Nucleon Spin Structure from Confinement to Asymptotic Freedom

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INTRODUCTION

The nucleon structure functions $F_1$, $F_2$, $g_1$ and $g_2$ depend on two Lorentz invariants. Although well-defined over all kinematics, their interpretation varies with momentum-transfer scale. They have been actively measured over several decades at a number of labs around the world.
Inelastic Scattering

Unpolarized Cross Section:

\[
\frac{d^2\sigma}{dxdQ^2} = \frac{8\pi\alpha^2y}{Q^4} \left[ y \frac{F_1}{2} + \frac{\xi}{2xy} F_2 \right]
\]

Polarized Cross Section:

\[
\frac{d^2\Delta\sigma}{dxdQ^2} = \frac{8\pi\alpha^2y}{Q^4} \left[ \cos \alpha \left\{ \left( \xi + \frac{y}{2} \right) g_1 - \frac{\gamma y}{2} g_2 \right\} - \sin \alpha \cos \phi \left\{ \frac{y}{2} \left( g_1 + g_2 \right) \right\} \right]
\]

\(\alpha\) = polar angle of target spin wrt the beam axis
\(\phi\) = azimuthal spin angle wrt the scattering plane

\(\alpha = 0^\circ\) (longitudinal); \(\alpha = 90^\circ, \phi = 0^\circ\) (transverse).

\(\gamma^2 = 4M^2x^2/Q^2 = Q^2/\nu^2\)

\(\xi = 1 - y - \gamma y^2/4\)

Parton Model:

\(F_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2(p^+(x) + q^+(x) + \bar{q}^+(x) + \bar{q}^+(x))\)

\(F_2(x, Q^2) = 2x F_1(x, Q^2)\)

\(g_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2(p^+(x) - q^+(x) + \bar{q}^+(x) - \bar{q}^+(x))\)
$F_2^p(x,Q^2)$ and $g_1^p(x,Q^2)$
Integrals over $x$

**Gottfried Sum Rule**

\[ \Phi_{1}^{p,n}(Q^2) = \int_{0}^{1} F_{1}^{p,n}(x, Q^2) dx \]

\[ F_{1}(x) = \frac{1}{2} \sum_{i} e_{i}^{2} q_{i}(x) \]

\[ \Phi_{1}^{p} - \Phi_{1}^{n} = \frac{1}{6} [u_{v} - d_{v} + 2\bar{u} - 2\bar{d}] \]

0.235(26) at $Q^2=4 \text{ GeV}^2$

**Bjorken Sum Rule**

\[ \Gamma_{1}^{p,n}(Q^2) = \int_{0}^{1} g_{1}^{p,n}(x, Q^2) dx \]

\[ g_{1}(x) = \frac{1}{2} \sum_{i} e_{i}^{2} \Delta q_{i}(x) \]

\[ \Gamma_{1}^{p} - \Gamma_{1}^{n} = \frac{1}{6} [\Delta u_{v} - \Delta d_{v} + 2\Delta \bar{u} - 2\Delta \bar{d}] \]

0.176(7) at $Q^2=5 \text{ GeV}^2$

**Complicating Factor**

\[ \Delta C_{NS}^{\bar{MS}} = 1 - \frac{\alpha_{S}}{\pi} - 3.583 \left( \frac{\alpha_{S}}{\pi} \right)^2 - 20.215 \left( \frac{\alpha_{S}}{\pi} \right)^3 + \ldots \]
regions of $Q^2$

scaling: $\ln Q^2$

higher twist: $(1/Q^2)^n$

no nice expansion

$\chi^\text{PT}: (Q^2)^n$
The Q^2 Map

- Infinite Q^2 Parton Model, PDF(x)
- Large Q^2 DIS, pQCD, PDF(x,log(Q^2))
- Medium Q^2 Higher twist, target mass correct.
- Low Q^2 Resonances (complexity)
- Tiny Q^2 Chiral perturbation theory
- Zero Q^2 Real photons

- Complexity, as measured by \( \gamma_0, \delta_{LT}, d_2 \) and \( \Gamma_1 \), disappears rapidly at high and low Q^2
The Major Spin Contributors

• CERN
  – EMC, SMC, COMPASS

• SLAC
  – E80, E130, E142, E143, E154, E155

• DESY
  – HERMES

• Jefferson Lab
  – Hall A, Hall B (CLAS), Hall C

• Brookhaven National Lab (RHIC)
  – STAR, PHENIX
EXPERIMENTS

Jefferson Lab’s Hall B (CLAS) is one of the current experiments that is significantly improving our knowledge of $g_1$ in the range $0.01 < Q^2 < 3.5$ GeV$^2$, where perturbative QCD breaks down.
Spin Structure with CLAS

• Long-standing program in Hall-B at JLab to measure longitudinal double spin asymmetries $A_\parallel$ on $^{15}$NH$_3$ and $^{15}$ND$_3$

  • EG1: $0.05<Q^2<3.5$ GeV$^2$
    – data (2001); anal (2008)
  • EG4: $0.01<Q^2<1$ GeV$^2$
    – data (2006); anal (2009)
  • EG1-DVCS: $1<Q^2<3.5$
    – data (2009); anal (2010)
  • EG12: $0.5<Q^2<7$ GeV$^2$
    – data (2012?); anal (2014)
Kinematics

EG1

EG4

EG12

EG1-DVCS (red)

• Overlapping colors correspond to different beam energies
• CLAS measures a large range in x at each fixed Q^2
• Different E_{beam} for fixed (x,Q^2) allows separation of A_1 & A_2
Formalism

\[ A_\parallel = \frac{\sigma_{\downarrow \uparrow} - \sigma_{\uparrow \uparrow}}{\sigma_{\downarrow \uparrow} + \sigma_{\uparrow \uparrow}} \]

\[ A_\parallel = D(A_1 + \eta A_2) \]

We can extract \( A_1 \) using a model for \( A_2 \) (small), or \( g_1 \) using a model for \( g_2 \) (small)

\[ A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} \]

\[ = \frac{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)}{F_1(x,Q^2)} \]

\[ A_2 = \frac{2\sigma_{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T} \]

\[ = \frac{\gamma [g_1(x,Q^2) + g_2(x,Q^2)]}{F_1(x,Q^2)} \]

We can extract \( A_1 \) and \( A_2 \) from \( A_\parallel \) at multiple values of \( \eta(E_{\text{beam}}) \)
$g_1^p(x, Q^2)$ before CLAS
$g_1^p(x,Q^2)$ with JLab CLAS
Presented graph shows the variation of Bjorken x with different values of Q^2, specifically highlighting Q^2 = 0.07 and Q^2 = 4. The analysis is preliminary in nature.

The graphs illustrate the behavior of g_1^p as a function of Bjorken x for various Q^2 values, with data points indicating experimental results and lines representing theoretical models.
$g_1^D$ for $Q^2 = 0.07$

$Q^2 = 0.07$

$g_1^D$ for $Q^2 = 0.13, 0.19$ GeV$^2$

$g_1^D$ for $Q^2 = 0.21, 0.32$ GeV$^2$

$g_1^D$ for $Q^2 = 0.27, 0.32$ GeV$^2$

$g_1^D$ for $Q^2 = 0.66, 0.77$ GeV$^2$

$g_1^D$ for $Q^2 = 0.10, 0.31$ GeV$^2$

$g_1^D$ for $Q^2 = 1.10, 1.55$ GeV$^2$

$g_1^D$ for $Q^2 = 2.23, 2.66$ GeV$^2$

$Q^2 = 4$

PRELIMINARY
The virtual photon asymmetry $A_1$ (approximately $g_1/F_1$) should scale with $Q^2$ if $g_1$ and $F_1$ evolve identically. It is very sensitive to resonance structure at moderate $Q^2$ and to PDFs at high $x$. 
Scaling starts at:

<table>
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<th>$x$</th>
<th>$Q^2$</th>
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</tbody>
</table>
Modeling the Resonance Region

Burkert arXiv:0908.3507

- Resonances induce huge variations in $A_1$ compared to the smooth deep-inelastic behavior (red)
- The world’s collected wisdom on resonance structure, encoded in MAID (green) does not describe the data well
$\Lambda_1$ Data from EG1

Close and Melnitchouk, PRC 68, 035210 (2003)


$g_1 / F_1$

Neutron

Proton

Deuteron

$W > 2; Q^2 > 1$
Quark polarization in the valence limit

\[ A_1(x, Q^2) = \frac{\sum e_i^2 \Delta q_i(x, Q^2)}{\sum e_i^2 q_i(x, Q^2)} \]

\[ R_u = \Delta u/u; \quad R_d = \Delta d/d \]

Simulated Data for EG12
Extracted from \( A_1^p, A_1^d \) and \( d/u \)
Moments

Accurate moments of structure functions (x-weighted integrals) can now be constructed from experimental data over a wide range of $Q^2$. These directly show the changing landscape with momentum transfer, and are often easier to compare with theoretical calculations.
Higher Twist

The Operator Product Expansion of QCD sorts quark-gluon correlations into higher twists, which fall off inversely with powers of $Q^2$. Measurements at intermediate $Q^2$ have been able to extract these higher-twist coefficients $d_2$ and $f_2$ and the related color electric and magnetic susceptibilities.
BJORKEN SUM & HIGHER TWIST

Bjorken Sum Rules:

\[ \Gamma_1^{(n)} = \int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2}, \quad n = 0, 2, 4, \ldots, \]

\[ \Gamma_2^{(n)} = \int_0^1 x^n g_2(x, Q^2) dx = \frac{1}{2} \frac{n}{n+1} (d_n - a_n), \quad n = 2, 4, \ldots, \]

Bjorken Sum Rule: CLAS: Deur

\[ \Gamma_1^{p-n} = \frac{g_A}{6} \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 \right] + \frac{\mu_4^{p-n}}{Q^2} + \ldots \]

\[ \mu_4^{p-n} = \frac{M^2}{9} \left( a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n} \right) \]

\[ d_2^{p-n} = \int_0^1 dx \ x^2 \left( 2g_1^{p-n} + 3g_2^{p-n} \right) \]

Fit \( \Gamma_1^{p-n} \) to powers of \( 1/Q^2 \) and extract \( f_2^{p-n} \).
Higher Twist $d_2$

\[
d_2(Q^2) = \int_0^1 dx \, x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right]
\]

CLAS EG1 (proton)
Osipenko, PRD71(05)054007
Model-dependent determination

Hall A (neutron)
E94-010
Amarian, PRL92(04)022301
Nachtmann Moments

\[
M_1(Q^2) = \int_0^1 dx \frac{\xi^2}{x^2} \left\{ g_1(x, Q^2) \left( \frac{x}{\xi} - \frac{1}{9} \frac{M^2 x \xi}{Q^2} \right) - g_2(x, Q^2) \frac{4}{3} \frac{M^2 x^2}{Q^2} \right\},
\]

\[
\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}
\]

\[
M_1(Q^2) = \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + \ldots
\]

\[
\mu_4(Q^2) = 4f_2(Q^2)/9M^2
\]

\[
f_2 = 0.039 \pm 0.022 \text{(stat)} \pm 0.000_{\text{sys}}^{0.018},
\]

\[
\pm 0.030 \text{(low } x) \pm 0.007_{0.011} \text{()}
\]

\[
d_2(Q^2) = \int_0^1 dx x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right]
\]

\[
\chi_E = \frac{2}{3}(2d_2 + f_2)
\]

\[
\chi_B = \frac{1}{3}(4d_2 - f_2)
\]

\[
21 \text{ September } 2009
\]

CLAS, Osipenko

PLB609(05)259
Higher Twist from $g_1$ in CLAS

\[
\left[ \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right] \exp = g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + h^{g_1}(x)/Q^2
\]

1 < $Q^2$ < 5 GeV$^2$, 2 < $W$ < 3.5 GeV

\[
\int_0^1 dx h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)
\]

- $F_1$ from NMC fit to $F_2$ and 1998 SLAC fit to $R$
- $g_1$ (leading twist) from NLO fit at high $Q^2$
- $h$ from fit to all data, especially CLAS in the pre-asymptotic region
- $d_2$: twist-3, $f_2$: twist-4

Leader, Sidorov, Stamenov, EPJST162(08)19
Color Polarizability

- Osipenko, CLAS, proton, PLB609(05)249
  \[ f_2 = 0.039(39) \quad \chi_E = 0.026(27) \quad \chi_B = -0.013(13) \]
- E94-010, Hall A, neutron
  \[ f_2 = 0.034(43) \quad \chi_E = 0.033(29) \quad \chi_B = -0.001(16) \]
- Deur, CLAS, Bjorken (p-n)
  \[ f_2 = -0.101(74) \quad \chi_E = -0.077(50) \quad \chi_B = 0.024(28) \]
- More accurate determinations are needed.
Duality

When structure functions are averaged over resonance peaks and valleys, they behave just like deep-inelastic scattering extrapolated into the resonance region. Local duality (one resonance region) and global duality (all resonances) tend to hold to 10% above $Q^2=2\text{ GeV}^2$, except for the polarized $\Delta$ resonance.
Duality

Duality - structure functions averaged over resonances behave according to DIS systematics
Global - all resonances
Local - one resonance
Polarized Duality

Hall C
RSS, Wesselmann, Slifer
$Q^2=1.379 \text{ GeV}^2$
Target Mass Corrections applied to PDFs
No duality for $\Delta$
PRL98(07)132003

Duality at CLAS (EG1)

Proton

Deuteron

Δ (1232)  \( S_{11} \) (1535)

Global

\( F_{15} \) (1680)

Δ (1232)  \( S_{11} \) (1535)

Global

\( F_{15} \) (1680)
The structure function $g_2$ in pQCD can be expressed as a convolution of $g_1$. Deviations from this Wandzura-Wilczek form measures higher twist. The Burkhardt-Cottingham sum rule states that the first moment of $g_2$ is zero. Precise data are now available to evaluate this claim.
RSS $g_2^p$

Wandzura-Wilczek

$$g_2^{WW} = -g_1 + \int x \frac{g_1}{y} \, dy$$

$Q^2 = 1.28 \text{ GeV}^2$

$$g_2 = g_2^{WW} - g_2$$

Hall C: Slifer et al. arXiv:0812.0031

Burkhardt-Cottingham Sum Rule

$$\Gamma_2 = \int_0^1 g_2(x, Q^2) \, dx = 0$$
Burkhardt Cottingham Sum

- Plot: K. Slifer
- Open points: Measured
- Solid points: Corrected for unmeasured regions and elastic contribution
- Green: Hall A E97-110
- Blue: Hall A E01-012
- Red: Hall C RSS
- Black: Hall A E94-010
- Brown: SLAC E155

Burkhardt & Cottingham, Ann. Phys. 56(70)453

\[ \int_{0}^{1} g_2(x, Q^2) \, dx = 0 \]
Spin Polarizabilities

The spin polarizability can be expressed in terms of moments of $g_1$ and $g_2$. How this evolves at low $Q^2$ from the real photon point provides a rigorous test of chiral perturbation theory.
\[ \gamma_0(Q^2) = C(Q^2) \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right\} \, dx \]

Error est. on \( g_2 \) (100%)

\[ C(Q^2) = 16\alpha M^2/Q^6 \]
\[ \gamma_0(Q^2) = C(Q^2) \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right\} \, dx \]

Error est. on \( g_2 \) (100%)

\[ C(Q^2) = \frac{16\alpha M^2}{Q^6} \]
\[ \delta_{LT}(Q^2) = \left( \frac{1}{2\pi^2} \right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu \]
\[ = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1(x, Q^2) + g_2(x, Q^2)] dx \]

Hall A
Amarian,
PRL93(04)1152301
A Deur
CLAS + Hall A

For isovector (p-n) case
$\Delta$ contribution cancels

\[
\gamma_0^{p-n} \text{ and } \gamma_0^{p+n}
\]
The GDH Sum Rule

At low $Q^2$, the evolution of the first moment of $g_1$ should be proportional to $Q^2$ and to the square of the anomalous magnetic moment of the nucleon. Data are now available to test this.
Global Properties

Energy-Weighted Sum Rule

\[ S(F) = \sum_a (E_a - E_0) |<a|F|0>|^2 = <0|[F,[H,F]|0> \]

GDH Sum Rule

\[ \int_{k_{\pi}}^{\infty} \frac{dk}{k} \Delta \sigma^{\gamma N}(k) = \frac{2\pi^2 \alpha \kappa^2}{M^2} \]

\[ \Delta \sigma^{\gamma N} = \sigma^{\gamma N}_{3/2} - \sigma^{\gamma N}_{1/2} \]

Sum over excited states is tied to property of ground state
\[ \Gamma_{1}^{p,d}(Q^2) = \int_{0}^{1} g_{1}^{p,d}(x, Q^2) dx \]
\[ \Gamma_{1}(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8 \]
\[ \Gamma_{1}^{d}(Q^2) = (1 - 1.5\omega_D) \{ \Gamma_{1}^{p}(Q^2) + \Gamma_{1}^{n}(Q^2) \} \]
\[ \Gamma_{1}^{p}(Q^2) = -\frac{\kappa_{p}^{2}}{8M_{p}^2} Q^2 + 3.89Q^4 + ... \]
\[ \Gamma_{1}^{n}(Q^2) = -\frac{\kappa_{n}^{2}}{8M_{n}^2} Q^2 + 3.15Q^4 + ... \]
EG4 $\Gamma_1^p$ Expected

Expected statistical accuracy on $\Gamma_1^p$ for EG4

- CLAS EG4 w/o large-w part (Simula param.)
- Full $\Gamma_1^p$ (Simula parameterization)
- CLAS EG1a
- CLAS EG1b preliminary
- Bernard et al, Xpt
- Ji et al, Xpt
- GDH slope
- Burkert-Ioffe
- Soffer-Teryaev (2004)
The Spin Crisis

Where does the spin of the nucleon come from? Only a quarter comes from the quark spins. The rest must be gluon polarization and orbital angular momentum; both of these are hard to measure. However, data are becoming available that suggest the gluon polarization is small.
The Spin Puzzle

\[
\frac{1}{2} = \frac{\Delta \Sigma}{2} + \Delta G + L_z
\]

Known

Could be constrained by QCD evolution but this requires an understanding of higher twist at moderate \( Q^2 \). Direct measures through photon-gluon fusion and pp reactions.

Transverse momentum dependent distributions and generalized parton distributions are sensitive to quark orbital angular momentum.
Photon-gluon fusion measurements show a relatively small polarized gluon distribution.
• Error envelopes for PDFs from LSS05 global analysis (green)
• CLAS EG1 data significantly improve errors on $\Delta u$, $\Delta d$, $\Delta x$ and $\Delta G$ (blue)
• CLAS EG12 (12 GeV upgrade) will especially improve $\Delta G$ (red)
NLO Fits to $A_1$

DeFlorian, Sassot, Stratmann & Vogelsang (DSSV)
PRD80(09)034030
Double polarized pp scattering occurs through qq, qg or gg scattering. Later two are sensitive to $\Delta g$. 

PHENIX
### Significant contributions from $x<0.001$

- $\Delta G$ vanishes with increasing $Q^2$

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z$$

- At $Q^2=4$ GeV$^2$, $L_z = 0.474$ (large)

- Errors on $\Delta G$ are still very large

<table>
<thead>
<tr>
<th>$x$ range in Eq. (35)</th>
<th>$Q^2$ [GeV$^2$]</th>
<th>$\Delta u + \Delta \bar{u}$</th>
<th>$\Delta d + \Delta \bar{d}$</th>
<th>$\Delta \bar{u}$</th>
<th>$\Delta \bar{d}$</th>
<th>$\Delta \bar{s}$</th>
<th>$\Delta g$</th>
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<td>-0.058</td>
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<td>0.238</td>
</tr>
</tbody>
</table>
Plot: Voutier, arXiv:0802.2499

[27] Mazouz et al., PRL 99(07)2425

[26] Airapetian et al., arXiv:0802.2499

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_q + L_g \]

\[ J_q = L_q + \frac{1}{2} \Delta \Sigma_q \]

\[ J_g = L_g + \Delta g \]

\[ J_q(x) = \frac{1}{2} x [q(x) + E_q(x)] \]

OAM accessed through GPDs via deeply virtual Compton scattering
The 21 cm line in hydrogen (ground-state hyperfine splitting) has been measured to 13 digits of accuracy. Theoretical calculations are limited to parts per million because of the nuclear physics that depends on \( g_1 \) and \( g_2 \). Recent data improves this calculation.
Hydrogen Hyperfine Splitting

Carlson, Nazaryan, Griffioen, PRA78(08)022517

\[ E_{\text{HFS}}(e^- p) = 1.4204057517667(9) \text{GHz} = (1 + \Delta_{QED} + \Delta_{p} + \Delta_{S})E_{F}^{p} \]

\[ \Delta_{S} = \Delta_{Z} + \Delta_{\text{pol}} \]

\[ \delta_{Z}^{\text{rad}} = \frac{\alpha}{3\pi} \left[ 2 \ln \frac{\Lambda^{2}}{m^{2}} - \frac{4111}{420} \right] \]

Zemach:

\[ \Delta_{Z} = -2\alpha m_{e} \langle r \rangle_{Z} (1 + \delta_{Z}^{\text{rad}}) \]

\[ \langle r \rangle_{Z} = -\frac{4}{\pi} \int_{0}^{\infty} \frac{dQ}{Q^{2}} \left[ G_{E}(Q^{2}) \frac{G_{M}(Q^{2})}{1 + \kappa} - 1 \right] \]

\[ \Delta_{S} = -38.62(16) \text{ ppm} \quad \Delta_{Z} = -41.0(5) \text{ ppm} \]

\[ \Delta_{\text{pol}} = 2.38(58) \text{ ppm} \]

\[ \Delta_{\text{pol}} = \frac{\alpha m_{e}}{2\pi(1 + \kappa)M} (\Delta_{1} + \Delta_{2}) = (0.2264798 \text{ ppm})(\Delta_{1} + \Delta_{2}) \]

\[ \tau = \frac{\nu^2}{Q^2} \]

\[ \Delta_{1} = \frac{9}{4} \int_{0}^{\infty} \frac{dQ^2}{Q^2} \left\{ F_{2}^{2}(Q^2) + \frac{8m_{p}^{2}}{Q^2} B_{1}(Q^2) \right\} = 8.85(278) \]

\[ \Delta_{2} = -24m_{p}^{2} \int_{0}^{\infty} \frac{dQ^2}{Q^4} B_{2}(Q^2). = -0.57(57) \]

\[ \Delta_{\text{pol}} = 1.88(64) \text{ ppm from CLAS} \]
COMMENSURATE SCALING

The QCD is scale-independent BOTH at high $Q^2$ and low $Q^2$. By defining an effective coupling constant consistent with the form from NLO pQCD, the relationship of these constants can be calculated.
Commensurate Scaling

Brodsky, Lu, PRD51(95)3652; Deur, PLB650(07)244

\[ \int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_g(Q)}{\pi} \right] \]

\[ R(Q) \equiv 3 \sum_j Q_j^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right] \]

\[ \frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{g_1}(Q^*)}{\pi} + \frac{3}{4} C_F \left( \frac{\alpha_{g_1}(Q^{**})}{\pi} \right)^2 \]

\[ + \left[ \frac{9}{16} C_F^2 + \left( \frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{Q^{**}}{\pi} \right] \left( \frac{\alpha_{g_1}(Q^{***})}{\pi} \right)^3, \]

\[ \ln \left( \frac{Q^*/Q}{\pi} \right) = -\frac{7}{4} + 2\zeta_3 + \left( \frac{11}{96} - \frac{7}{3} \zeta_3 + 2\zeta_3 + \frac{\pi^2}{24} \right) \left( \frac{11}{3} C_A - \frac{2}{3} f \right) \frac{\alpha_{g_1}(Q)}{\pi}, \]

\[ \ln \left( \frac{Q^{**}/Q}{\pi} \right) = -\frac{233}{216} + \frac{64}{9} \zeta_3 - \frac{3}{2} \zeta_5 + \left( \frac{13}{54} + \frac{2}{9} \zeta_3 \right) \frac{C_A}{C_F}. \]

R(e^+e^- -> more than 2 hadrons)
CONCLUSIONS

Although much is known, there is still much to be learned about nucleon spin that will keep us busy for at least another decade.
Conclusions

• What’s still missing:
  – high x: \( A_{1p,d} \) for polarized PDFs (CLAS12)
  – \( g_2^p \) (transverse target); SANE (Hall C) covers \( Q^2 > 1 \) GeV\(^2\);
    nobody’s measuring \( Q^2 < 1 \) GeV\(^2\)
  – low \( Q^2 \) evolution of \( g_{1p,d} \) (EG4)
  – gluon spin: evolution for \( 1 < Q^2 < 10 \) (CLAS12); direct measurements?
  – orbital angular momentum (GPD experiments)

• What’s gained:
  – understanding three regions
    • \( Q^2 \) near 0 (\( \chi \)PT)
    • \( Q^2 \) from 0.1-10 GeV\(^2\) (TMC, higher twists, resonances, the transition)
    • \( Q^2 \) near infinity (pQCD)