\int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2}\bar{\beta}_1(\tau)g_1(\nu,Q^2)\right\}$$

$$\Delta_2 = -12M\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{\rm th}}^\infty \frac{d\nu}{\nu^2}\beta_2(\tau)g_2(\nu,Q^2)$$

in which

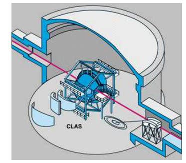
- $\nu_{\rm th} = m_\pi + \frac{m_\pi^2+Q^2}{2M}$
- $F_2(Q^2)$ is the Pauli form factor
- $\tau = \frac{\nu^2}{Q^2}$
- g_1 and g_2 are the polarized structure functions
- and $\beta_{1,2}$ are kinematic functions

x Integrals

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8M^2}{Q^2}\int_0^{x_{\rm th}} dx\beta_1(\tau)g_1(x,Q^2)\right\}$$

$$\Delta_2 = -24M^2\int_0^\infty \frac{dQ^2}{Q^4}\int_0^{x_{\rm th}} dx\beta_2(\tau)g_2(x,Q^2)$$

- $x_{\rm th} = \frac{Q^2}{Q^2+m_\pi^2+2Mm_\pi}$
- Advantage: experiments evaluate $\int f(x)g_{1,2}dx$, so error analysis is simplified.
- Disadvantage: large, canceling integrands as $Q^2 \to 0$.

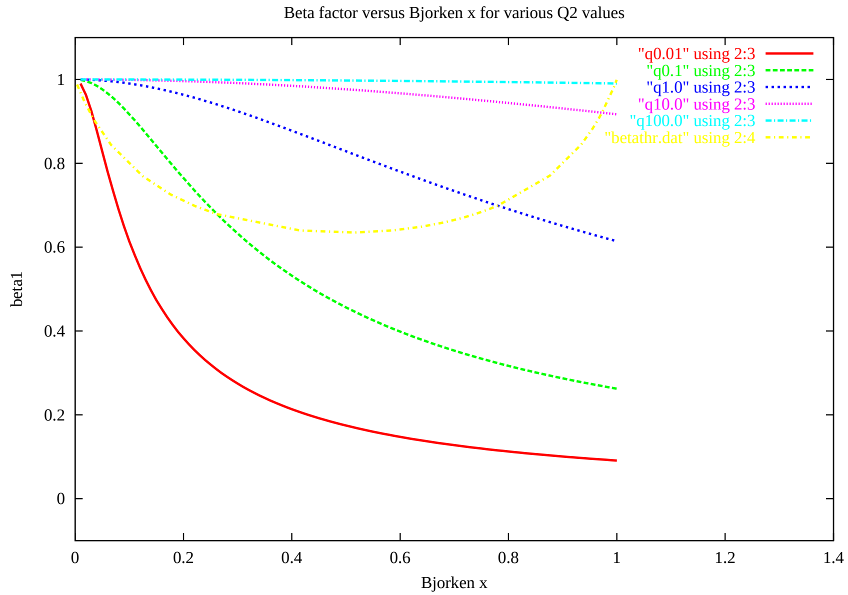


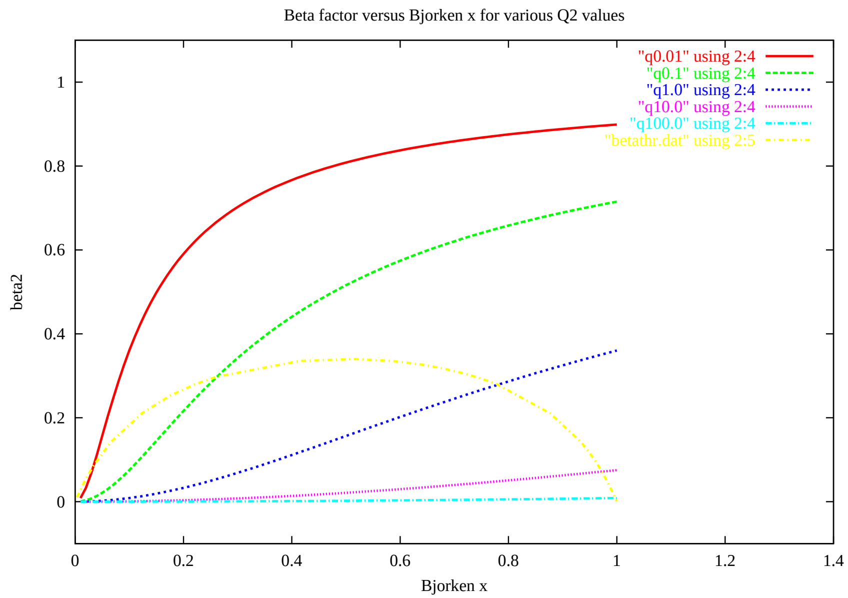
$\beta_1(\tau)$ and $\beta_2(\tau)$

- $\tau = \frac{\nu^2}{Q^2} = \frac{Q^2}{4M^2x^2}$
- $\beta_1(\tau) =$
$\frac{4}{9}\left[-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}\right]$
- $\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$

- $\beta_1(\tau) \to 0$ as $\tau \to 0$
- $\beta_1(\tau) \to 1$ as $\tau \to \infty$
- $\beta_2(\tau) \to 1$ as $\tau \to 0$
- $\beta_2(\tau) \to 1/4\tau$ as $\tau \to \infty$

- $\int \beta_1 g_1 dx \sim (0.8 - 1.0) \times \Gamma_1$
- $\int \beta_2 g_2 dx \sim (0.0 - 0.2) \times \Gamma_2$

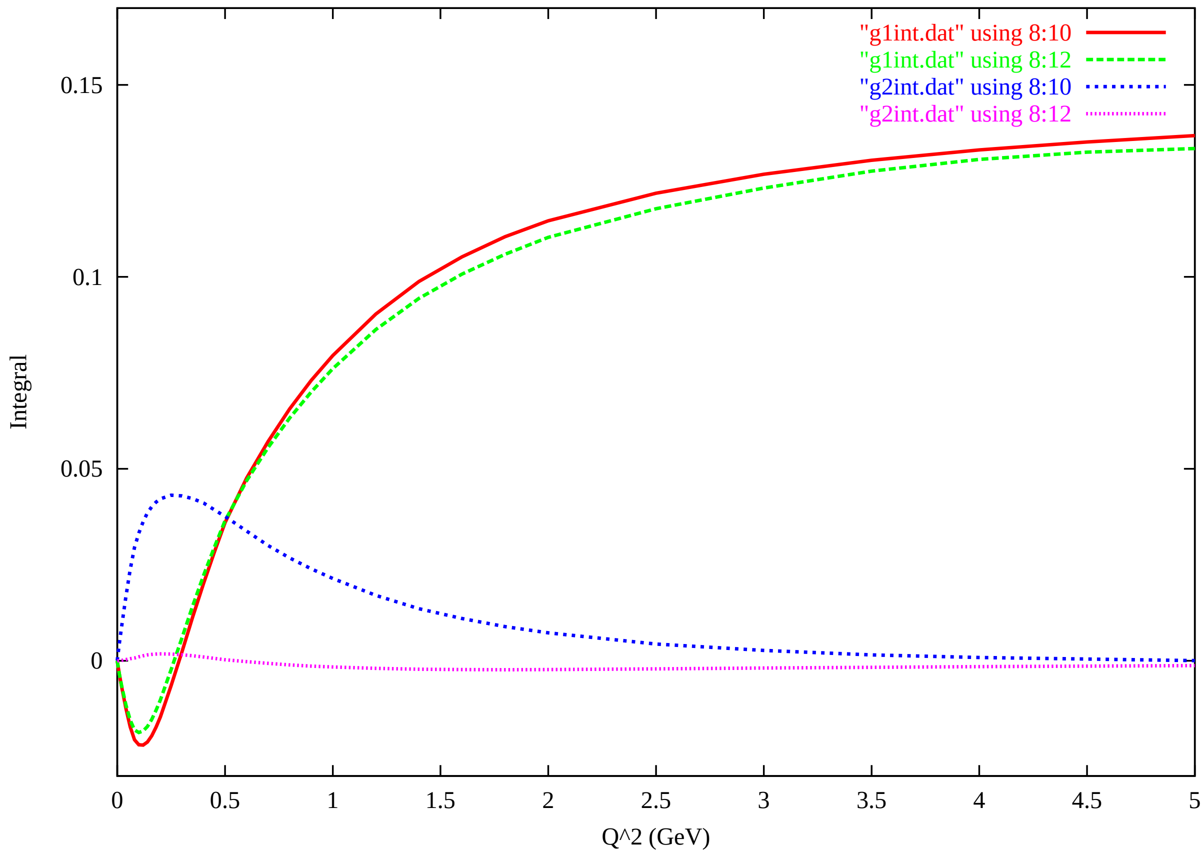


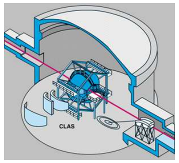


Integrals

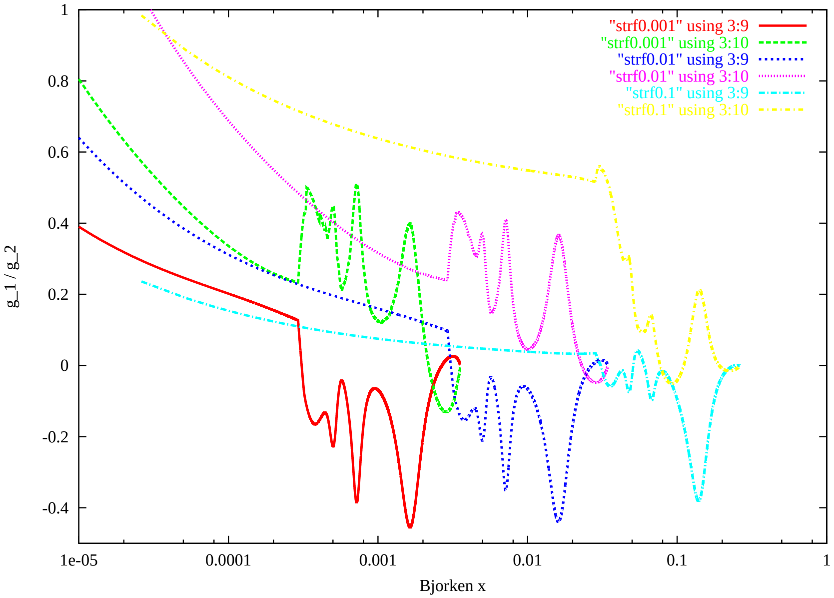
Comparisons between $\Gamma_1 = \int g_1 dx$ and $B_1 = \int \beta_1 g_1 dx$ and between $\Gamma_2 = \int g_2 dx$ and $B_2 = \int \beta_2 g_2 dx$

- $B_1 \approx \Gamma_1$
- $B_2 \approx 0$
- Experimentally, errors on Γ_1 are understood; we exploit this fact.
- $\Gamma_2 = \int g_2 dx \neq 0$ at low Q^2.

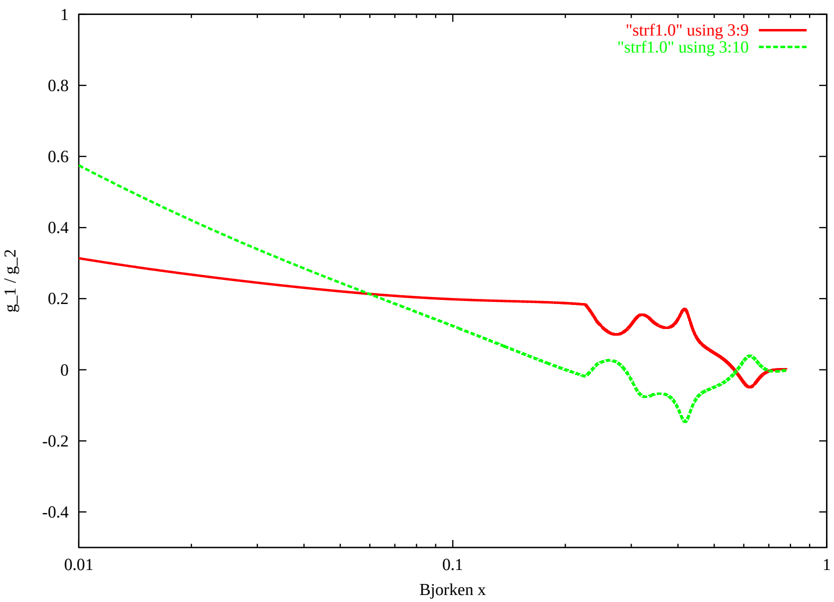


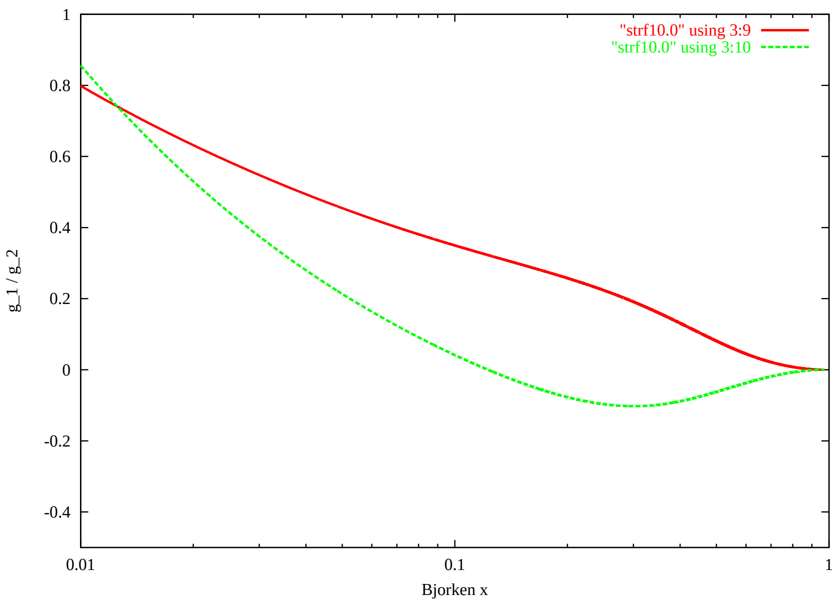


Model g_1 *and* g_2

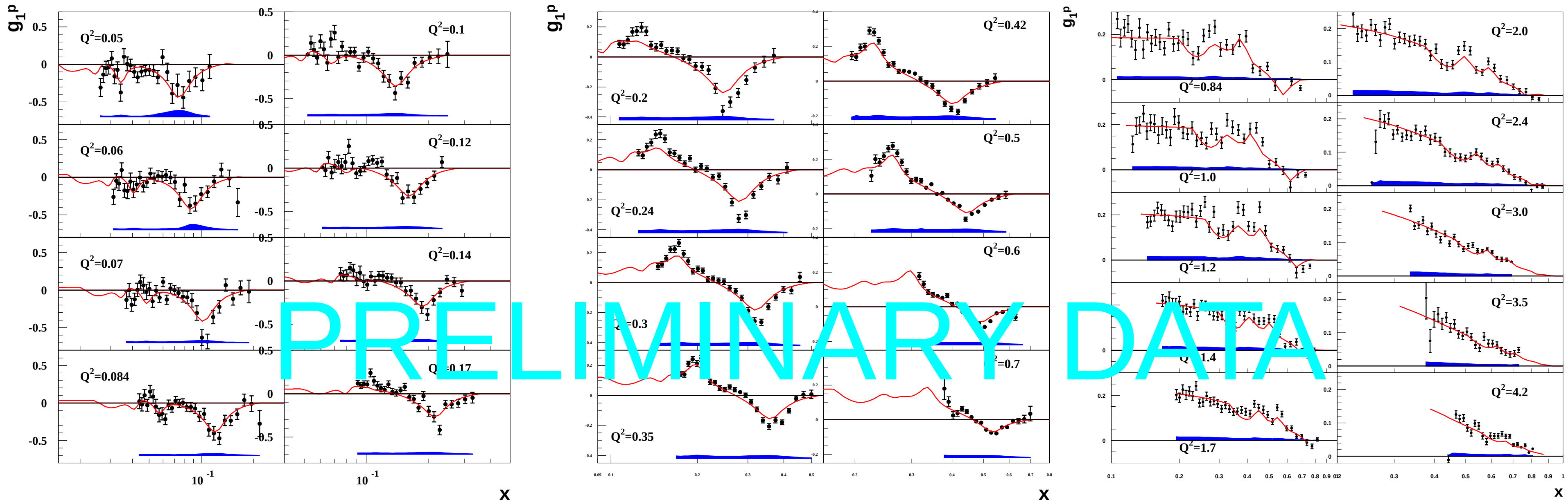


- MAID parameterization in resonance region
- E155 fit in DIS region
- g_2^{WW} in DIS region
- $Q^2 = 0.001, 0.01, 0.1, 1.0, 10.0$



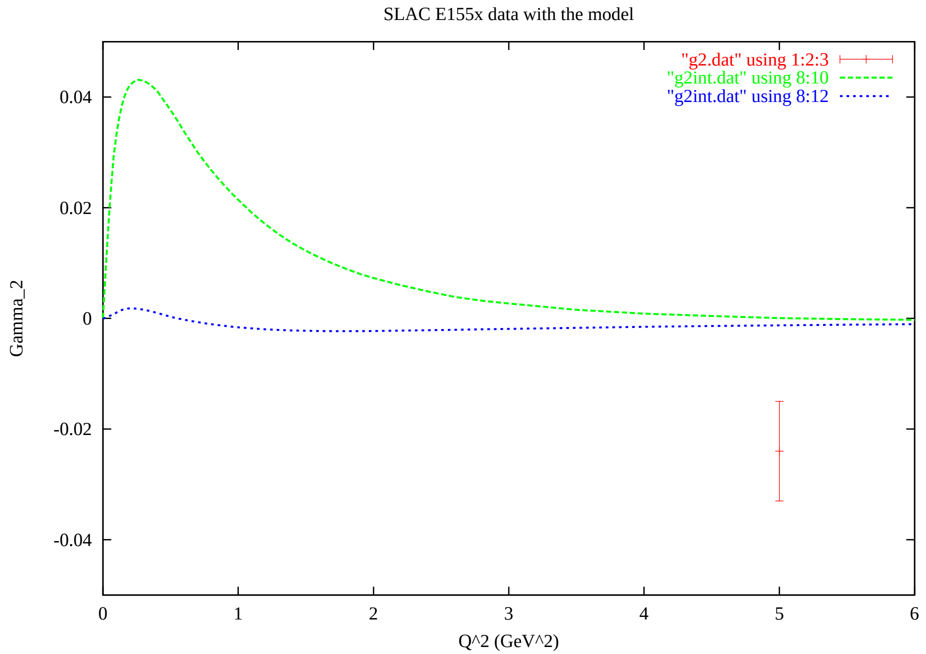


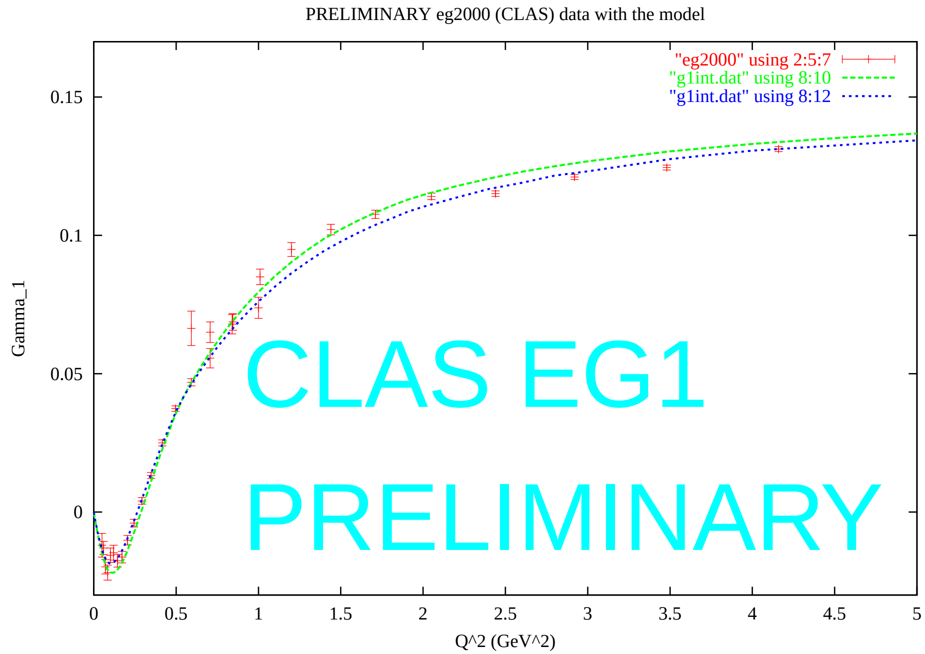
CLAS g_1 with Model



- Preliminary CLAS g_1 data
- $0.05 < Q^2 < 4.2$ GeV^2
- Red line: Model
- Model reproduces the data quite well over the full range kinematics.

$\Gamma_{1,2}$ **Data**

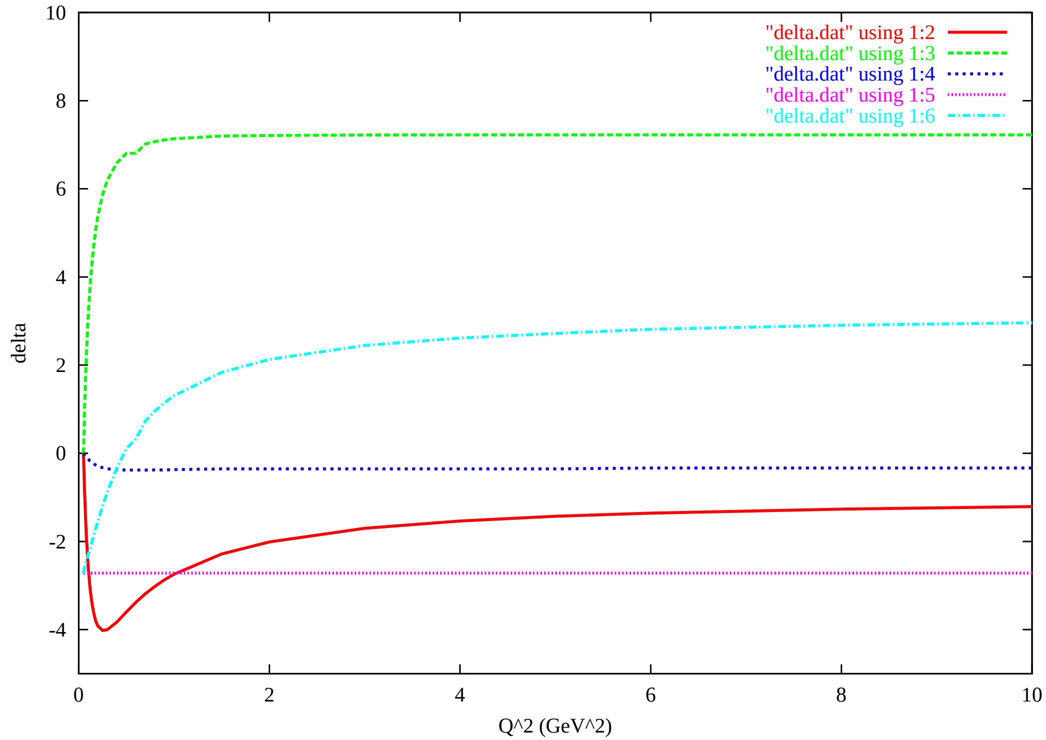




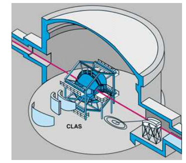
- Left plot: E155x data for $\Gamma_2 = \int g_2(x, Q^2)dx$ with model (green, upper curve) and $B_2 = \int \beta_2 g_2 dx$ (blue, lower curve)
- Right plot: CLAS data for $\Gamma_1 = \int g_1(x, Q^2)dx$ with model (green, upper curve) and $B_1 = \int \beta_1 g_1 dx$ (blue, lower curve)

Contributions to $\Delta_{\rm pol}$

- Running integrals over Q^2
- Magenta: $\Delta_{\rm pol}$ up to $Q^2 = 0.05$ GeV^2
- Red: $\Delta_1^{g_1}$ for $[0.05, Q^2]$
- Blue: Δ_2 for $[0.05, Q^2]$
- Green: $\Delta_1^{F_2}$ for $[0.05, Q^2]$
- Cyan: $\Delta_{\rm pol} = \Delta_1^{g_1} + \Delta_2 + \Delta_1^{F_2}$



Δ_1 *at low* Q^2



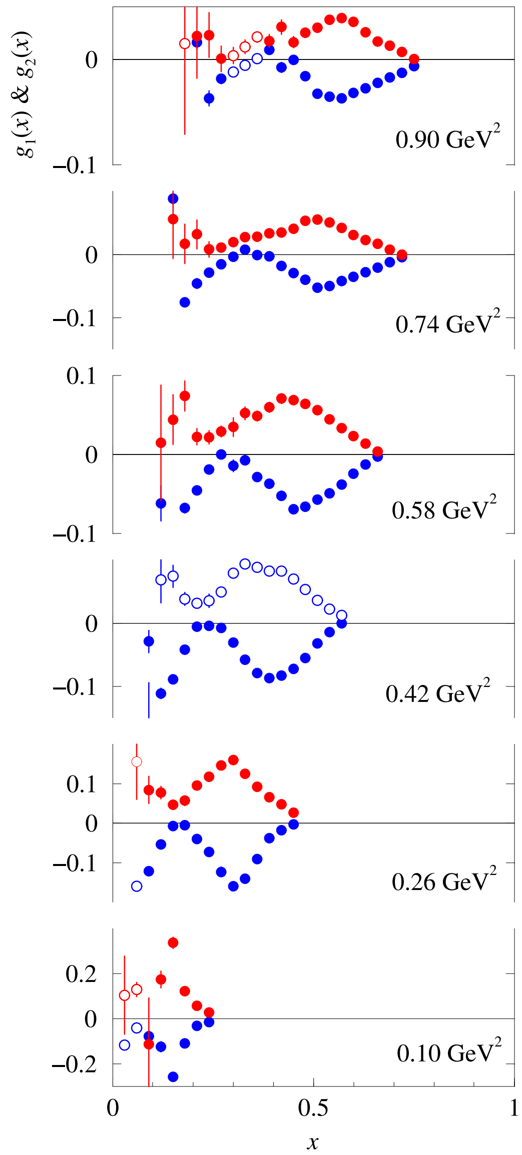
- $G_E = F_1 - \frac{Q^2}{4M^2} F_2$ $\qquad$ $G_M = F_1 + F_2$
- $F_2(0) = \kappa$ $\qquad$ $F_1(0) = 1$ $\qquad$ $G_E(0) = 1$ $\qquad$ $G_M(0) = 1 + \kappa$
- $\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2}|_0$ $\qquad$ $\langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{dG_M(Q^2)}{dQ^2}|_0$
- $\frac{dF_2}{dQ^2}|_0 = \frac{dG_M}{dQ^2}|_0 - \frac{dG_E}{dQ^2}|_0 - \frac{\kappa}{4M^2}$
- Friar and Sick:

$\langle r_E^2 \rangle = (0.895 \pm 0.018 \text{ fm})^2$ $\qquad$ $\langle r_M^2 \rangle = (0.855 \pm 0.035 \text{ fm})^2$

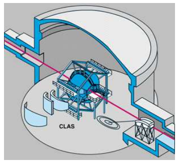
- GDH Sum Rule: $\frac{\Gamma_1}{Q^2} = -\frac{\kappa^2}{8M^2}$ as $Q^2 \to 0$
- $\Delta_1^{[0,0.05]} = \frac{9}{4} \int_0^{0.05} \frac{dQ^2}{Q^2} \left\{ \kappa^2 + 2\kappa \frac{dF_2}{dQ^2}|_0 Q^2 - \kappa^2 \right\}$
- $\kappa = 1.79284739(6)$ $\qquad$ $M = 0.938272029(80)$ GeV
- $\Delta_1^{[0,0.05]} = -2.35 \pm 0.30$ $\qquad$ (-2.07) in 2nd order
- Bosted form factor fit: $\Delta_1^{[0,0.05]} = -2.44301$

Δ_2 at low Q^2

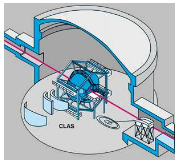
- Hall A ^{3}He data show $g_2 \approx -g_1$ for the neutron at low Q^2.
- $g_1 + g_2 \propto \sigma_{LT}$ which should go to zero as $Q^2 \to 0$.
- $\beta_2(\tau) \to \frac{1}{4\tau}$ as $\tau \to \infty$ with $\tau = \frac{Q^2}{4M^2x^2}$. Therefore, $\beta_2 = 0$ at $x = 0$ and $\beta_2 = \frac{M^2Q^2}{(Q^2+m^2)^2}$ at $x_{\rm th}$, with $m^2 = m_\pi^2 + 2Mm_\pi$
- Take average β_2 and $g_2 = -g_1$
- $\Delta_2^{[0,0.05]} =$ $-24M^2 \int_0^{0.05} \frac{dQ^2}{Q^4} \frac{M^2Q^2}{2(Q^2+m^2)^2} \left(\frac{\kappa^2}{8M^2}Q^2\right)$ $= -2.276$ (numerically incorrect, but integral converges!)



Comments on $\langle r \rangle_Z$



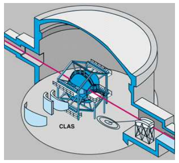
- $\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$
- Unless G_E and G_M go as $1 + \epsilon Q^2$, the Zemach radius diverges.
- Bosted fit, PRC51(95)409:
$G_E = 1/(1 + 0.14Q + 3.01Q^2 + 0.02Q^3 + 1.20Q^4 + 0.32Q^5)$ and $G_M = (1+\kappa)G_E$ fits all data well; yet the Zemach integral diverges.
- JLab fit, ARNPS54(04)217,
$(1+\kappa)G_E/G_M = 1 - 0.13(Q^2 - 0.29)$ yields a divergent $\langle r \rangle_Z$.
- Friar and Sick's analysis assumes a convergent Q^2 dependence (reasonable); however, data alone are consistent with $\langle r \rangle_Z = \infty$.



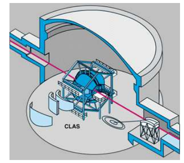
Results

term	Q^2 (GeV^2)	value	component
Δ_1	$[0, 0.05]$	-2.44 ± 1.2	
	$[0.05, 20]$	7.22 ± 0.72	F_2
		-1.10 ± 0.55	g_1
	$[20, \infty]$	0.00 ± 0.01	F_2
		0.12 ± 0.01	g_1
total		3.80 ± 1.5	(3.55 ± 1.27)
Δ_2	$[0, 0.05]$	-0.28 ± 0.28	
	$[0.05, 20]$	-0.33 ± 0.33	
	$[20, \infty]$	0.00 ± 0.01	
total		-0.61 ± 0.61	(-1.86 ± 0.36)
Δ_{pol}		0.72 ± 0.37 ppm	(0.38 ± 0.37)

Comments on $\Delta_{\rm pol}$



- $\Delta_{\rm pol}$ is dominated by F_2 with a smaller (canceling) contribution from g_1, and a small contribution from g_2.
- Most of $\Delta_{\rm pol}$ comes from $Q^2 < 1$ GeV^2.
- Unless $F_2 \to \kappa + \epsilon Q^2$ and $\Gamma_1 = -\kappa^2 Q^2/8M^2$ (generalized GDH Sum Rule) as $Q^2 \to 0$, Δ_1, Δ_Z diverge.
- If $\Gamma_2 \to \kappa^2 Q^2/8M^2$ ($g_2 = -g_1$ and GDH) as $Q^2 \to 0$, Δ_2 converges.
- $\Delta_{\rm pol} = 0.7 \pm 0.4$ ppm is small compared to $\Delta_{\rm pol} = 3.3 \pm 0.6$ ppm from the HFS+Zemach analysis.
- Discrepancy most likely lies in the low-Q^2 dependencies of g_1, g_2, G_E and G_M.



Conclusions

- Determination of $\Delta_{\rm pol}$ can be improved only by precision data for g_1, g_2 and F_2 with $Q^2 < 1$ GeV^2
- The behavior of g_1, g_2, and F_2 for $Q^2 < 0.05$ is crucial, since a large part of $\Delta_{\rm pol}$ comes from this region.
- Although beautiful g_1 data exist from CLAS at JLab over a large kinematic region, the errors on this part are dominated by the lowest Q^2 data.
- Finite hyperfine splittings imply: $\Gamma_1 \to -\kappa^2 Q^2/8M^2$ $g_2 \to -g_1$, $F_2 \to \kappa - \epsilon Q^2$, $G_E \to 1 - \epsilon_E Q^2$, and $G_M/(1+\kappa) \to 1 - \epsilon_M Q^2$ as $Q^2 \to 0$.
- Higher orders (Q^4, Q^6, etc.) are crucial at low Q^2 for an accurate determination of $\Delta_{\rm pol}$.