



Nucleon Spin Structure from Confinement to Asymptotic Freedom

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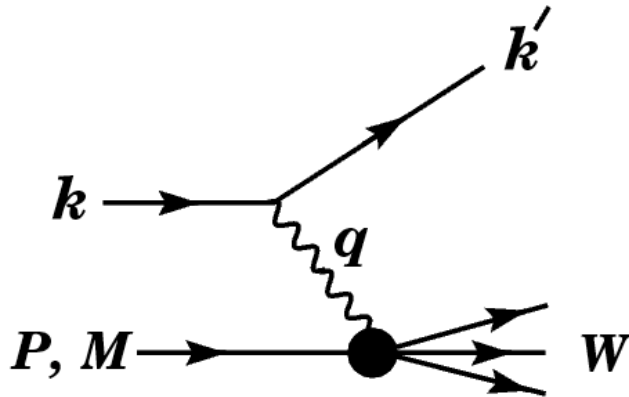


INTRODUCTION

The nucleon structure functions F_1 , F_2 , g_1 and g_2 depend on two Lorentz invariants. Although well-defined over all kinematics, their interpretation varies with momentum-transfer scale. They have been actively measured over several decades at a number of labs around the world.

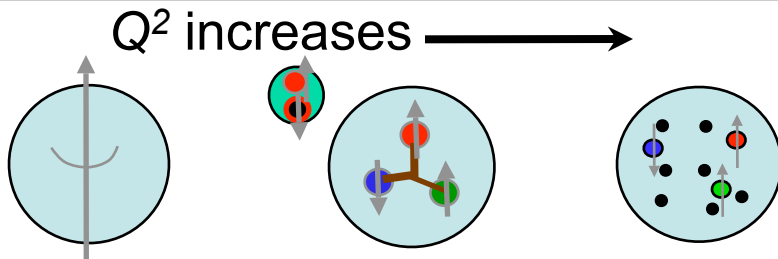


Inelastic Scattering



Lorentz invariants:

$$\begin{aligned} \nu &= p \cdot q / M = (E - E')_{\text{lab}} \\ Q^2 &= -q \cdot q = (4EE' \sin^2 \frac{\theta}{2})_{\text{lab}} \\ x &= -q \cdot q / 2p \cdot q = (Q^2 / 2M\nu)_{\text{lab}} \\ y &= p \cdot q / p \cdot k = (\nu / E)_{\text{lab}} \\ W^2 &= (p + q)^2 = (M^2 + 2M\nu - Q^2)_{\text{lab}} \\ s &= (k + p)^2 = (2EM + M^2)_{\text{lab}} \end{aligned}$$



Unpolarized Cross Section:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{8\pi\alpha^2 y}{Q^4} \left[\frac{y}{2} F_1 + \frac{\xi}{2xy} F_2 \right]$$

Polarized Cross Section:

$$\begin{aligned} \frac{d^2\Delta\sigma}{dx dQ^2} &= \frac{8\pi\alpha^2 y}{Q^4} \left[\cos\alpha \left\{ \left(\xi + \frac{y}{2} \right) g_1 - \frac{\gamma y}{2} g_2 \right\} - \right. \\ &\quad \left. \sin\alpha \cos\phi \left\{ \frac{y}{2} g_1 + g_2 \right\} \right] \end{aligned}$$

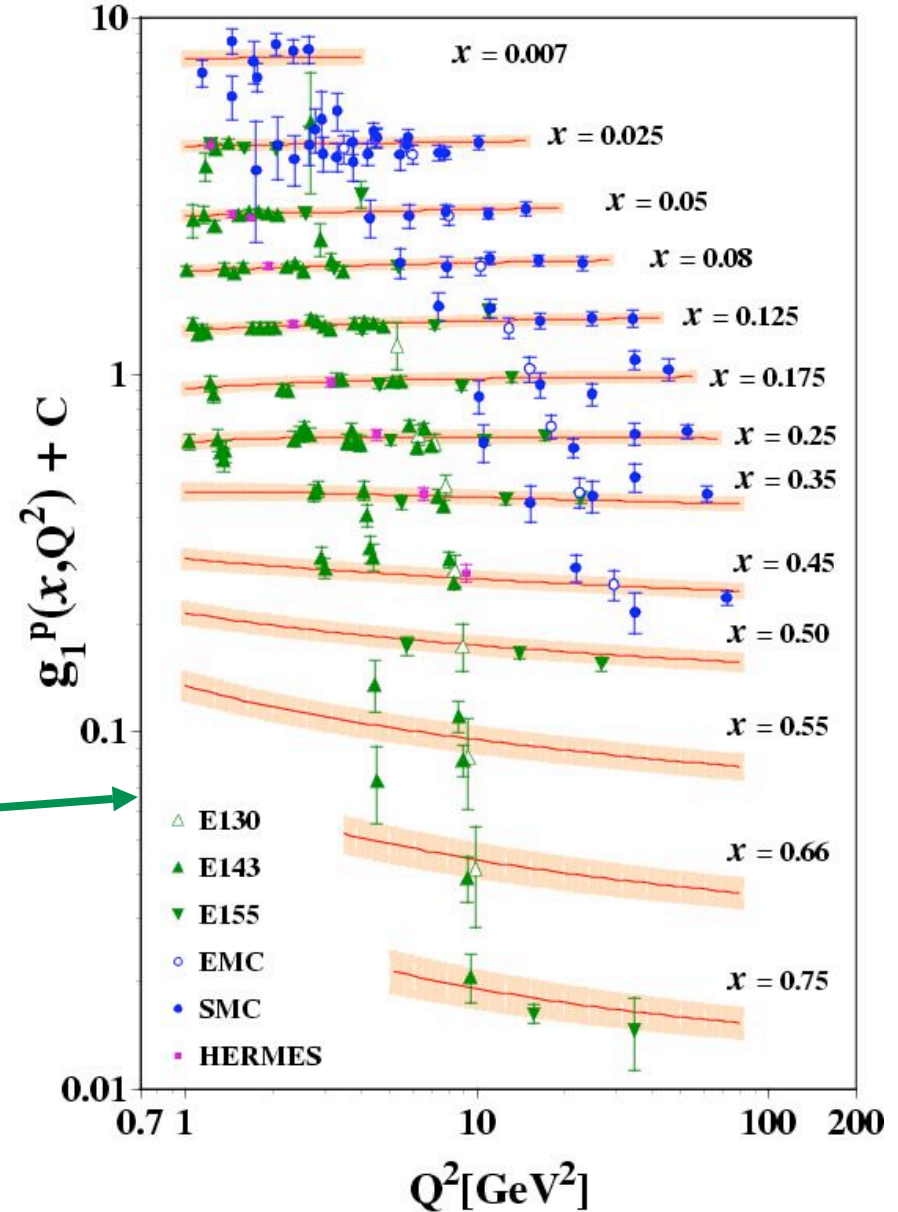
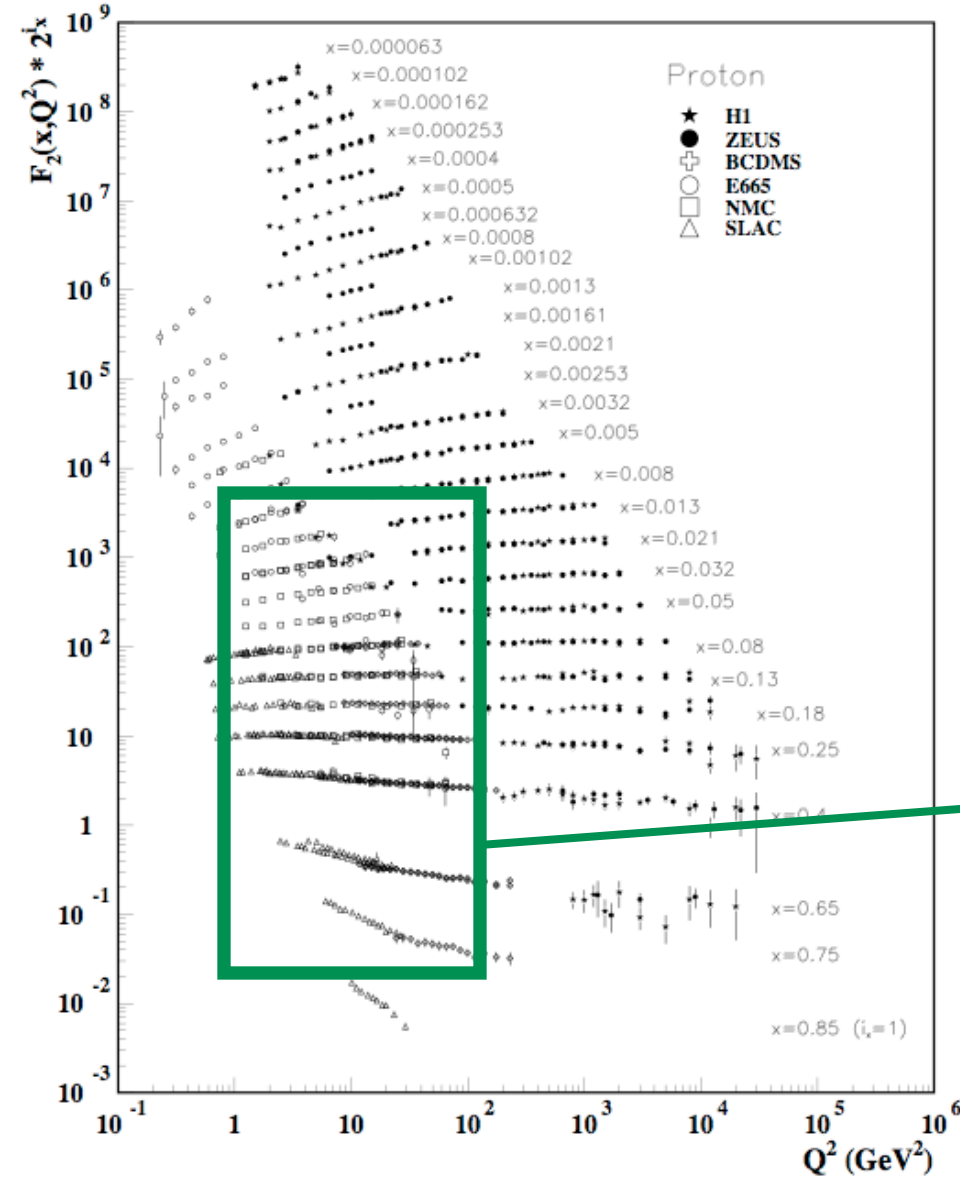
α = polar angle of target spin wrt the beam axis
 ϕ = azimuthal spin angle wrt the scattering plane
 $\alpha = 0^\circ$ (longitudinal); $\alpha = 90^\circ, \phi = 0^\circ$ (transverse).
 $\gamma^2 = 4M^2 x^2 / Q^2 = Q^2 / \nu^2$
 $\xi = 1 - y - \gamma y^2 / 4$

Parton Model:

$$\begin{aligned} F_1(x, Q^2) &= \frac{1}{2} \sum_i e_i^2 (q^\uparrow(x) + q^\downarrow(x) + \bar{q}^\uparrow(x) + \bar{q}^\downarrow(x)) \\ F_2(x, Q^2) &= 2x F_1(x, Q^2) \\ g_1(x, Q^2) &= \frac{1}{2} \sum_i e_i^2 (q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)) \end{aligned}$$



$F_2^p(x, Q^2)$ and $g_1^p(x, Q^2)$





Gottfried Sum Rule

0.235(26) at $Q^2=4 \text{ GeV}^2$

$$\Phi_1^{p,n}(Q^2) = \int_0^1 F_1^{p,n}(x, Q^2) dx$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x)$$

$$\Phi_1^p - \Phi_1^n = \frac{1}{6} [u_v - d_v + 2\bar{u} - 2\bar{d}]$$

Bjorken Sum Rule

0.176(7) at $Q^2=5 \text{ GeV}^2$

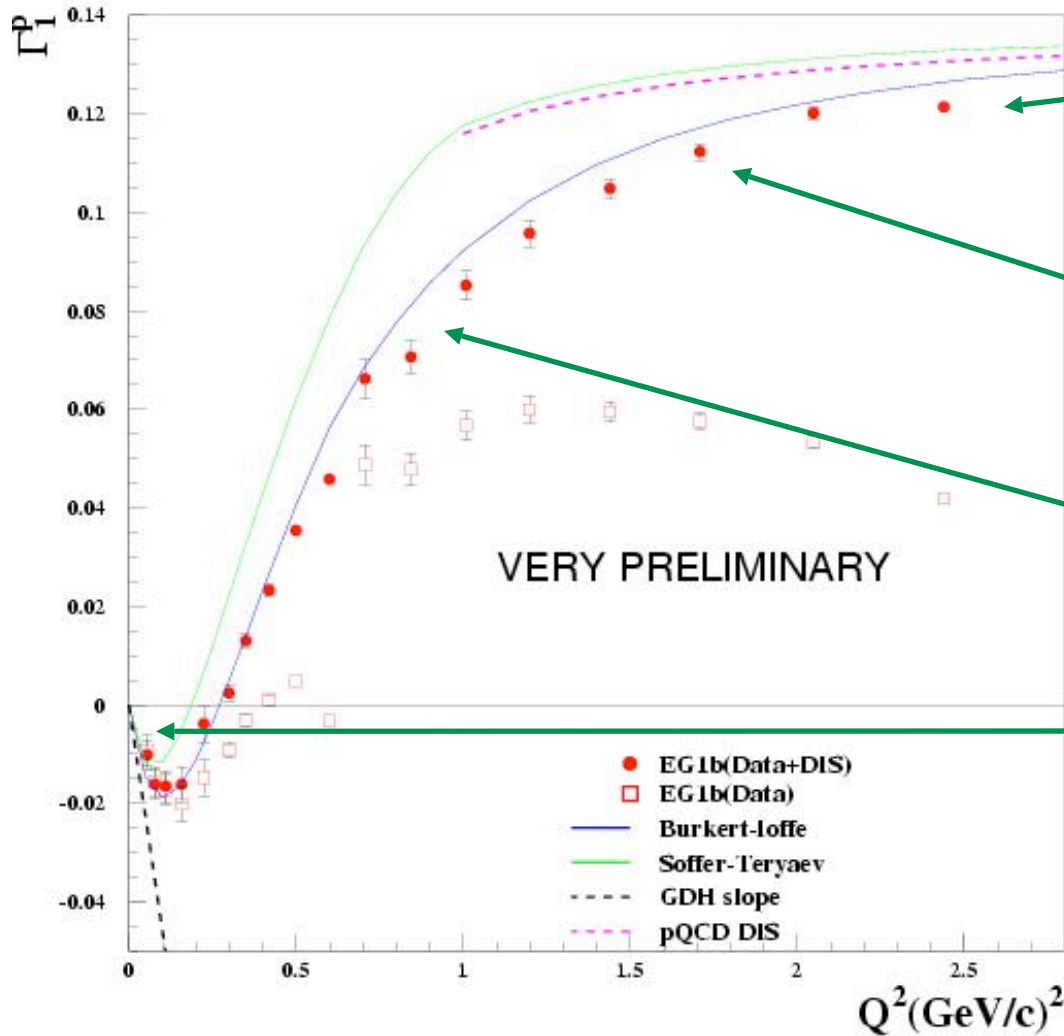
$$\Gamma_1^{p,n}(Q^2) = \int_0^1 g_1^{p,n}(x, Q^2) dx$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x)$$

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} [\Delta u_v - \Delta d_v + 2\Delta\bar{u} - 2\Delta\bar{d}]$$

Complicating Factor

$$\Delta C_{NS}^{\bar{M}S} = 1 - \frac{\alpha_S}{\pi} - 3.583 \left(\frac{\alpha_S}{\pi}\right)^2 - 20.215 \left(\frac{\alpha_S}{\pi}\right)^3 + \dots$$



scaling: $\ln Q^2$

higher twist: $(1/Q^2)^n$

no nice expansion

χ_{PT} : $(Q^2)^n$



- Infinite Q^2 Parton Model, PDF(x)
- Large Q^2 DIS, pQCD, PDF(x, log(Q^2))
- Medium Q^2 Higher twist, target mass correct.
- Low Q^2 Resonances (complexity)
- Tiny Q^2 Chiral perturbation theory
- Zero Q^2 Real photons
- Complexity, as measured by γ_0 , δ_{LT} , d_2 and Γ_1 , disappears rapidly at high and low Q^2



- CERN
 - EMC, SMC, COMPASS
- SLAC
 - E80, E130, E142, E143, E154, E155
- DESY
 - HERMES
- Jefferson Lab
 - Hall A, Hall B (CLAS), Hall C
- Brookhaven National Lab (RHIC)
 - STAR, PHENIX



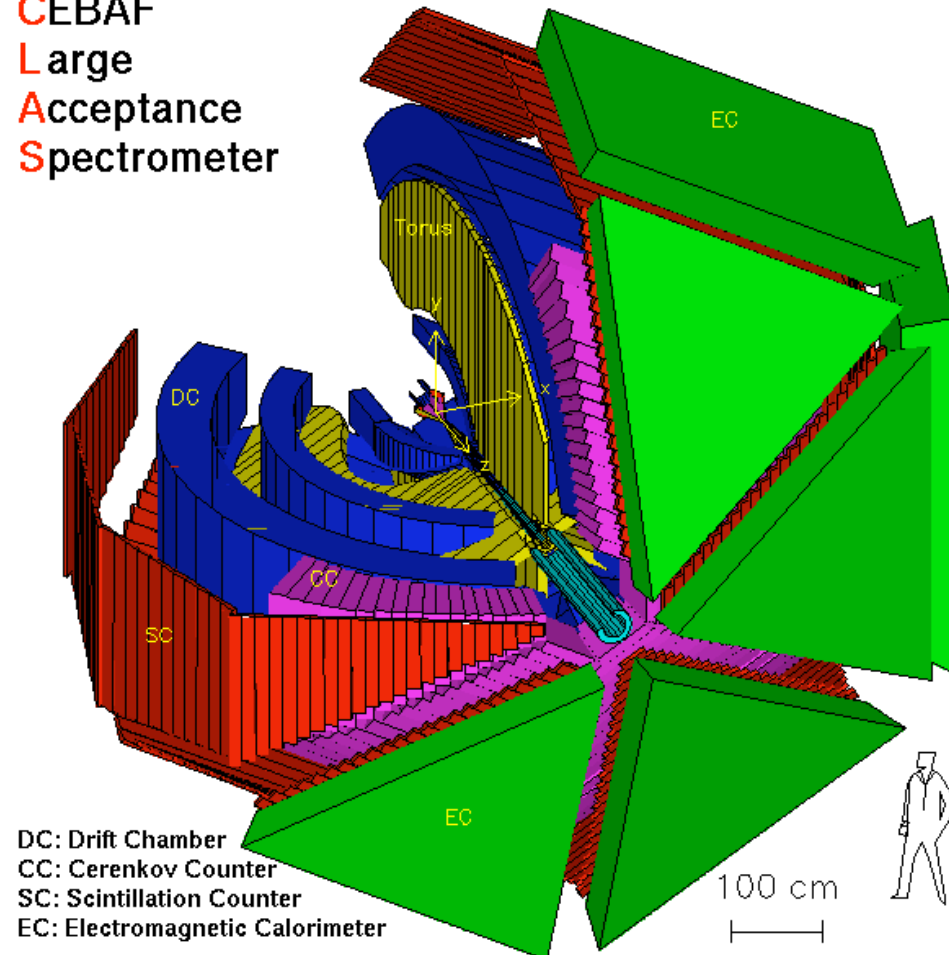
EXPERIMENTS

Jefferson Lab's Hall B (CLAS) is one of the current experiments that is significantly improving our knowledge of g_1 in the range $0.01 < Q^2 < 3.5 \text{ GeV}^2$, where perturbative QCD breaks down.



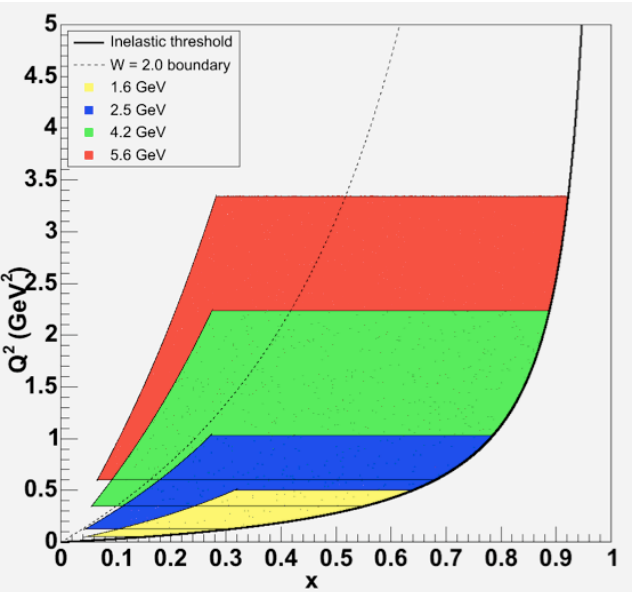
- Long-standing program in Hall-B at JLab to measure longitudinal double spin asymmetries $A_{||}$ on $^{15}\text{NH}_3$ and $^{15}\text{ND}_3$
- EG1: $0.05 < Q^2 < 3.5 \text{ GeV}^2$
 - data (2001); anal (2008)
- EG4: $0.01 < Q^2 < 1 \text{ GeV}^2$
 - data (2006); anal (2009)
- EG1-DVCS: $1 < Q^2 < 3.5$
 - data (2009); anal (2010)
- EG12: $0.5 < Q^2 < 7 \text{ GeV}^2$
 - data (2012?); anal (2014)

CEBAF
Large
Acceptance
Spectrometer

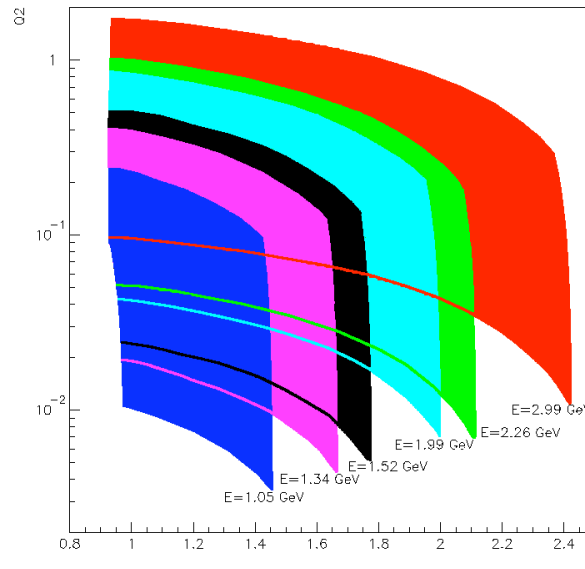




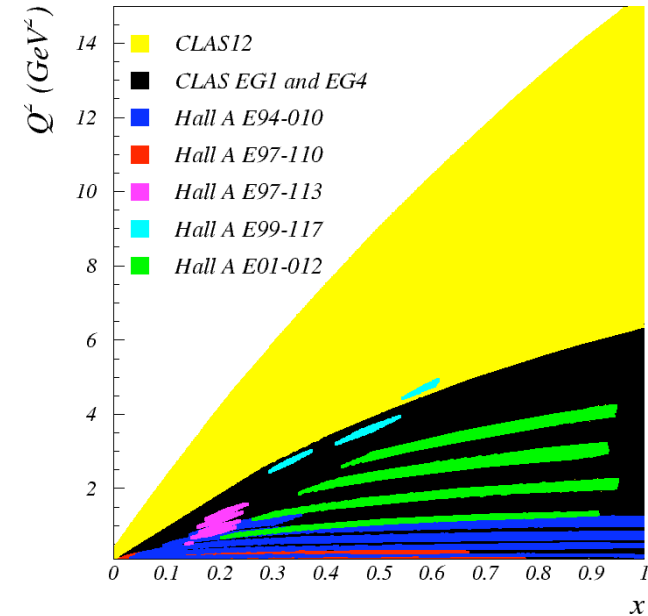
EG1



EG4



EG12



EG1-DVCS (red)

- Overlapping colors correspond to different beam energies
- CLAS measures a large range in x at each fixed Q^2
- Different E_{beam} for fixed (x, Q^2) allows separation of A_1 & A_2



$$A_{\parallel} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}}$$

$$A_{\parallel} = D(A_1 + \eta A_2)$$

We can extract A_1 using a model for A_2 (small), or g_1 using a model for g_2 (small)

We can extract A_1 and A_2 from A_{\parallel} at multiple values of $\eta(E_{\text{beam}})$

$$A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T}$$

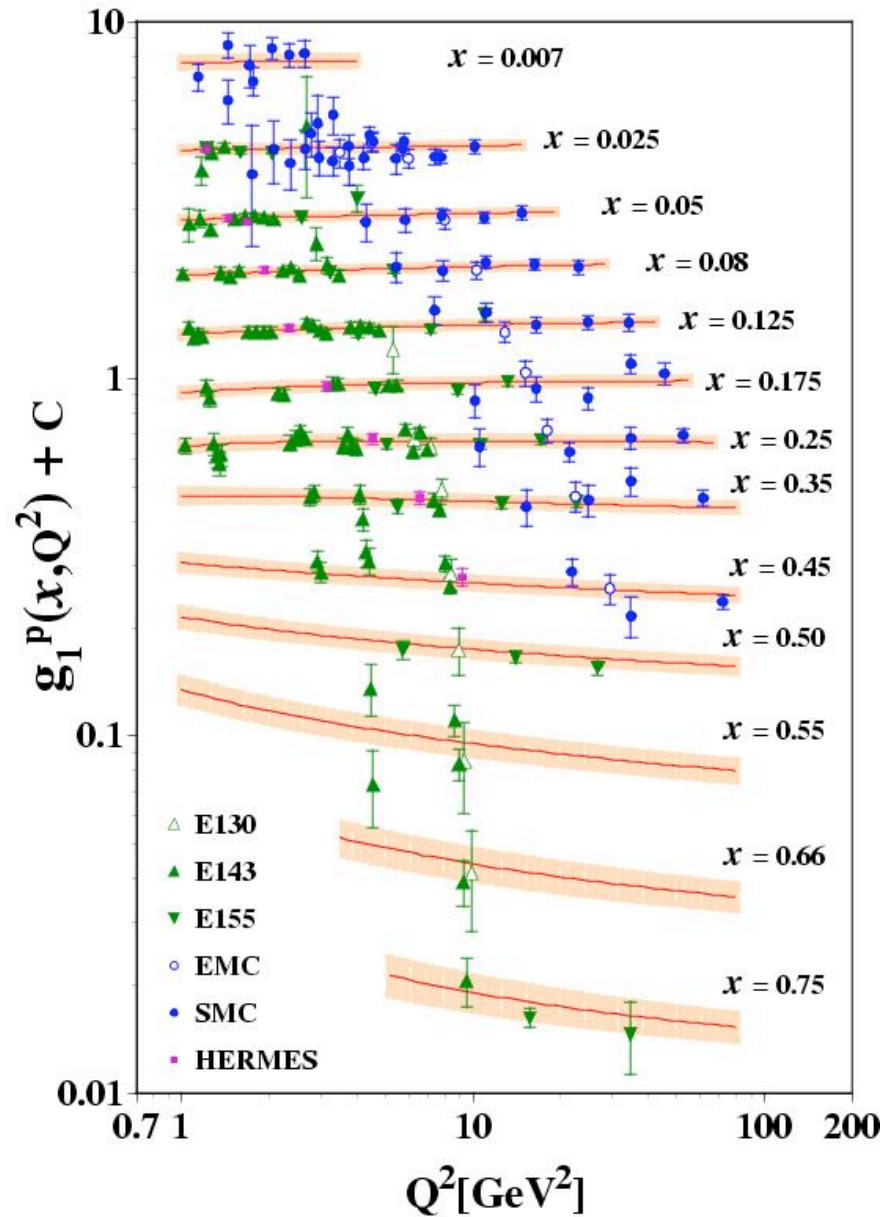
$$= \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{F_1(x, Q^2)}$$

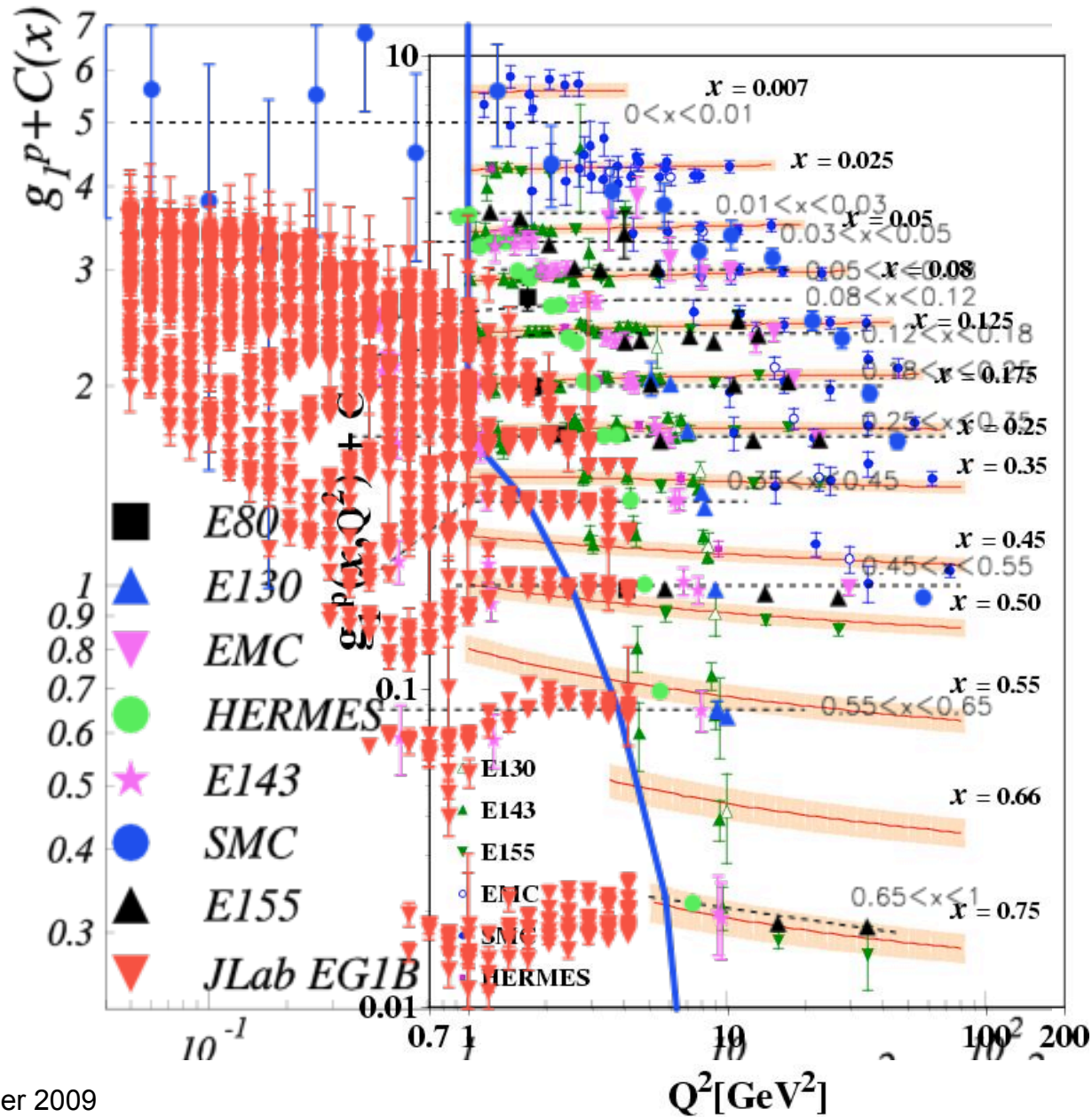
$$A_2 = \frac{2\sigma_{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T}$$

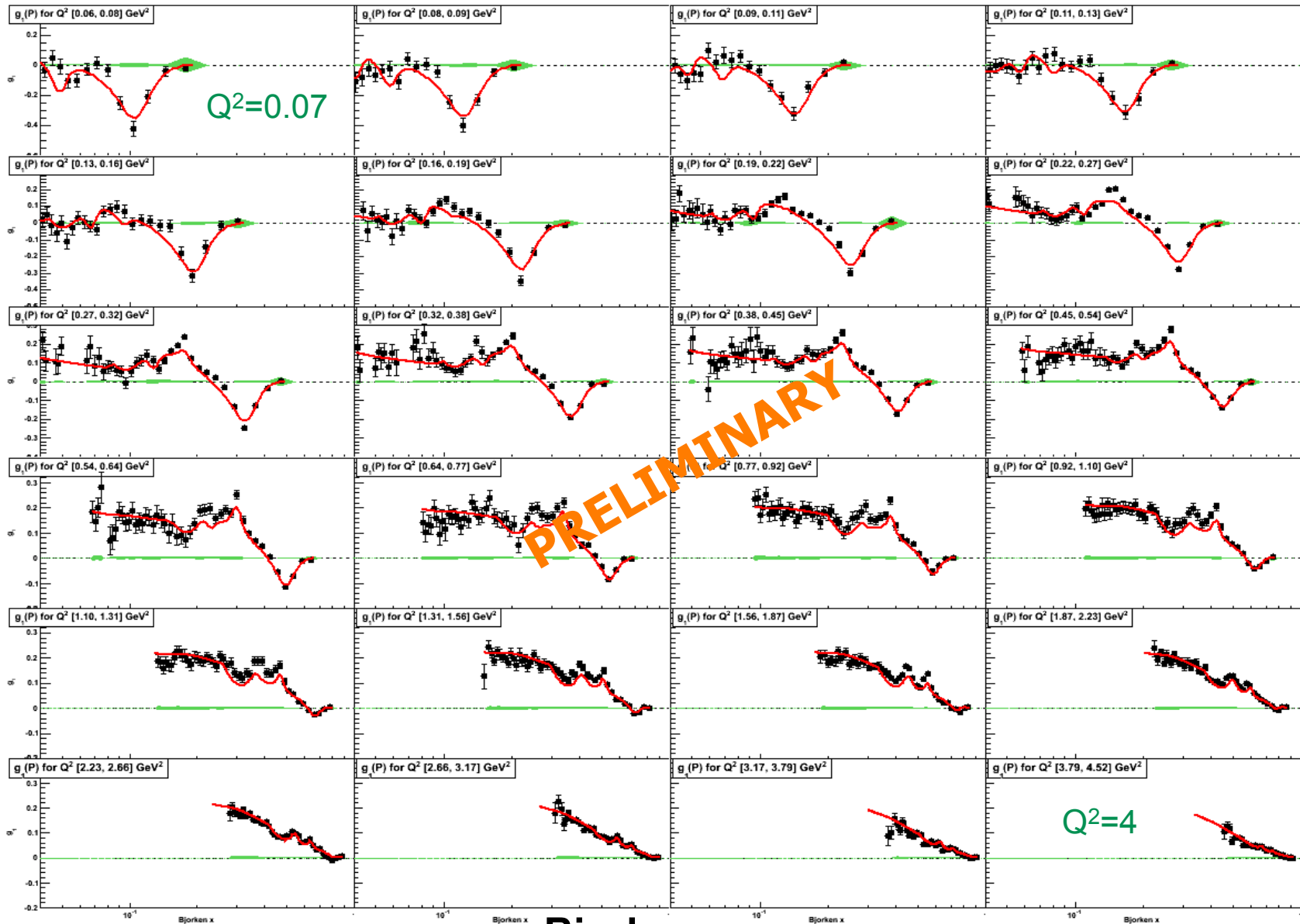
$$= \frac{\gamma[g_1(x, Q^2) + g_2(x, Q^2)]}{F_1(x, Q^2)}$$



$g_1^p(x, Q^2)$ before CLAS



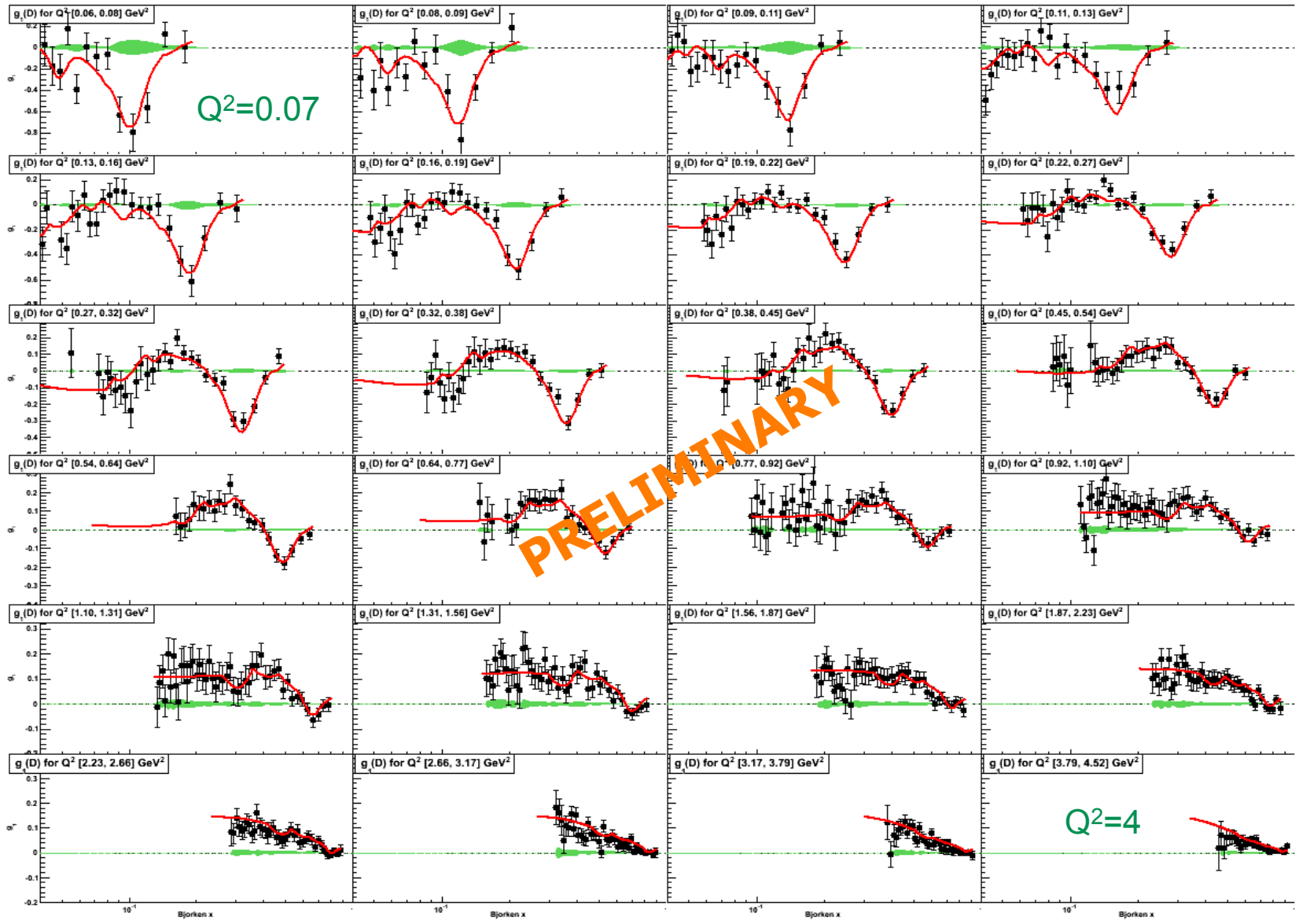




Bjorken x
QNP09 Beijing



g_1^D

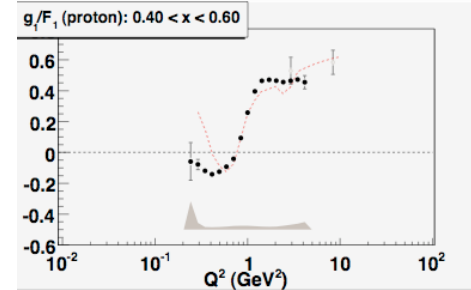
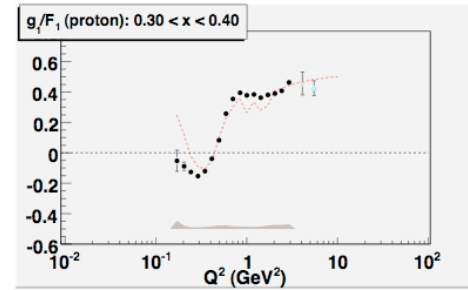
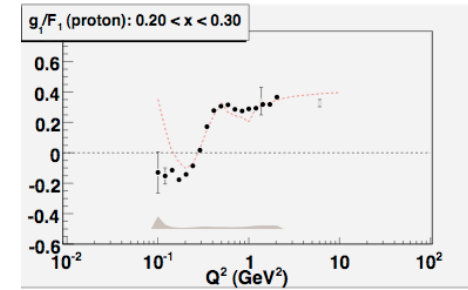
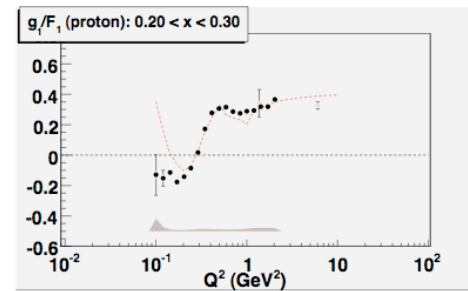
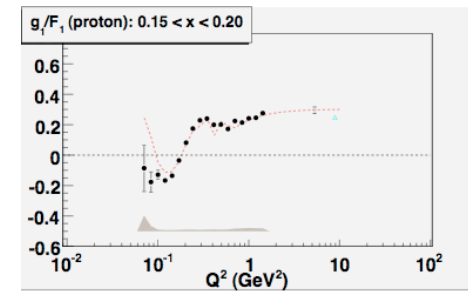
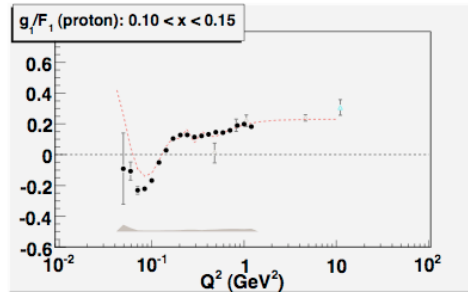
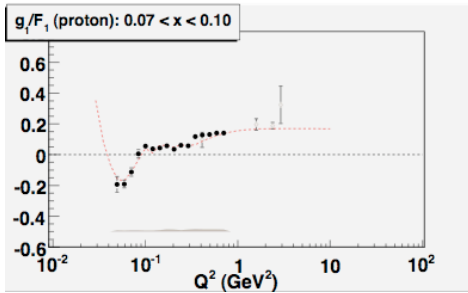
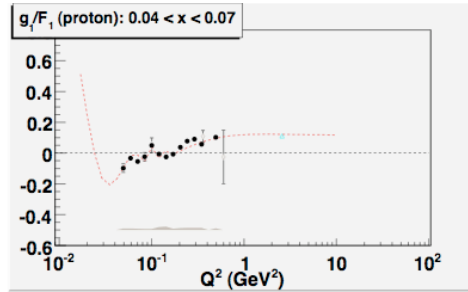
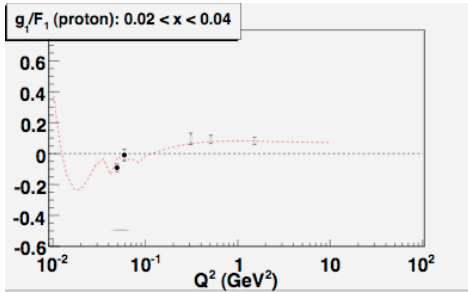


Bjorken x
QNP09 Beijing



A_1

The virtual photon asymmetry A_1 (approximately g_1/F_1) should scale with Q^2 if g_1 and F_1 evolve identically. It is very sensitive to resonance structure at moderate Q^2 and to PDFs at high x .

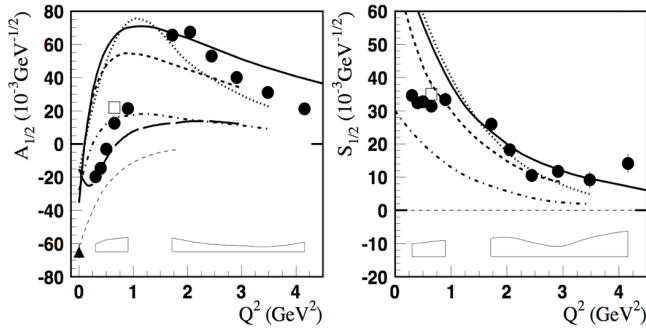


Scaling starts at:

x	Q^2
0.085	0.1
0.125	0.2
0.175	0.3
0.250	0.6
0.350	0.8
0.500	1.0

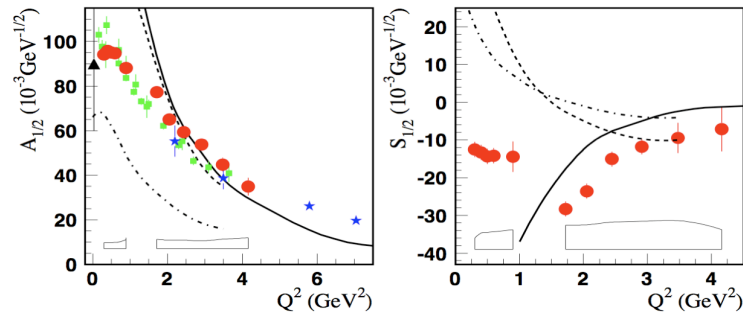
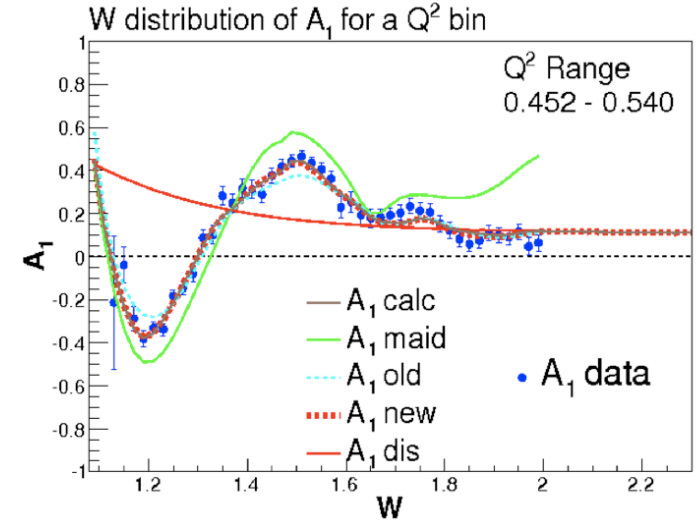


Burkert arXiv:0908.3507



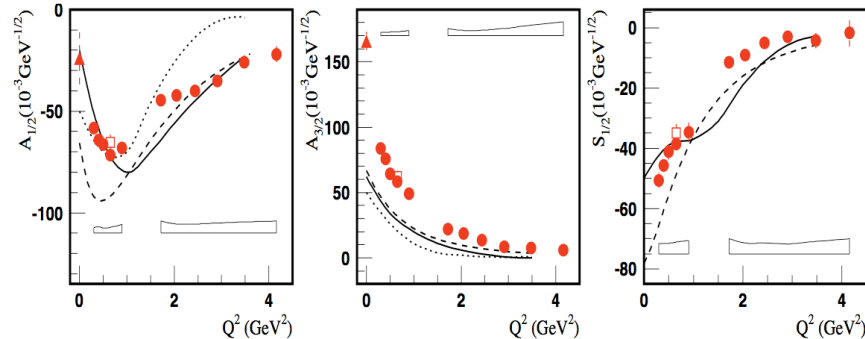
$$A_1 = \frac{A_{1/2}^2 - A_{3/2}^2}{A_{1/2}^2 + A_{3/2}^2}$$

P_{11}



S_{11}

- Resonances induce huge variations in A_1 compared to the smooth deep-inelastic behavior (red)

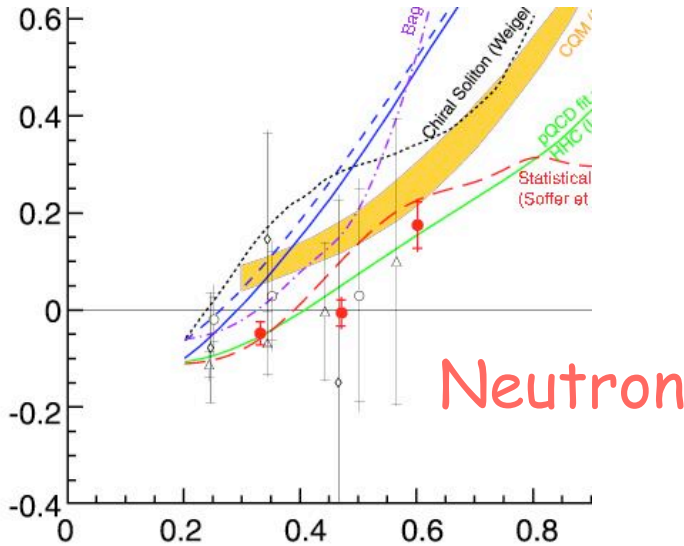


D_{13}

- The world's collected wisdom on resonance structure, encoded in MAID (green) does not describe the data well



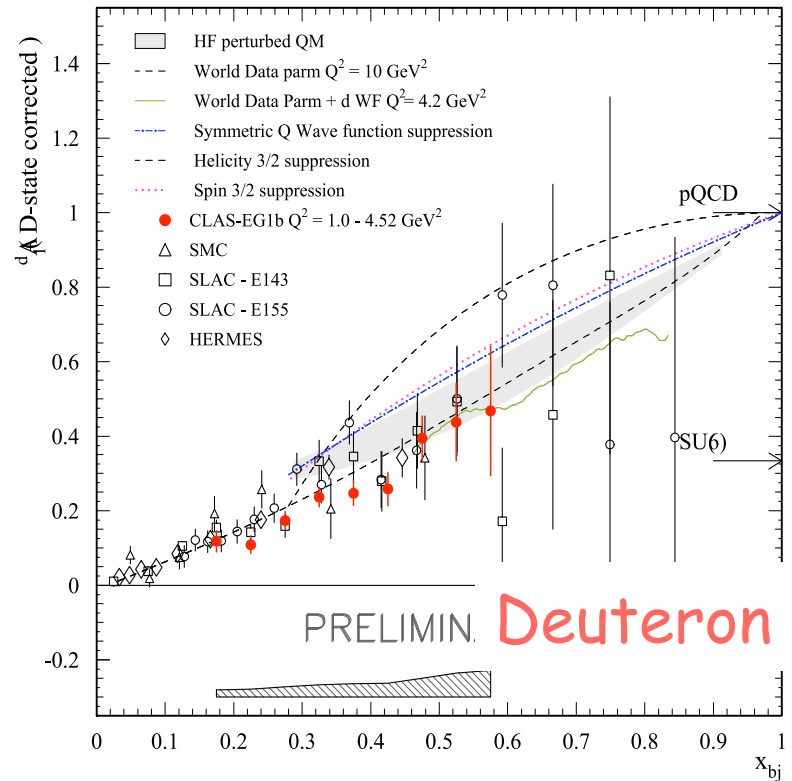
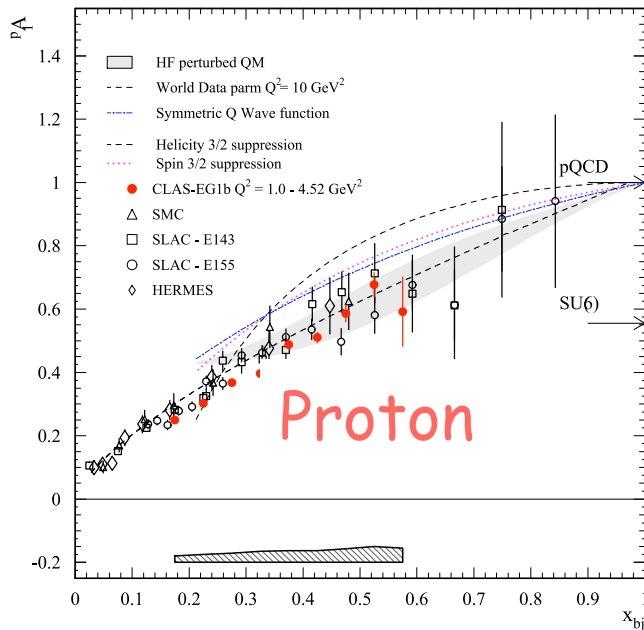
A₁ Data from EG1



$$\sim g_1/F_1$$

Close and Melnitchouk, PRC
68, 035210 (2003)

Isgur, PRD **59, 034013 (2003)**

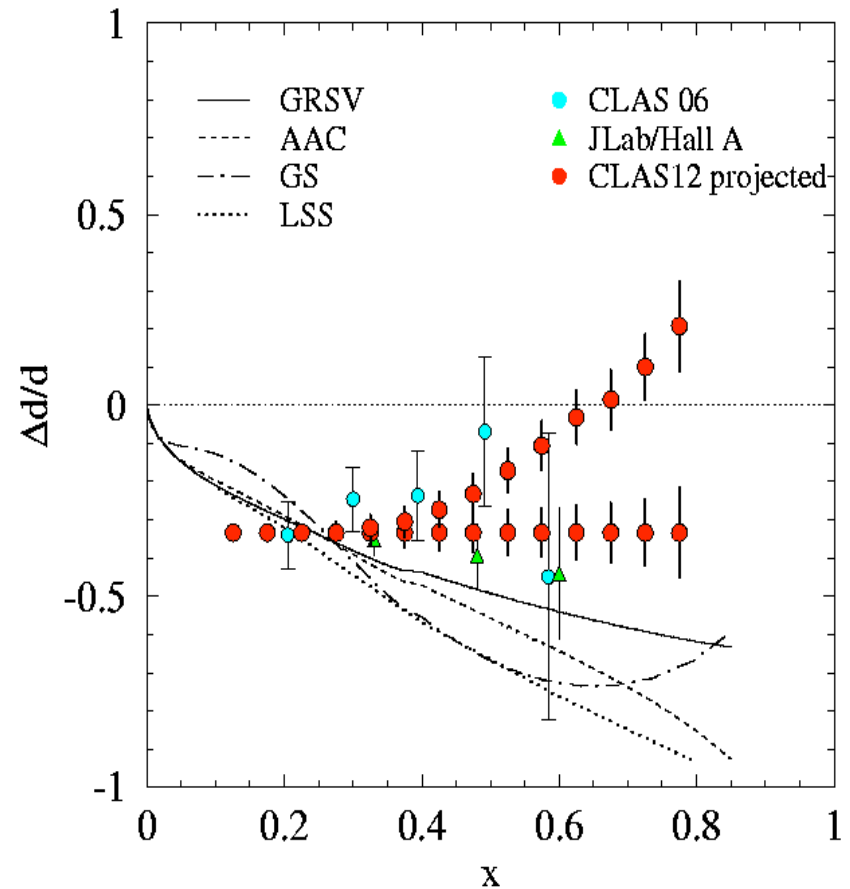
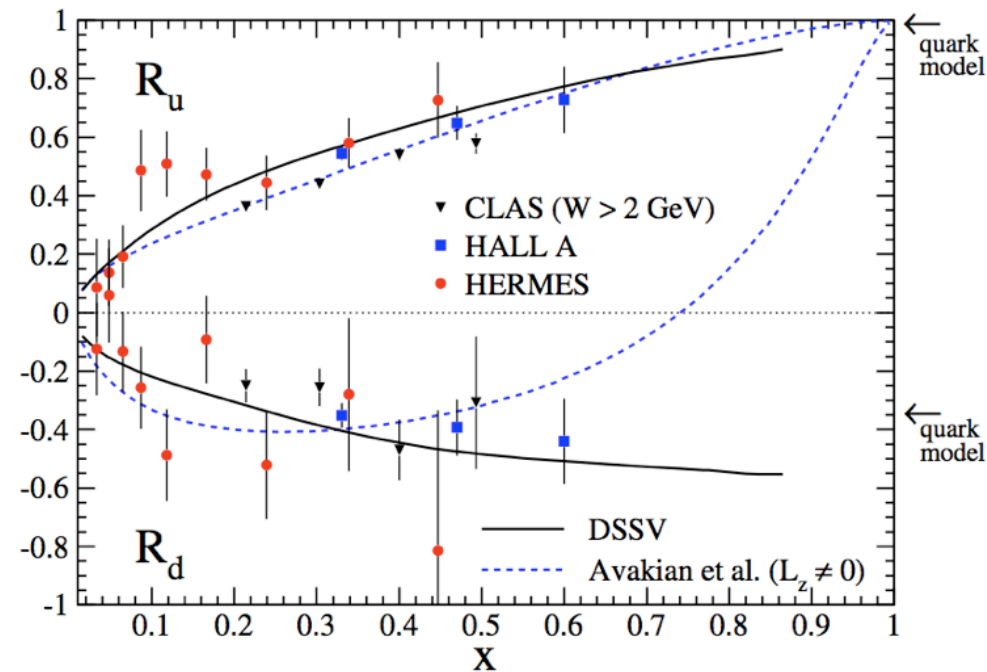




$$A_1(x, Q^2) = \frac{\sum e_i^2 \Delta q_i(x, Q^2)}{\sum e_i^2 q_i(x, Q^2)}$$

$$R_u = \Delta u/u; R_d = \Delta d/d$$

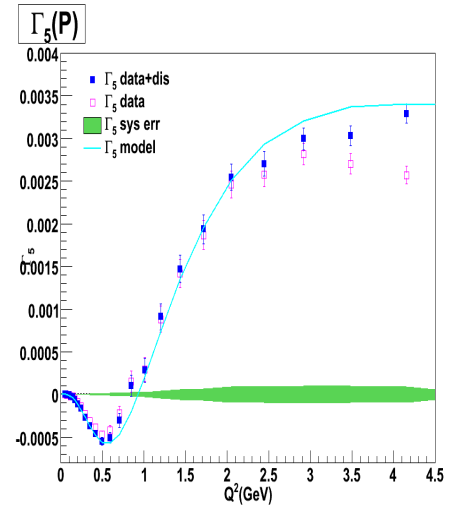
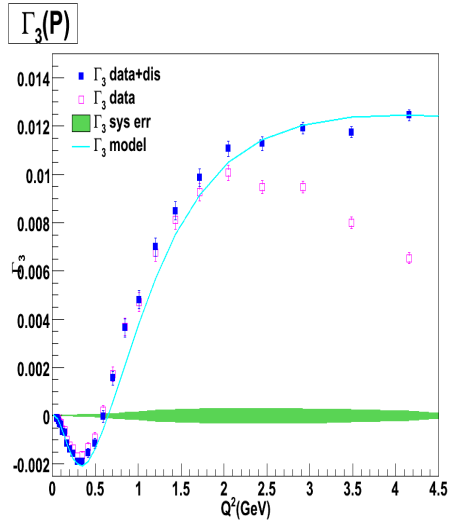
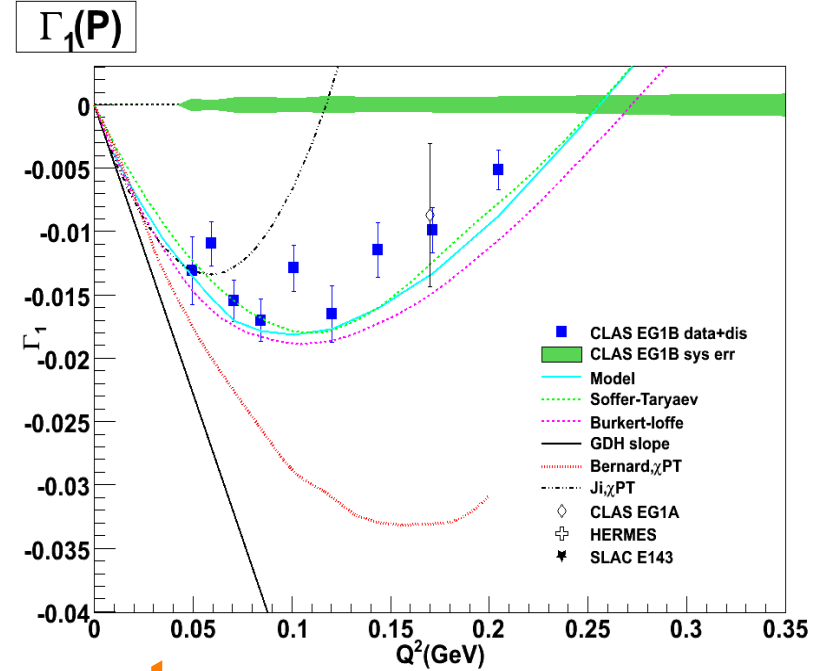
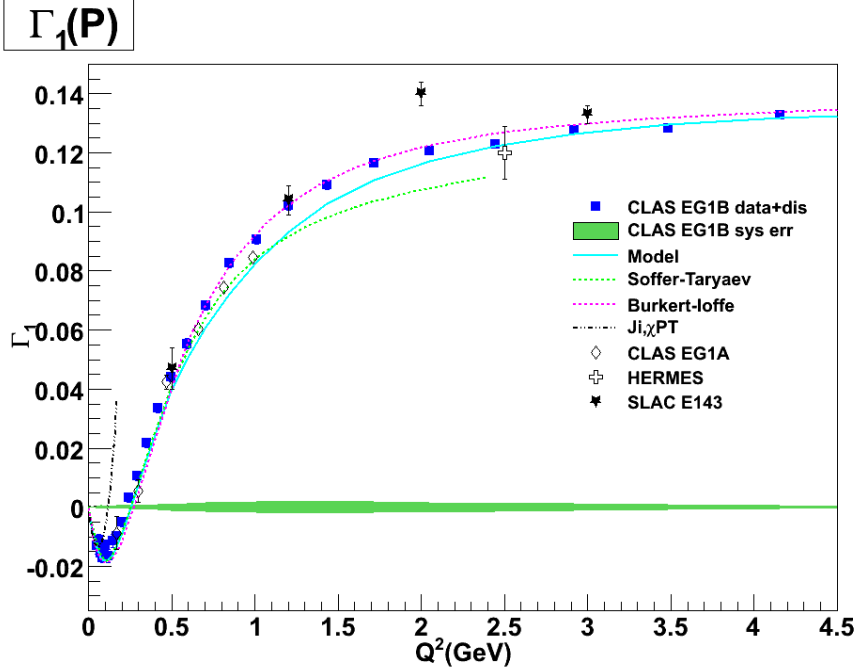
Simulated Data for EG12
Extracted from A_1^p , A_1^d and d/u





Moments

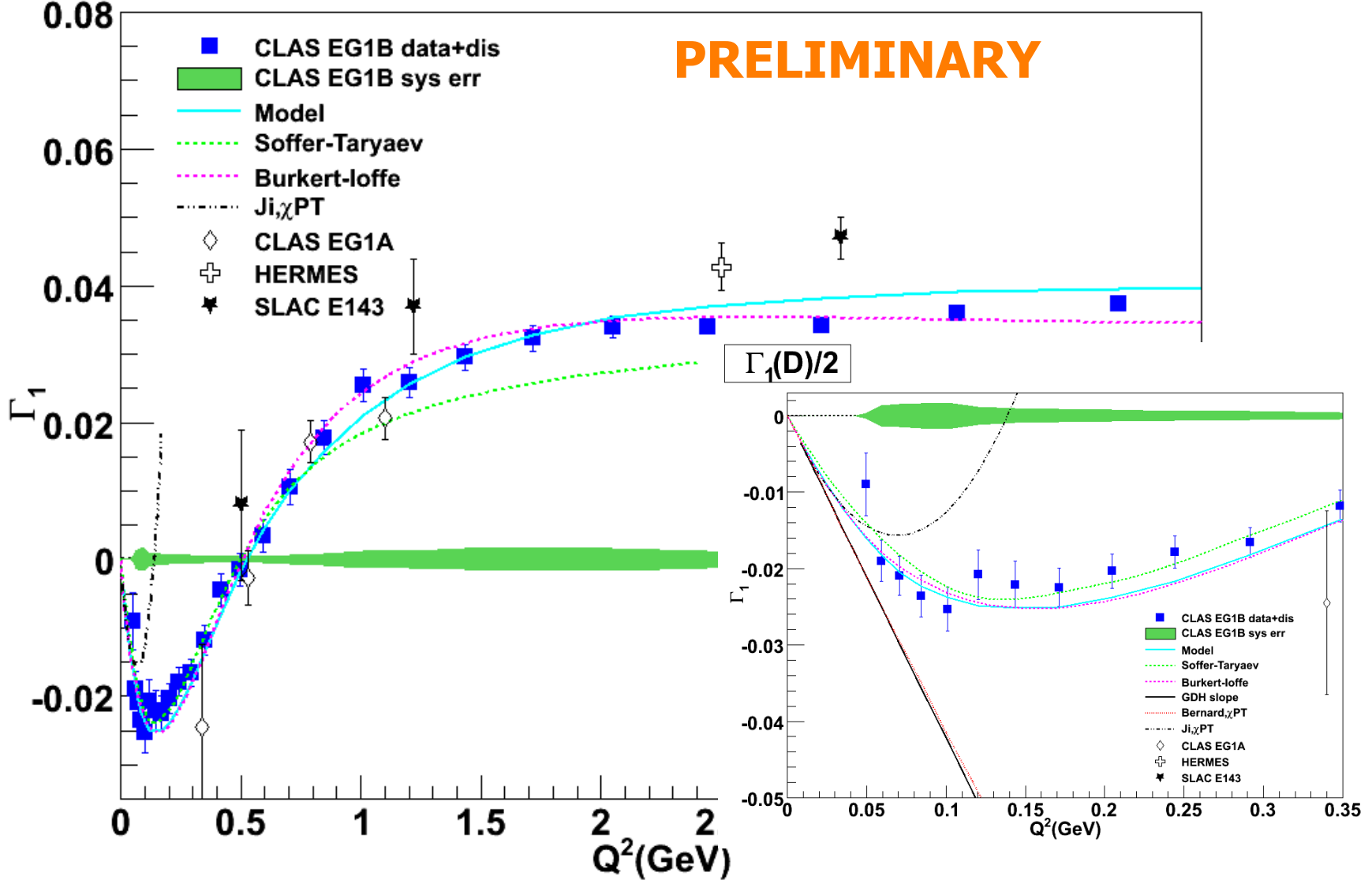
Accurate moments of structure functions (x-weighted integrals) can now be constructed from experimental data over a wide range of Q^2 . These directly show the changing landscape with momentum transfer, and are often easier to compare with theoretical calculations.



PRELIMINARY



$\Gamma_1(D)/2$





Higher Twist

The Operator Product Expansion of QCD sorts quark-gluon correlations into higher twists, which fall off inversely with powers of Q^2 . Measurements at intermediate Q^2 have been able to extract these higher-twist coefficients d_2 and f_2 and the related color electric and magnetic susceptibilities.

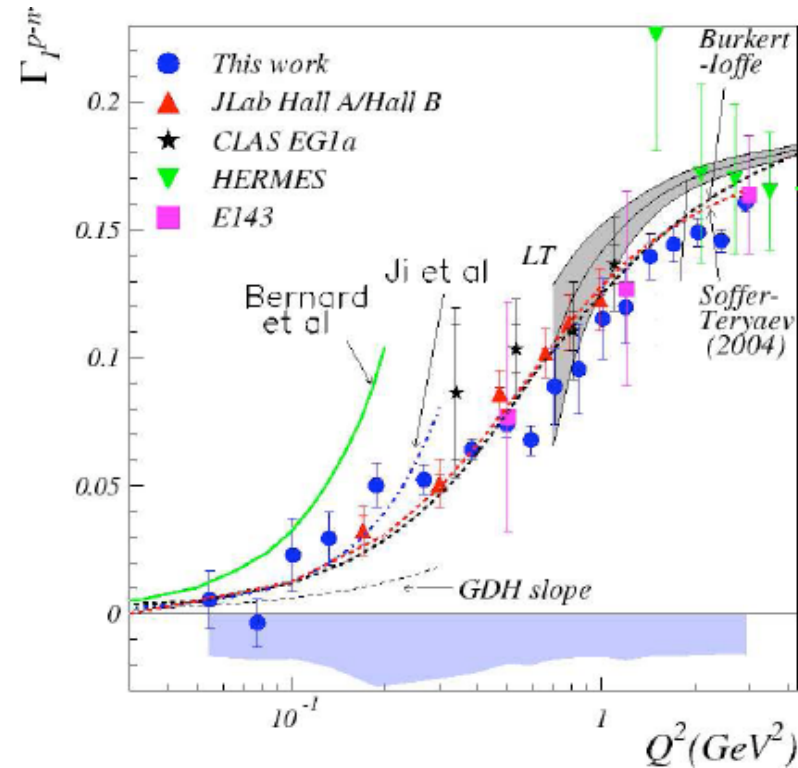


$$\Gamma_1^{(n)} = \int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2}, \quad n=0,2,4,\dots,$$

$$\Gamma_2^{(n)} = \int_0^1 x^n g_2(x, Q^2) dx = \frac{1}{2} \frac{n}{n+1} (d_n - a_n), \quad n=2,4,\dots,$$

Bjorken Sum Rule:

CLAS: Deur

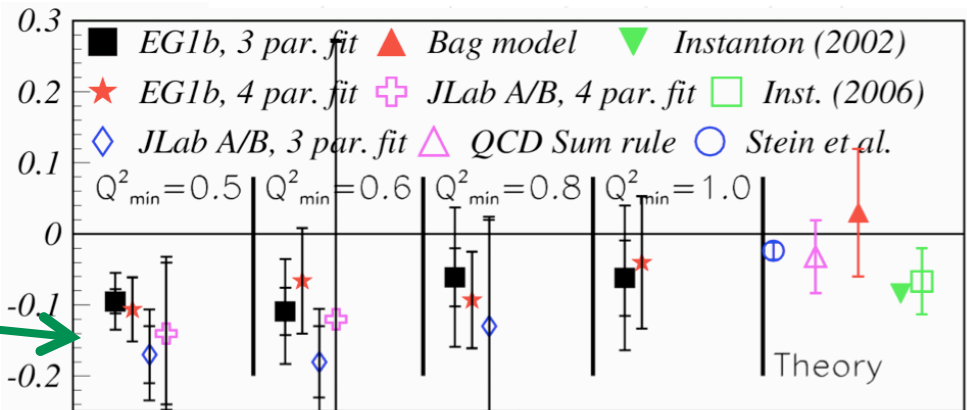


$$\Gamma_1^{p-n} = \frac{g_A}{6} \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s}{\pi} \right)^3 \right] + \frac{\mu_4^{p-n}}{Q^2} + \dots$$

$$\mu_4^{p-n} = \frac{M^2}{9} (a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n})$$

$$d_2^{p-n} = \int_0^1 dx x^2 (2g_1^{p-n} + 3g_2^{p-n})$$

Fit Γ_1^{p-n} to powers of $1/Q^2$ and extract f_2^{p-n}



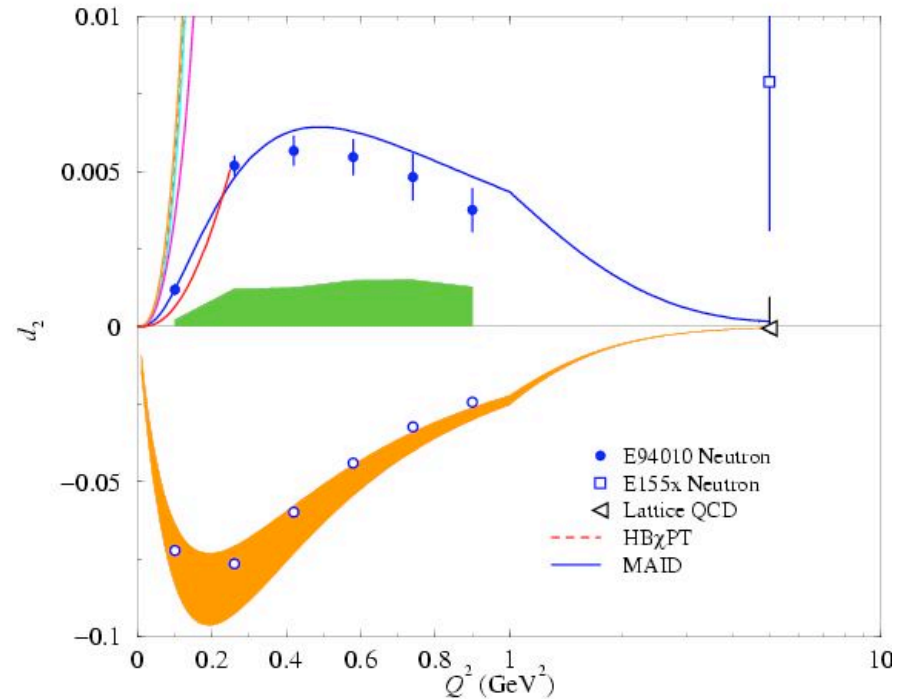
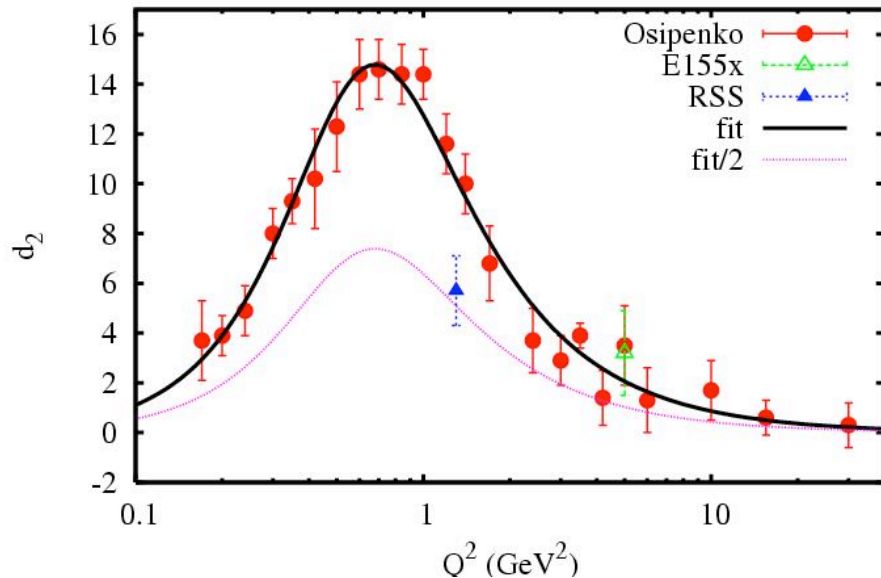


$$d_2(Q^2) = \int_0^1 dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)]$$

CLAS EG1 (proton)
Osipenko, PRD71(05)054007
Model-dependent determination

Hall A (neutron)
E94-010
Amarian, PRL92(04)022301

d_2 fit to $aQ^b/(1+cQ^d)$





CLAS, Osipenko

PLB609(05)259

$$M_1(Q^2) = \int_0^1 dx \frac{\xi^2}{x^2} \left\{ g_1(x, Q^2) \left(\frac{x}{\xi} - \frac{1}{9} \frac{M^2 x \xi}{Q^2} \right) - g_2(x, Q^2) \frac{4}{3} \frac{M^2 x^2}{Q^2} \right\},$$

$$\xi = 2x / (1 + \sqrt{1 + 4M^2 x^2 / Q^2})$$

$$M_1(Q^2) = \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + \dots$$

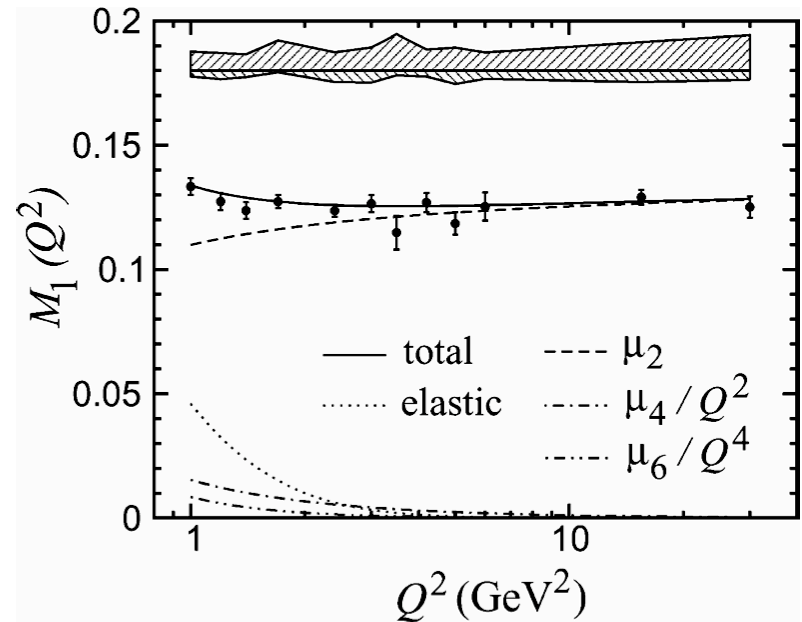
$$\mu_4(Q^2) = 4f_2(Q^2)/9M^2$$

$$f_2 = 0.039 \pm 0.022(\text{stat}) \pm 0.018^{0.000}(\text{sys}) \\ \pm 0.030(\text{low } x) \pm 0.011^{0.007}(\alpha_s),$$

$$d_2(Q^2) = \int_0^1 dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)]$$

$$\chi_E = \frac{2}{3} (2d_2 + f_2)$$

$$\chi_B = \frac{1}{3} (4d_2 - f_2)$$

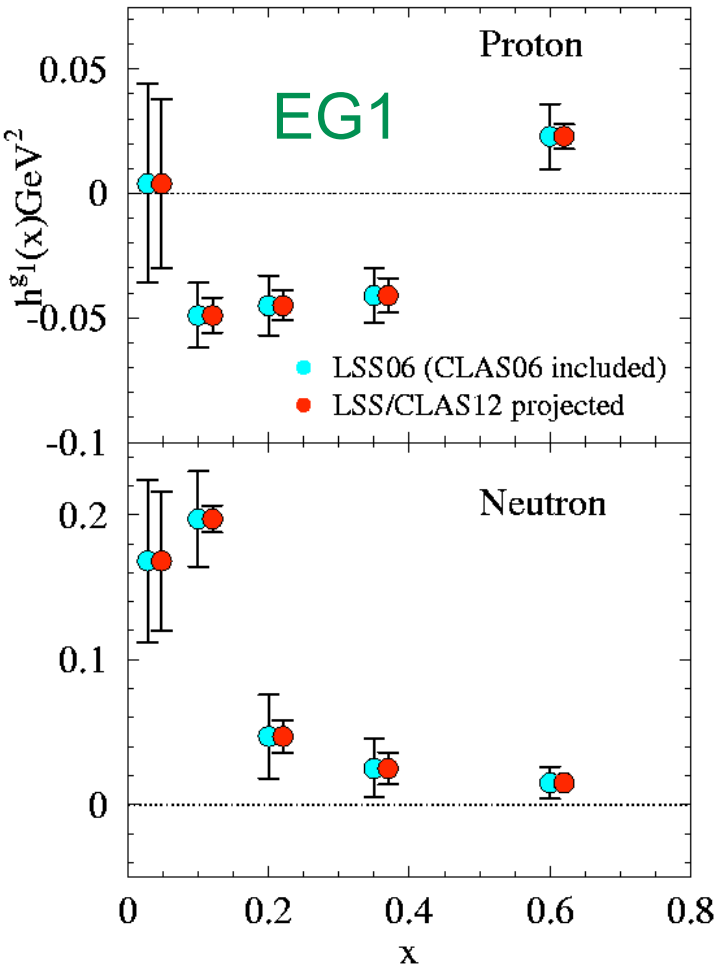


$$\chi_E = 0.026 \pm 0.015(\text{stat}) \pm 0.024^{0.021}(\text{sys}),$$

$$\chi_B = -0.013 \mp 0.007(\text{stat}) \mp 0.012^{0.010}(\text{sys})$$



$$\left[\frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right]_{\text{exp}} F_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + h^{g_1}(x)/Q^2$$



$$1 < Q^2 < 5 \text{ GeV}^2, \quad 2 < W < 3.5 \text{ GeV}$$

$$\int_0^1 dx h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)$$

- F_1 from NMC fit to F_2 and 1998 SLAC fit to R
- g_1 (leading twist) from NLO fit at high Q^2
- h from fit to all data, especially CLAS in the pre-asymptotic region
- d_2 : twist-3, f_2 : twist-4

Leader, Sidorov, Stamenov, EPJST**162**(08)19



- Osipenko, CLAS, proton, PLB609(05)249
– $f_2 = 0.039(39)$ $\chi_E = 0.026(27)$ $\chi_B = -0.013(13)$
- E94-010, Hall A, neutron
– $f_2 = 0.034(43)$ $\chi_E = 0.033(29)$ $\chi_B = -0.001(16)$
- Deur, CLAS, Bjorken (p-n)
– $f_2 = -0.101(74)$ $\chi_E = -0.077(50)$ $\chi_B = 0.024(28)$
- More accurate determinations are needed.



Duality

When structure functions are averaged over resonance peaks and valleys, they behave just like deep-inelastic scattering extrapolated into the resonance region. Local duality (one resonance region) and global duality (all resonances) tend to hold to 10% above $Q^2=2$ GeV^2 , except for the polarized Δ resonance.



Hall C

PRL85(00)1182

Global duality to 10%

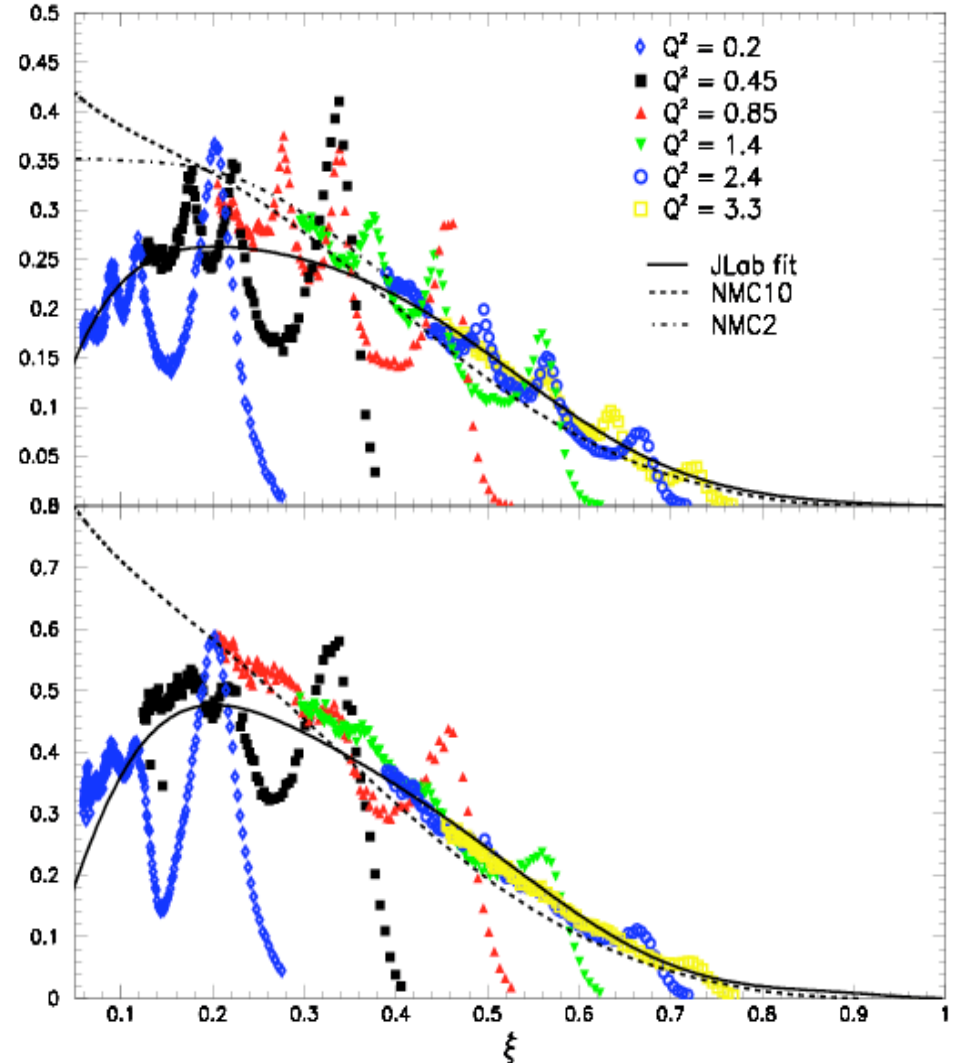
Local duality to 10%

$W=1.232, 1.535, 1.680$ GeV

Duality - structure functions averaged over resonances behave according to DIS systematics

Global - all resonances

Local - one resonance





Hall C

RSS, Wesselmann, Slifer

$Q^2=1.379 \text{ GeV}^2$

Target Mass Corrections
applied to PDFs

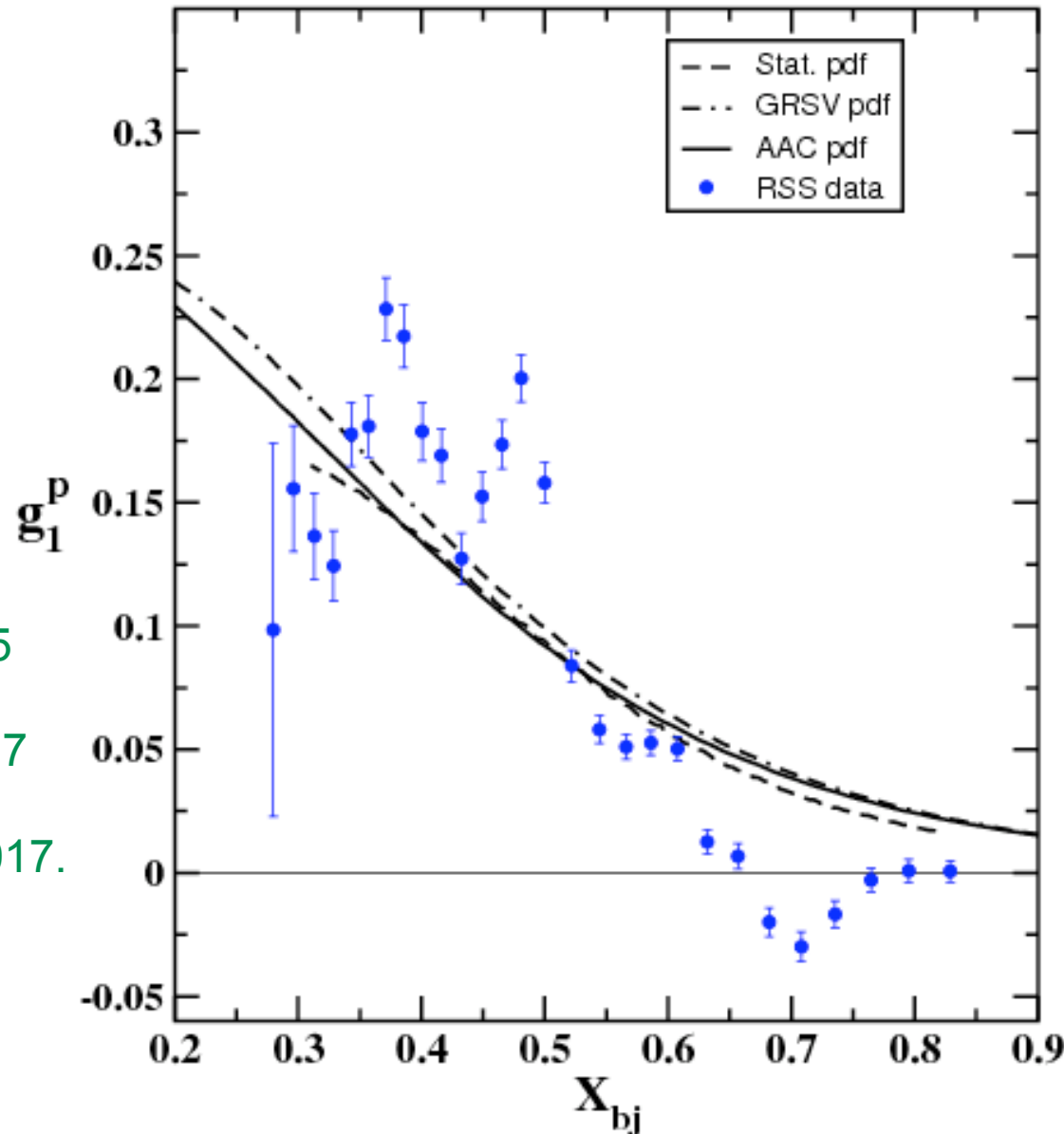
No duality for Δ

PRL98(07)132003

GRSV: Phys. Rev. D 53, (1996) 4775

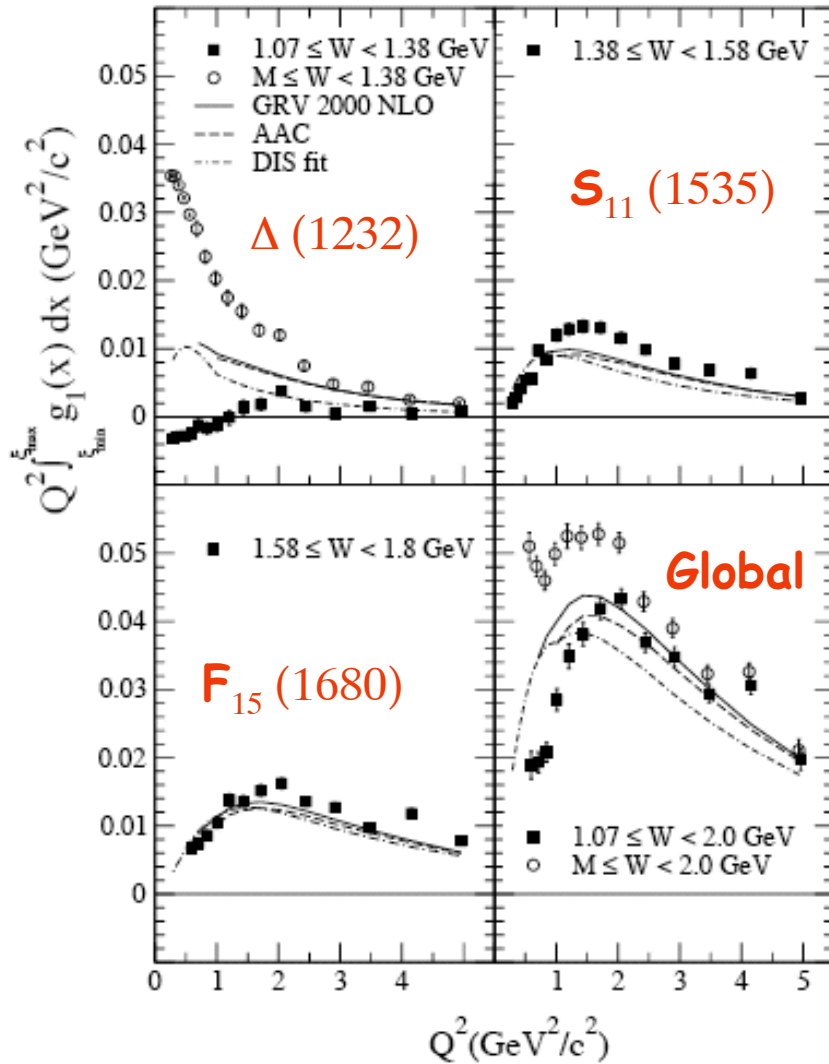
BSB : Eur. Phys. J. C 41, (2005) 327

AAC : Phys. Rev. D 62, (2000) 034017.

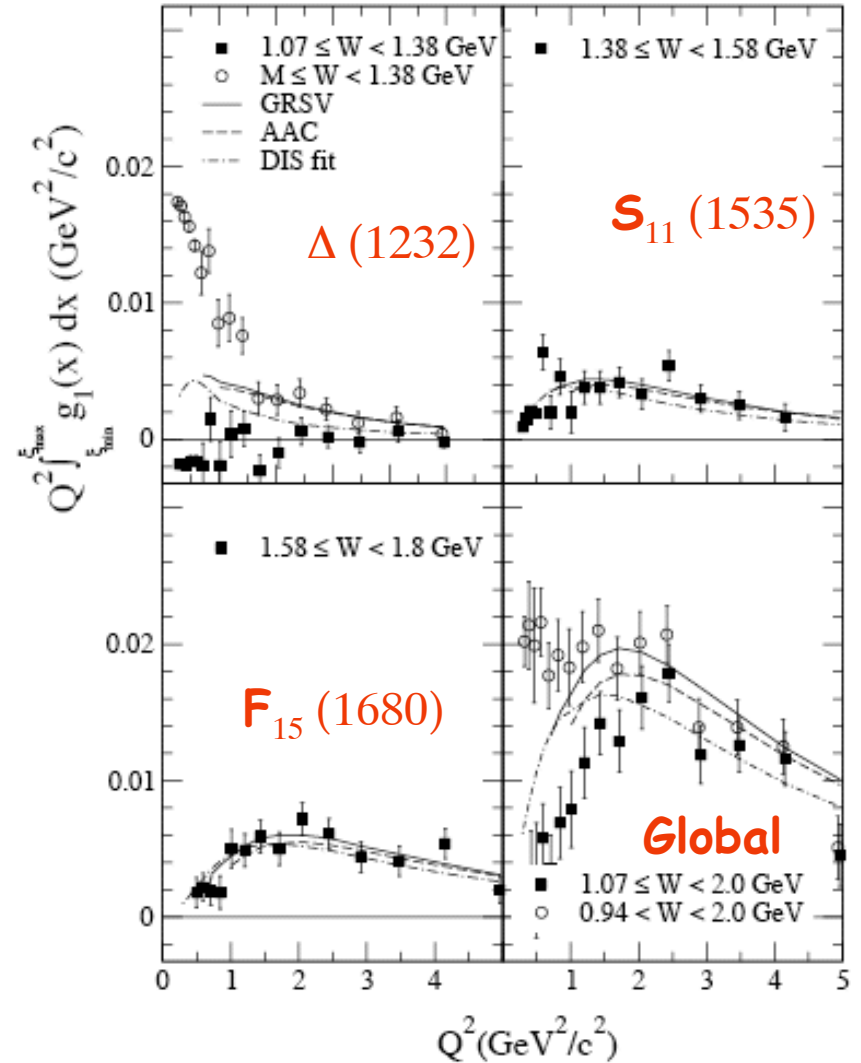




Proton



Deuteron





g_2

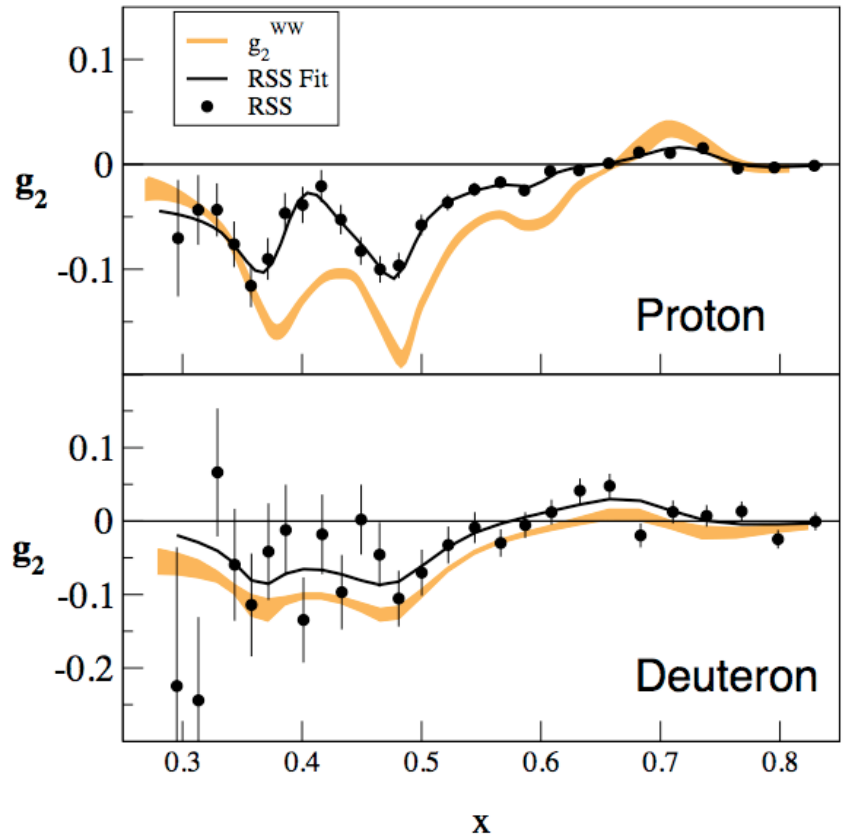
The structure function g_2 in pQCD can be expressed as a convolution of g_1 . Deviations from this Wandzura-Wilczek form measures higher twist. The Burkhardt-Cottingham sum rule states that the first moment of g_2 is zero. Precise data are now available to evaluate this claim.



Wandzura-Wilczek

$$g_2^{WW} = -g_1 + \int_x^1 \frac{g_1}{y} dy$$

$$Q^2=1.28 \text{ GeV}^2 \quad g_2 = g_2^{WW} + g_2^-$$

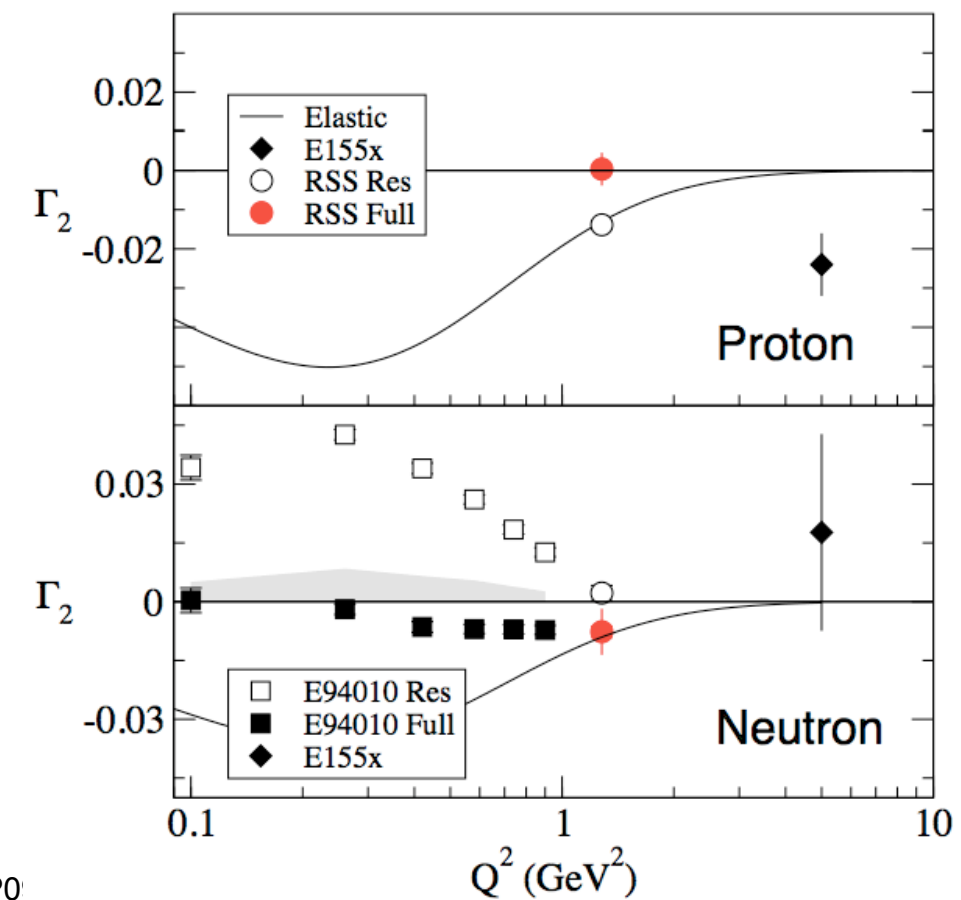


RSS g_2^p

Burkhardt-Cottingham Sum Rule

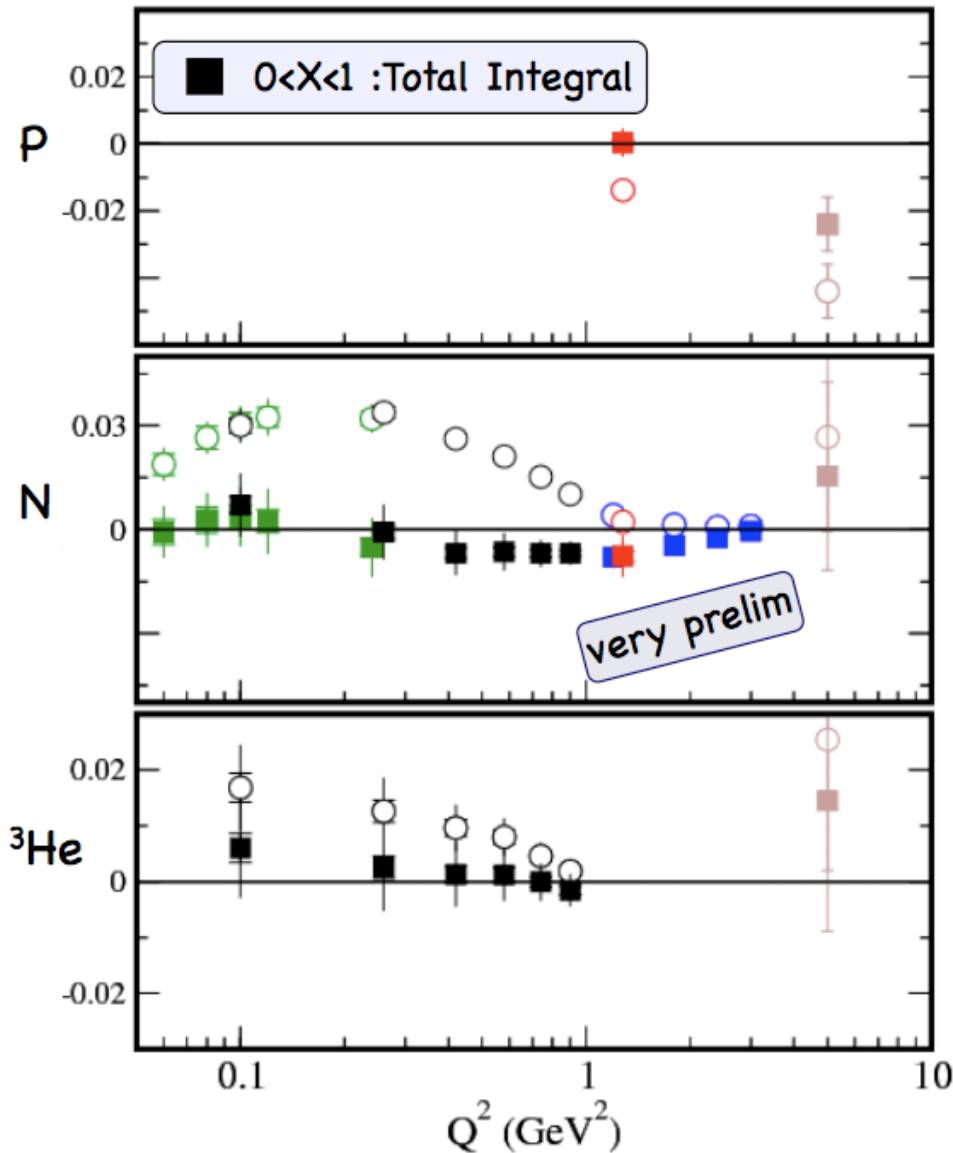
$$\Gamma_2 = \int_0^1 g_2(x, Q^2) dx = 0$$

Hall C: Slifer et al. arXiv:0812.0031





Burkhardt Cottingham Sum



- Plot: K. Slifer
- Open points: Measured
- Solid points: Corrected for unmeasured regions and elastic contribution
- Green: Hall A E97-110
- Blue: Hall A E01-012
- Red: Hall C RSS
- Black: Hall A E94-010
- Brown: SLAC E155

Burkhardt & Cottingham,
Ann. Phys. **56**(70)453

$$\int_0^1 g_2(x, Q^2) dx = 0$$



Spin Polarizabilities

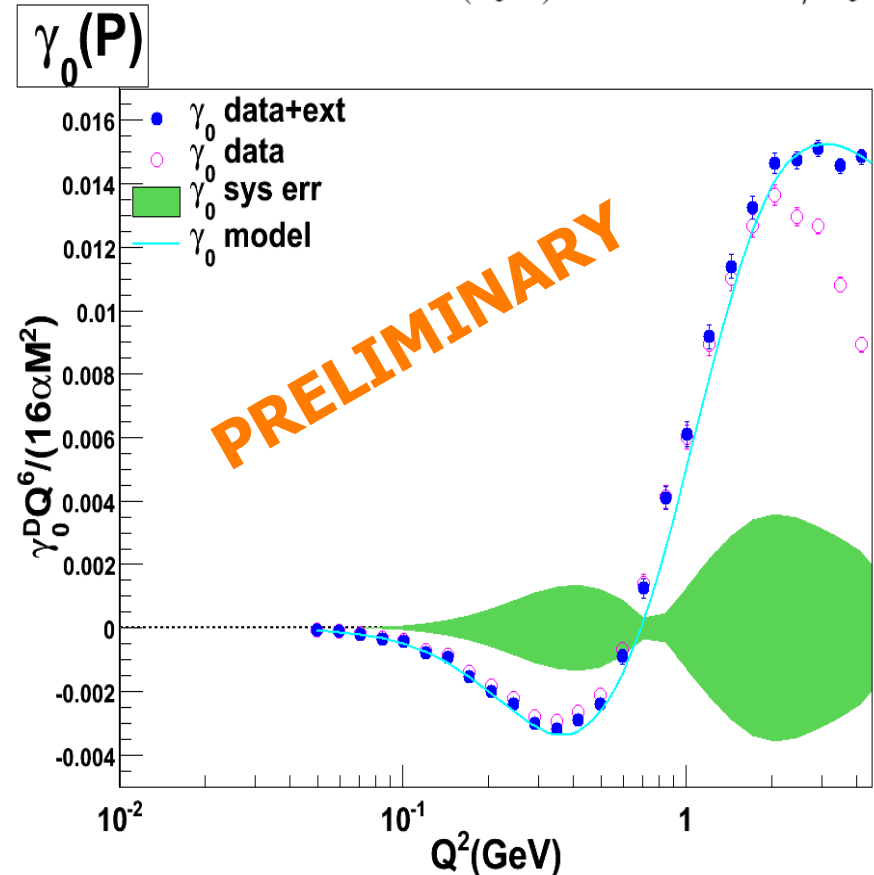
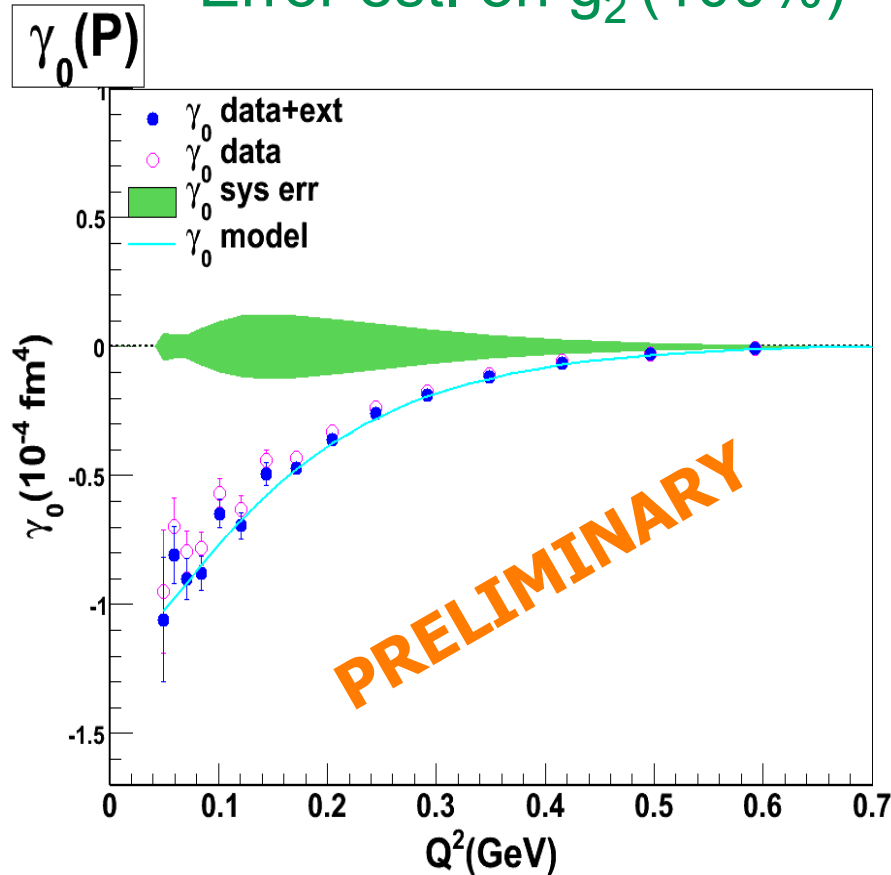
The spin polarizability can be expressed in terms of moments of g_1 and g_2 . How this evolves at low Q^2 from the real photon point provides a rigorous test of chiral perturbation theory.



$$\gamma_0(Q^2) = C(Q^2) \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right\} dx$$

Error est. on g_2 (100%)

$$C(Q^2) = 16\alpha M^2 / Q^6$$



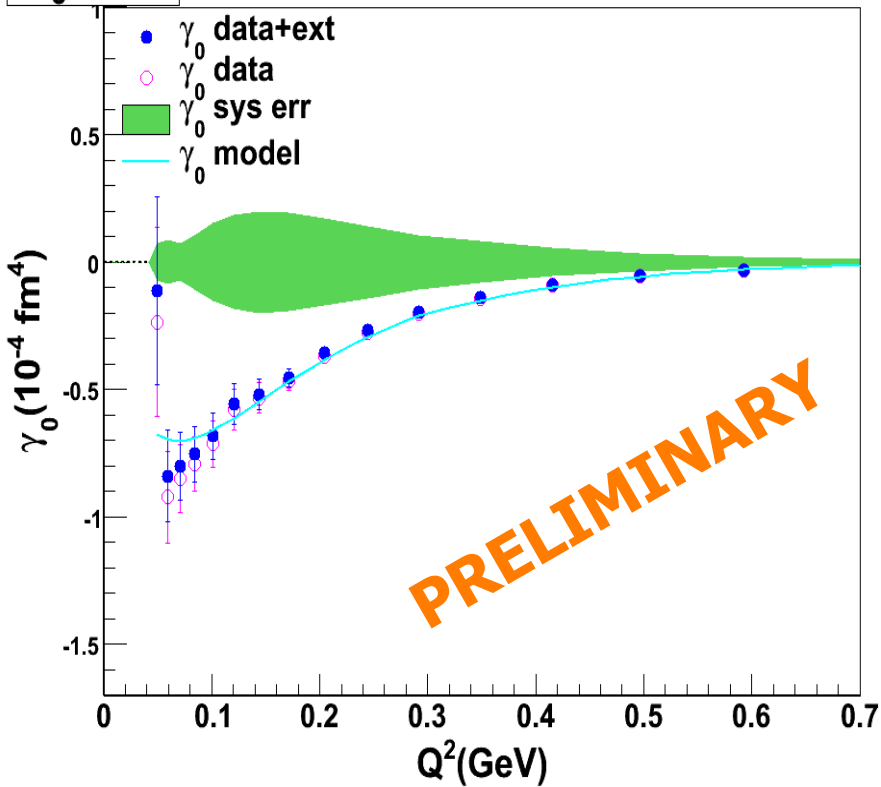


$$\gamma_0(Q^2) = C(Q^2) \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right\} dx$$

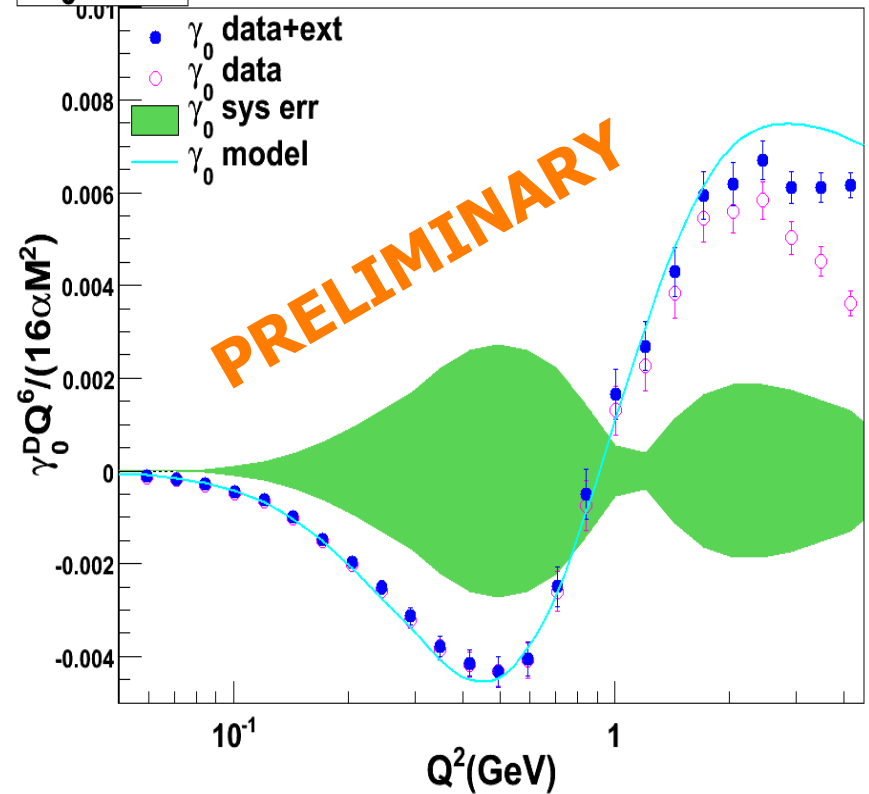
$$C(Q^2) = 16\alpha M^2 / Q^6$$

Error est. on g_2 (100%)

$\gamma_0(D)/2$



$\gamma_0(D)/2$

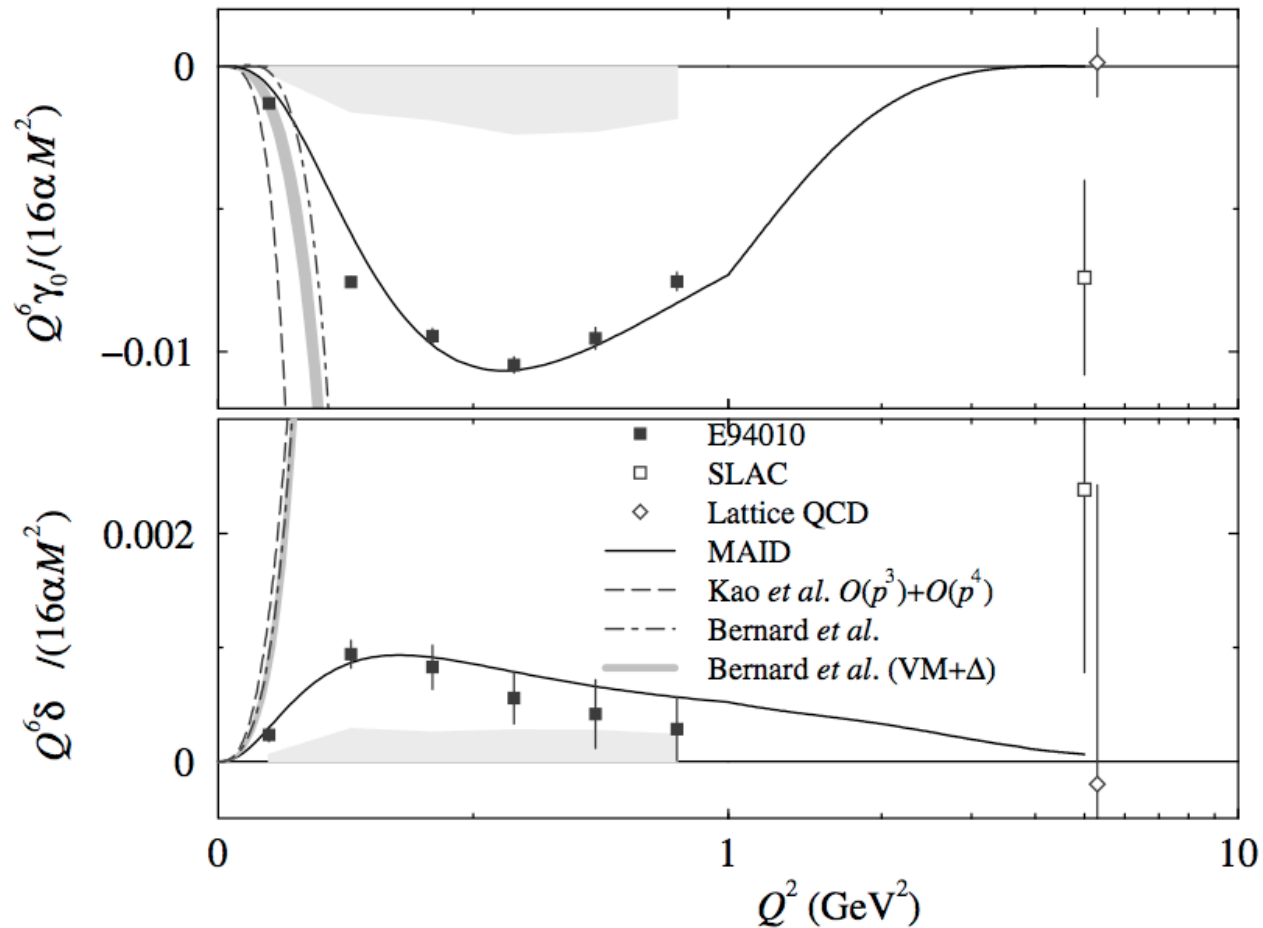




$$\delta_{LT}(Q^2) = \left(\frac{1}{2\pi^2}\right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu$$

$$= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1(x, Q^2) + g_2(x, Q^2)] dx$$

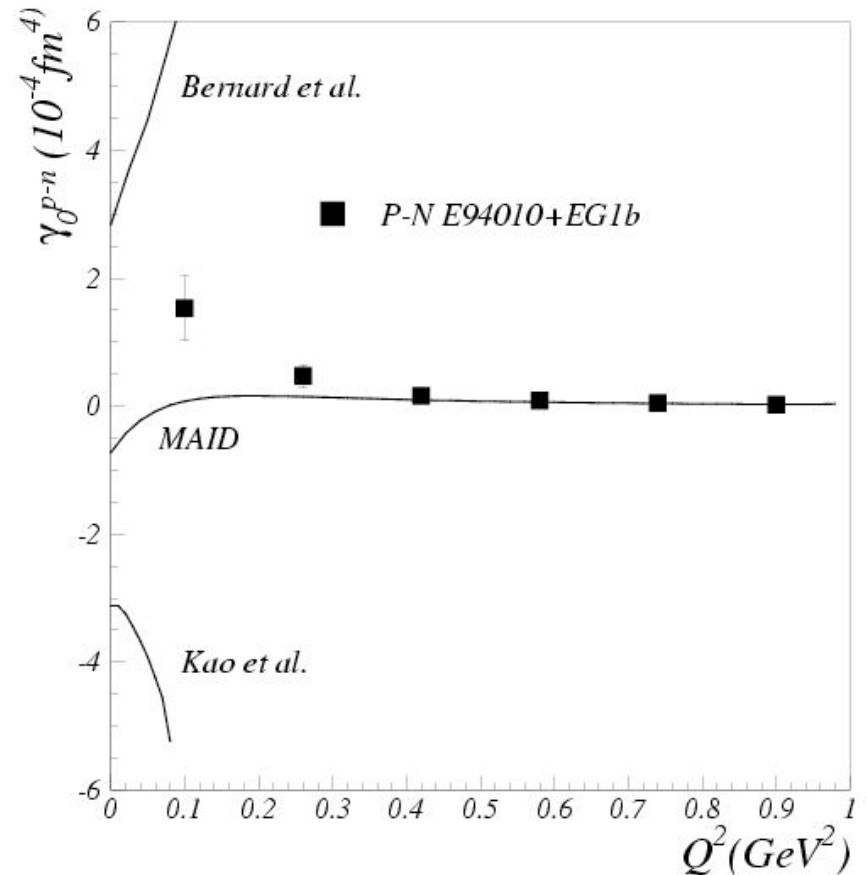
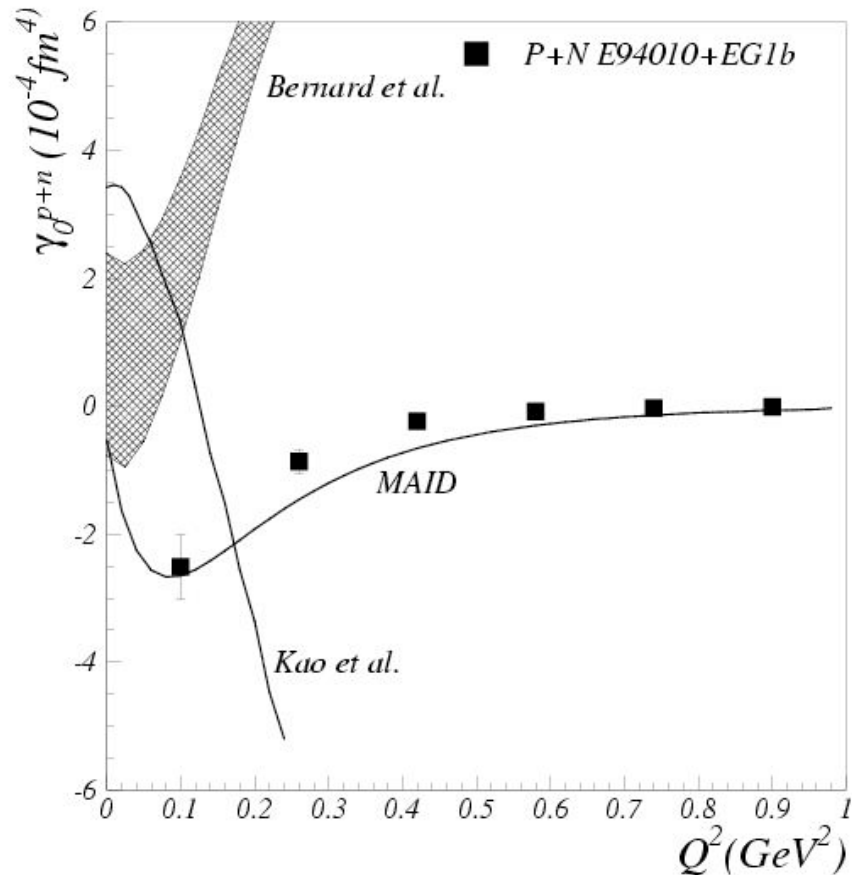
Hall A
Amarian,
PRL93(04)1152301





A Deur
CLAS + Hall A

For isovector (p-n) case
 Δ contribution cancels





The GDH Sum Rule

At low Q^2 , the evolution of the first moment of g_1 should be proportional to Q^2 and to the square of the anomalous magnetic moment of the nucleon. Data are now available to test this.



Energy-Weighted Sum Rule

$$S(F) = \sum_a (E_a - E_0) |\langle a | F | 0 \rangle|^2 = \langle 0 | [F, [H, F]] | 0 \rangle$$

GDH Sum Rule

$$\int_{k_\pi}^{\infty} \frac{dk}{k} \Delta \sigma^{\gamma N}(k) = \frac{2\pi^2 \alpha \kappa^2}{M^2}$$

$$\Delta \bar{\sigma}^{\gamma N} = \sigma_{3/2}^{\gamma N} - \sigma_{1/2}^{\gamma N}$$

Sum over excited states is tied to property of ground state



$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x, Q^2) dx \quad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

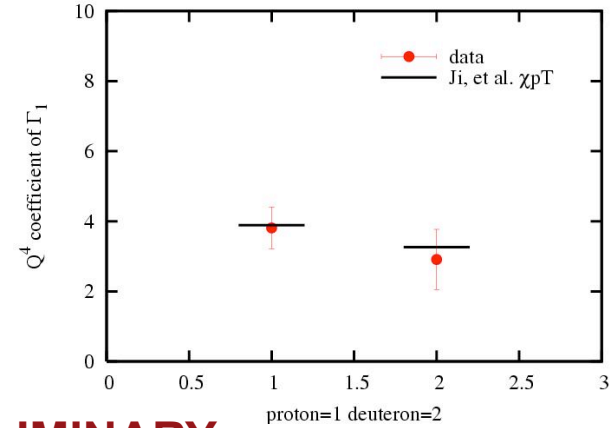
$$\Gamma_1^d(Q^2) = (1 - 1.5\omega_D) \{ \Gamma_1^p(Q^2) + \Gamma_1^n(Q^2) \}$$

$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^z}{8M^2} Q^2 + 3.89Q^4 + \dots$$

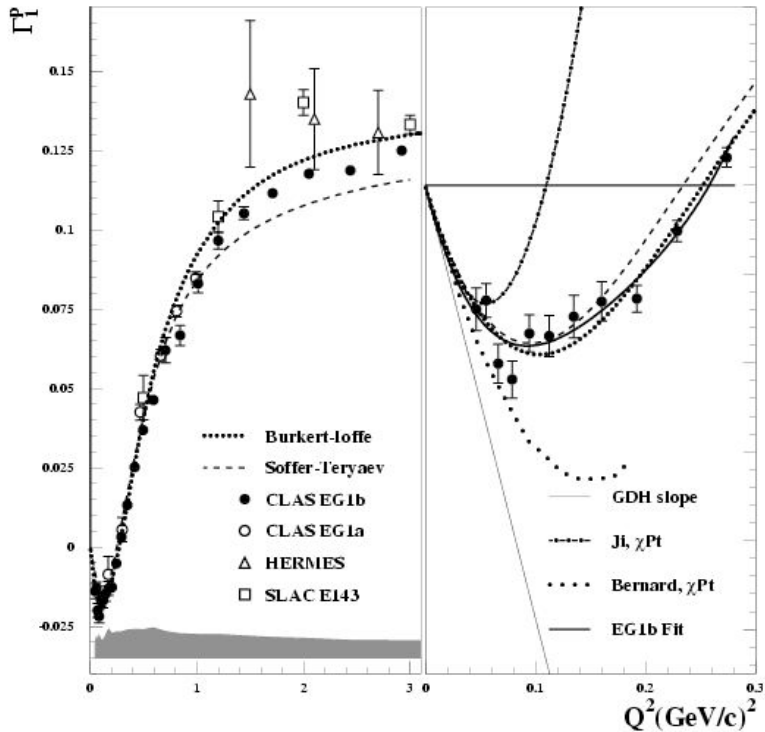
$$\Gamma_1^n(Q^2) = -\frac{\kappa_n^z}{8M^2} Q^2 + 3.15Q^4 + \dots$$

low Q^2 fit

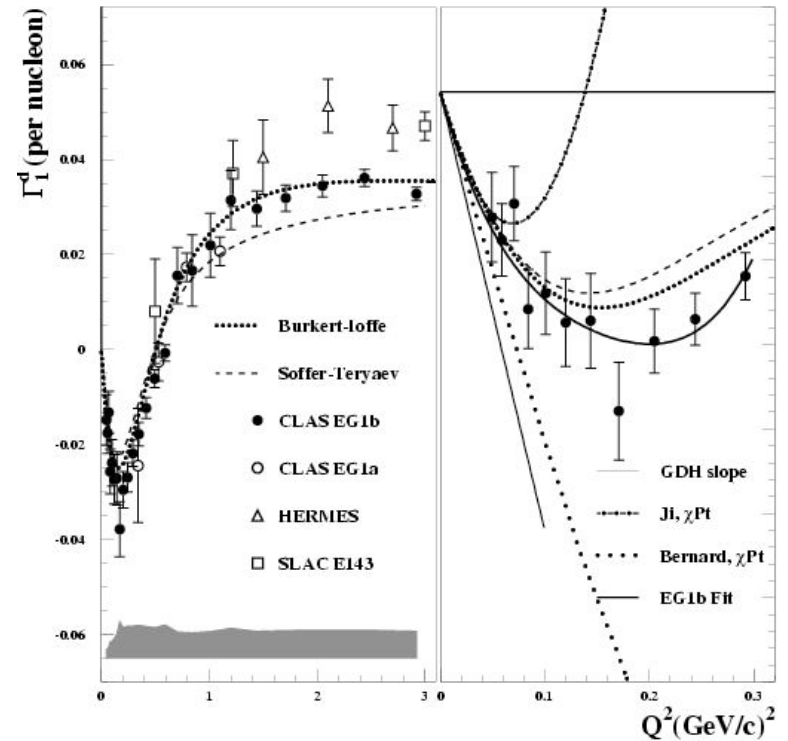
GDH + χpT

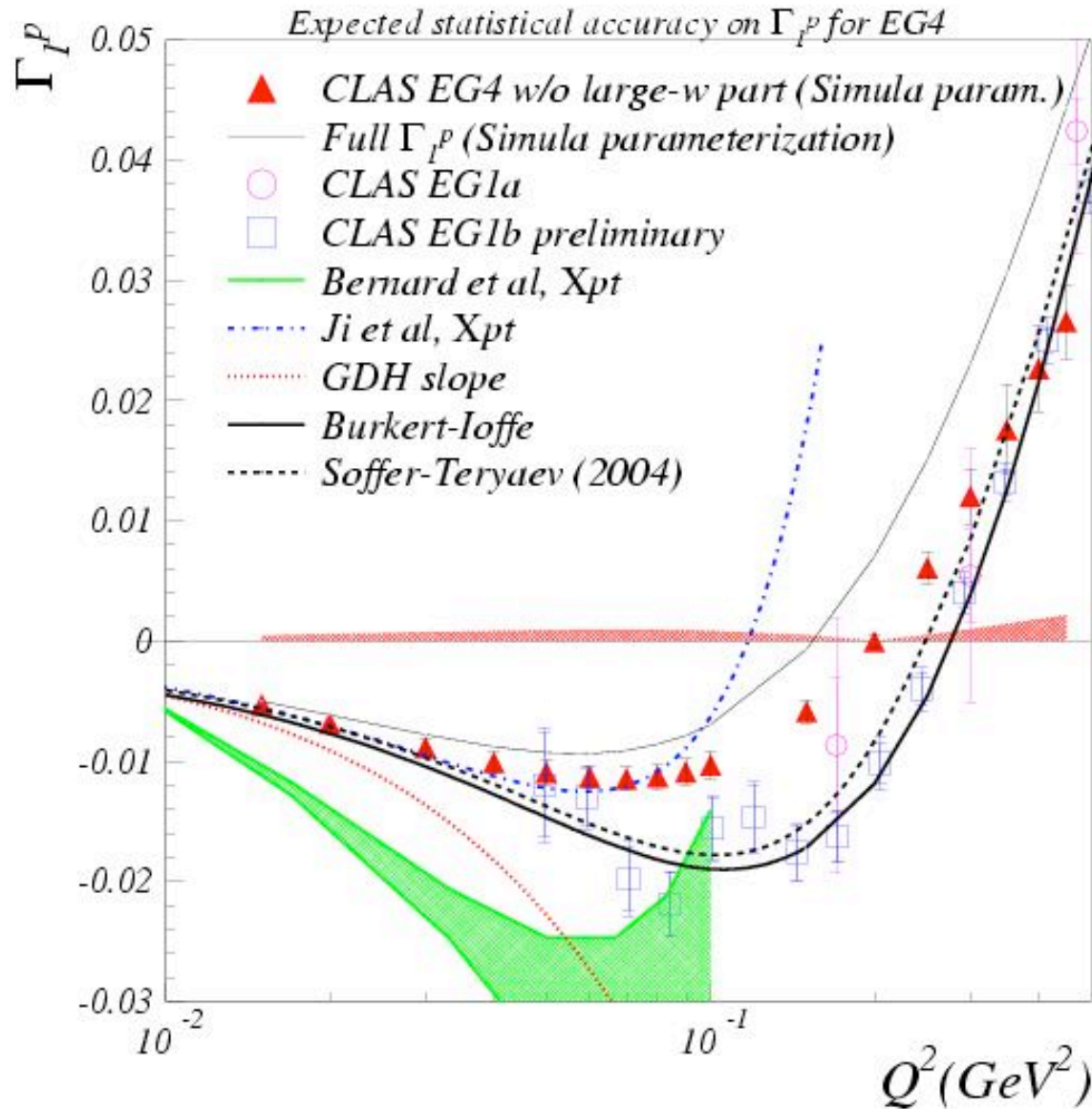


PRELIMINARY



PRELIMINARY







The Spin Crisis

Where does the spin of the nucleon come from? Only a quarter comes from the quark spins. The rest must be gluon polarization and orbital angular momentum; both of these are hard to measure. However, data are becoming available that suggest the gluon polarization is small.

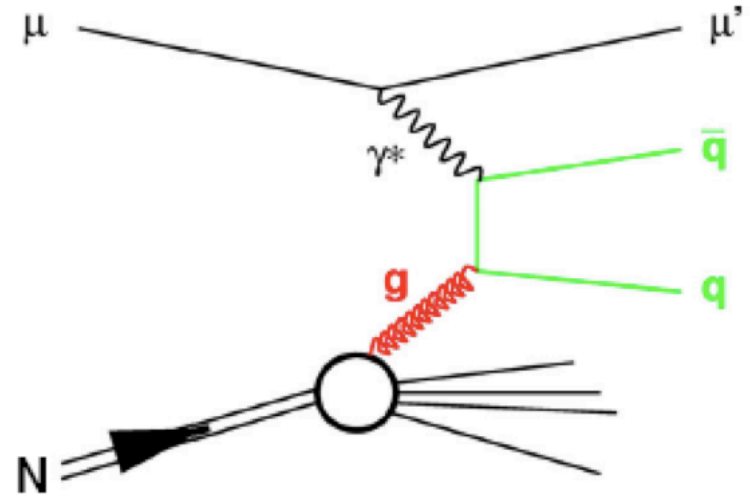
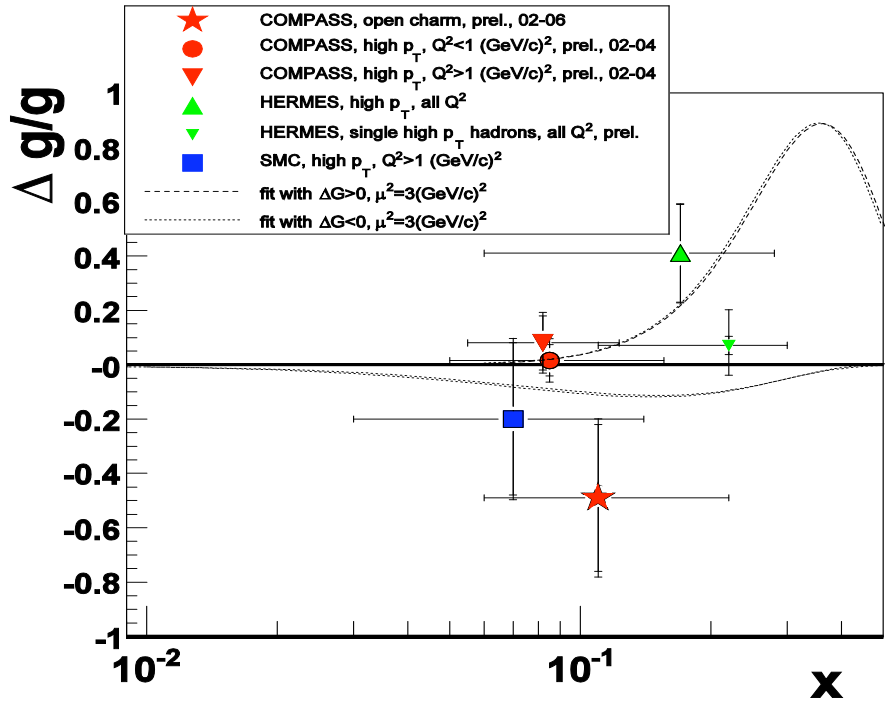


$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

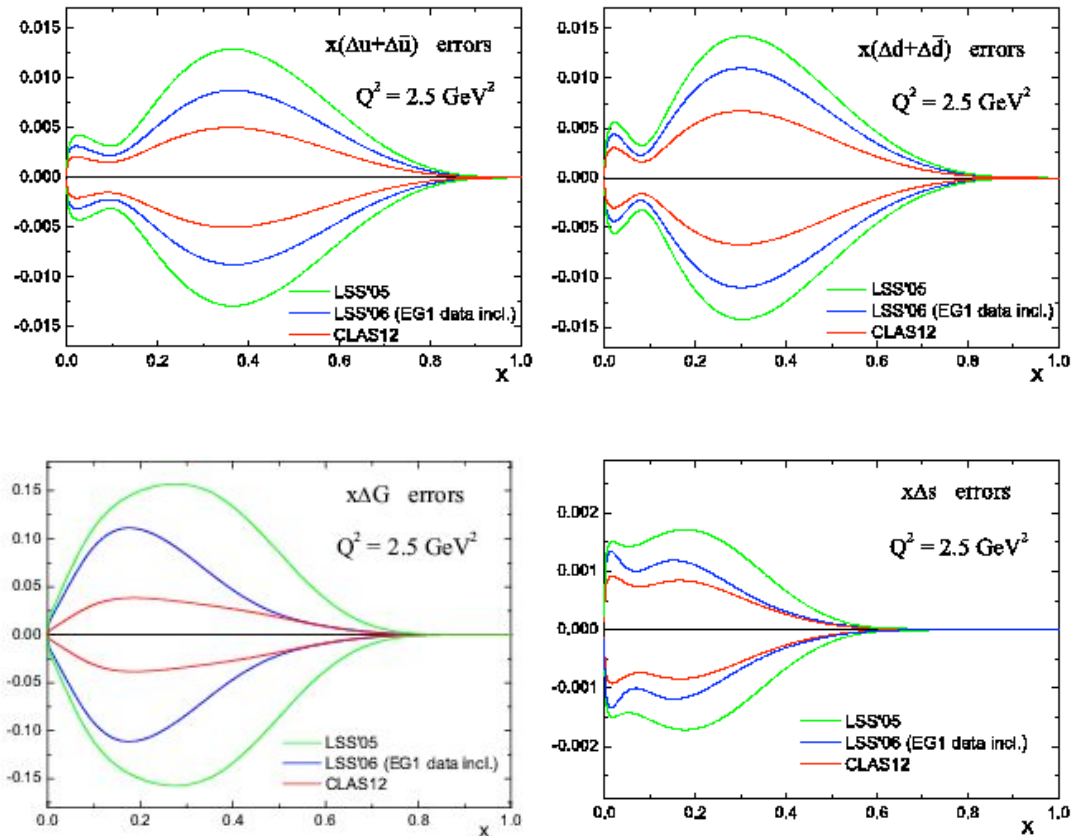
Known →

Could be constrained by QCD evolution but this requires an understanding of higher twist at moderate Q^2 . Direct measures through photon-gluon fusion and pp reactions.

Transverse momentum dependent distributions and generalized parton distributions are sensitive to quark orbital angular momentum



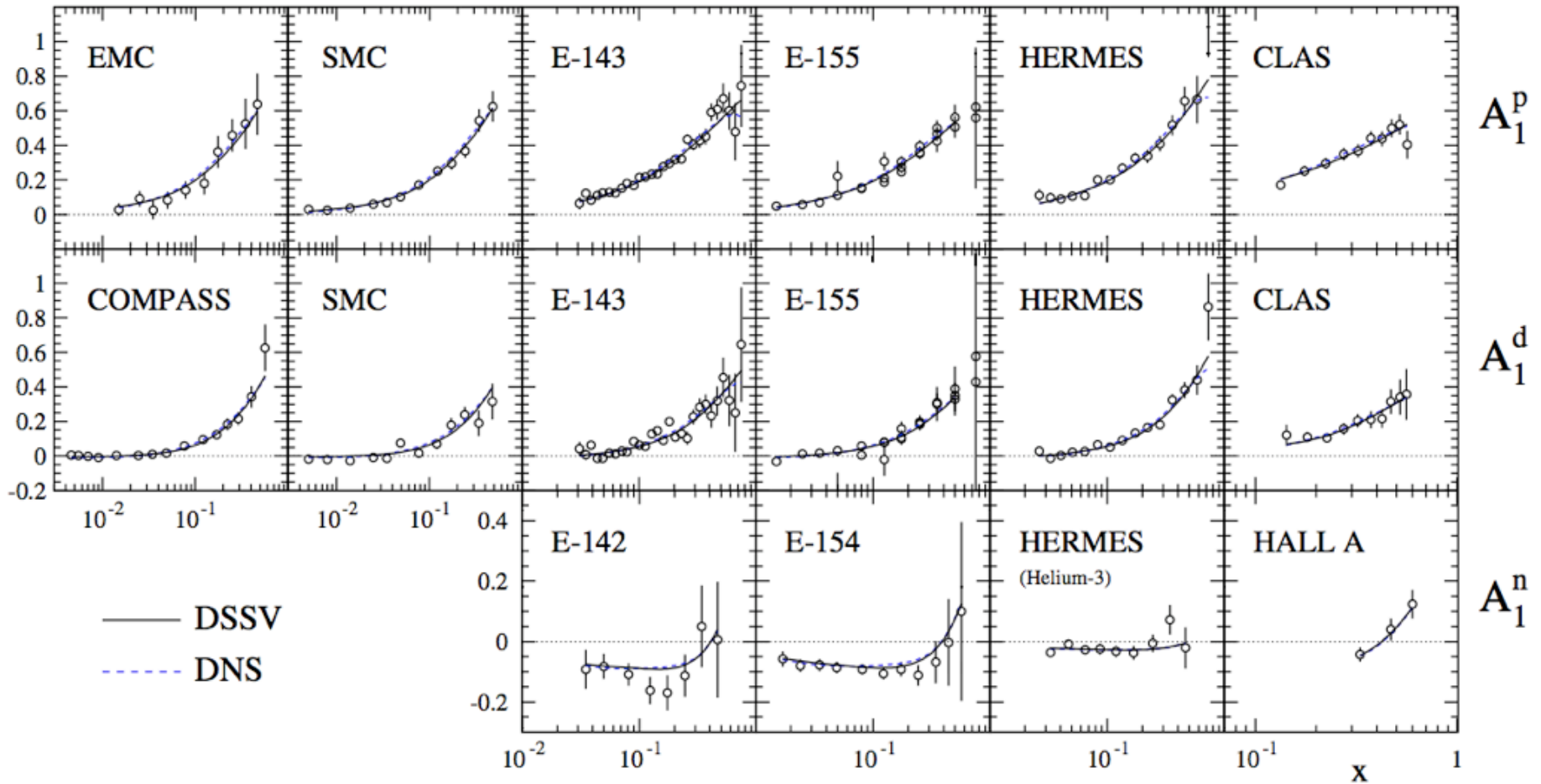
Photon-gluon fusion measurements show a relatively small polarized gluon distribution.



- Error envelopes for PDFs from LSS05 global analysis (green)
- CLAS EG1 data significantly improve errors on Δu , Δd , Δx and ΔG (blue)
- CLAS EG12 (12 GeV upgrade) will especially improve ΔG (red)



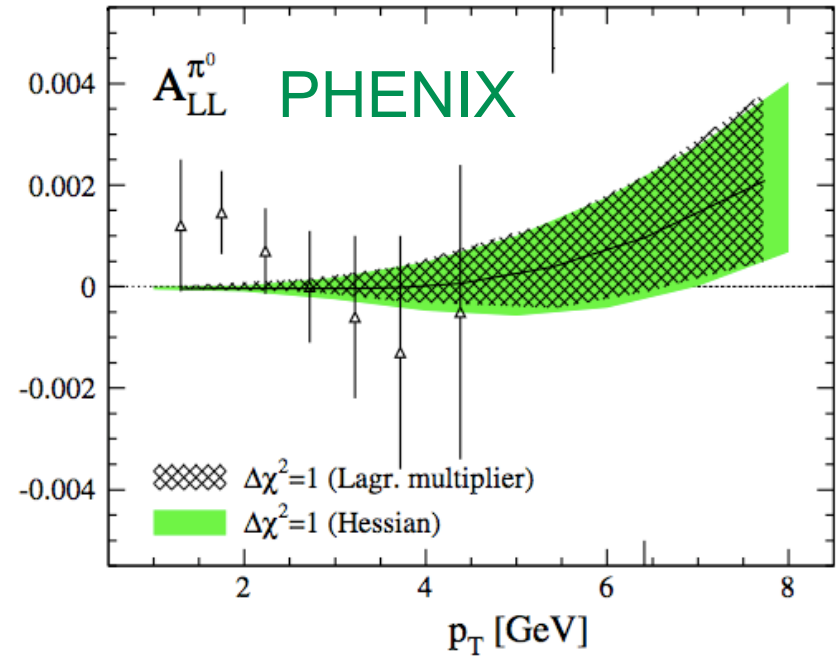
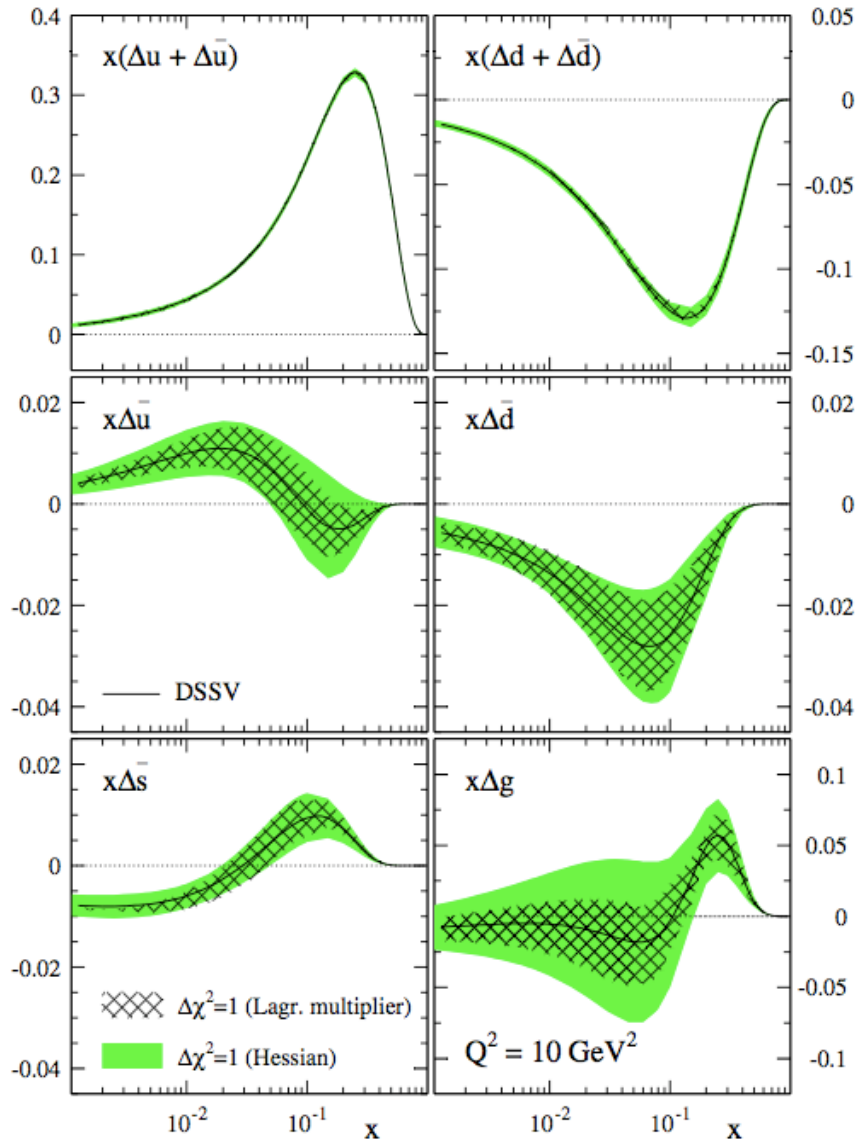
NLO Fits to A_1



DeFlorian, Sassot, Stratmann & Vogelsang (DSSV)
PRD80(09)034030



Polarized PDFs



Double polarized
pp scattering occurs through
qq, qg or gg scattering.
Later two are sensitive to Δg



x range in Eq. (35)	Q^2 [GeV ²]	$\Delta u + \Delta \bar{u}$	$\Delta d + \Delta \bar{d}$	$\Delta \bar{u}$	$\Delta \bar{d}$	$\Delta \bar{s}$	Δg	$\Delta \Sigma$
0.001–1.0	1	0.809	−0.417	0.034	−0.089	−0.006	−0.118	0.381
	4	0.798	−0.417	0.030	−0.090	−0.006	−0.035	0.369
	10	0.793	−0.416	0.028	−0.089	−0.006	0.013	0.366
	100	0.785	−0.412	0.026	−0.088	−0.005	0.117	0.363
0.0–1.0	1	0.817	−0.453	0.037	−0.112	−0.055	−0.118	0.255
	4	0.814	−0.456	0.036	−0.114	−0.056	−0.096	0.245
	10	0.813	−0.458	0.036	−0.115	−0.057	−0.084	0.242
	100	0.812	−0.459	0.036	−0.116	−0.058	−0.058	0.238

- Significant contributions from $x < 0.001$
- ΔG vanishes with increasing Q^2
- $1/2 = (1/2)\Delta\Sigma + \Delta G + L_z$ implies:
- At $Q^2 = 4 \text{ GeV}^2$, $L_z = 0.474$ (large)
- Errors on ΔG are still very large



Orbital Angular Momentum

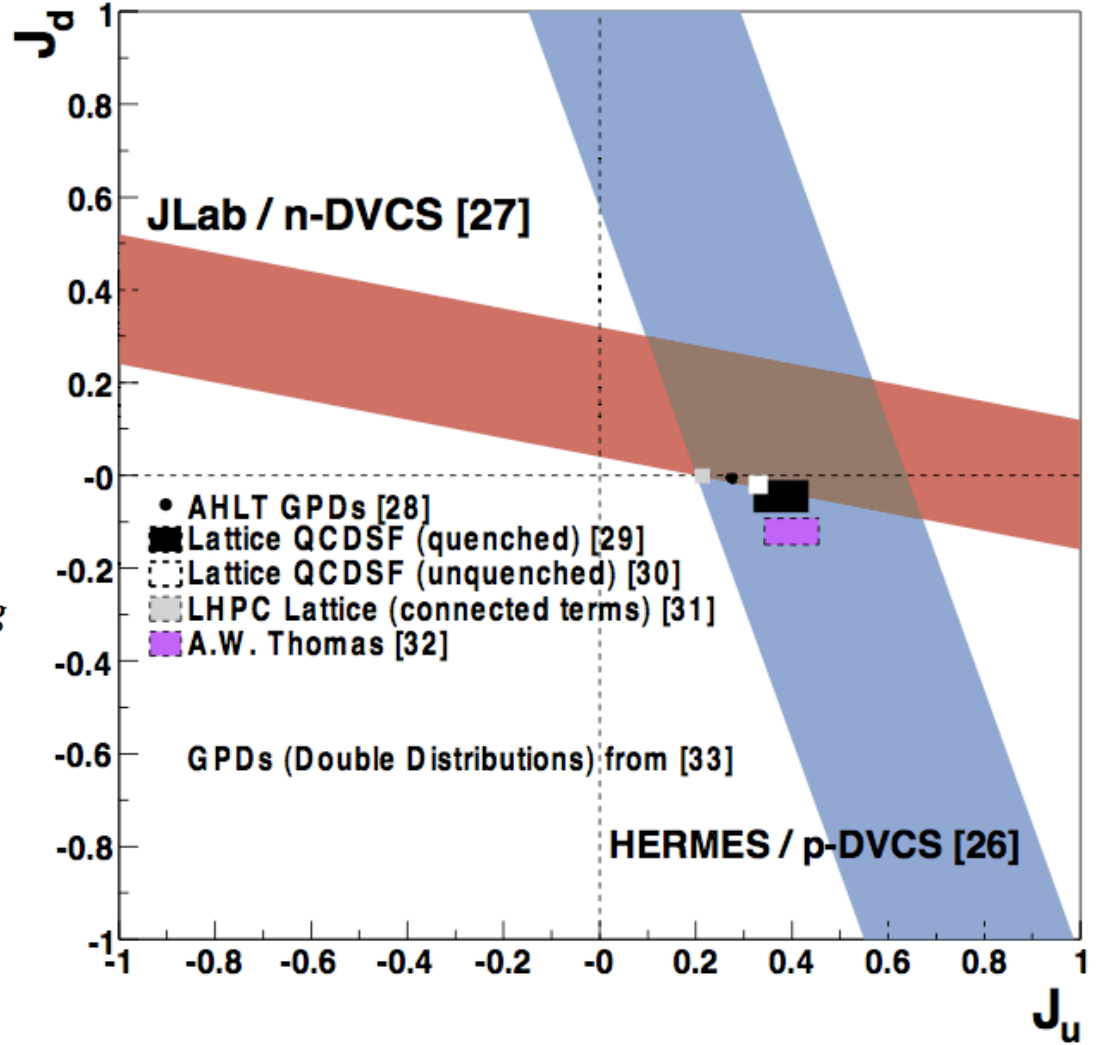
Plot: Voutier,
arXiv:0802.2499
[27] Mazouz et al.,
PRL **99**(07)2425
[26] Airapetian et al.,
arXiv:0802.2499

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_q + L_g$$

$$J_q = L_q + \frac{1}{2} \Delta \Sigma_q$$

$$J_g = L_g + \Delta g$$

$$J_q(x) = \frac{1}{2} x [q(x) + E_q(x)]$$



OAM accessed through GPDs via deeply virtual Compton scattering



HYPERFINE

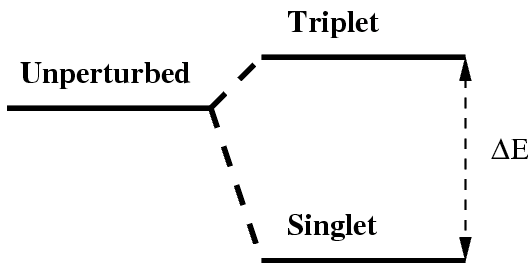
The 21 cm line in hydrogen (ground-state hyperfine splitting) has been measured to 13 digits of accuracy. Theoretical calculations are limited to parts per million because of the nuclear physics that depends on g_1 and g_2 . Recent data improves this calculation.



Hydrogen Hyperfine Splitting

Carlson, Nazaryan, Griffioen, PRA78(08)022517

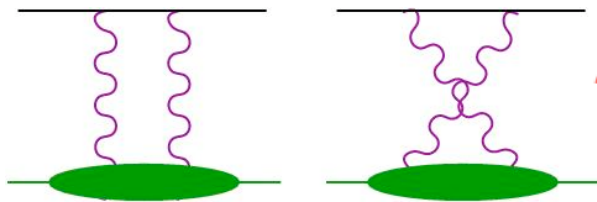
$$E_{\text{HFS}}(e^- p) = 1.4204057517667(9) \text{ GHz} = (1 + \Delta_{\text{QED}} + \Delta_R^p + \Delta_S) E_F^p$$



$$\Delta_S = \Delta_Z + \Delta_{\text{pol}} \quad \delta_Z^{\text{rad}} = \frac{\alpha}{3\pi} \left[2 \ln \frac{\Lambda^2}{m^2} - \frac{4111}{420} \right]$$

Zemach: $\Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\text{rad}})$

$$\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$$



$$\Delta_S = -38.62(16) \text{ ppm} \quad \Delta_Z = -41.0(5) \text{ ppm}$$

$$\Delta_{\text{pol}} = 2.38(58) \text{ ppm}$$

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{2\pi(1+\kappa)M} (\Delta_1 + \Delta_2) = (0.2264798 \text{ ppm})(\Delta_1 + \Delta_2)$$

$$\tau = \nu^2 / Q^2$$

$$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + \frac{8m_p^2}{Q^2} B_1(Q^2) \right\} = 8.85(278)$$

$$\Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2) = -0.57(57)$$

$$B_1 = \int_0^{x_{\text{th}}} dx \beta(\tau) g_1(x, Q^2),$$

$$B_2 = \int_0^{x_{\text{th}}} dx \beta_2(\tau) g_2(x, Q^2),$$

$$\beta(\tau) = \frac{4}{9} \left(-3\tau + 2\tau^2 + 2(2 - \tau)\sqrt{\tau(\tau + 1)} \right)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)},$$

$$\Delta_{\text{pol}} = 1.88(64) \text{ ppm from CLAS}$$



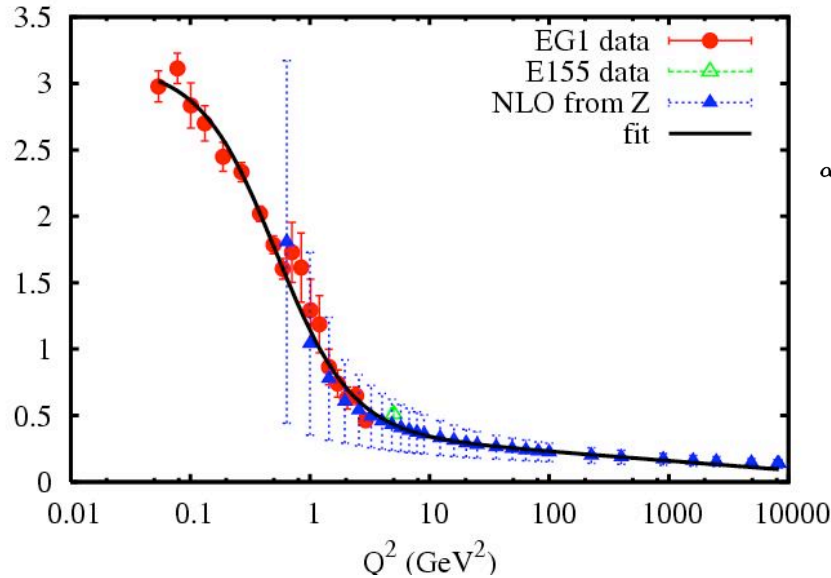
COMMENSURATE SCALING

The QCD is scale-independent BOTH at high Q^2 and low Q^2 . By defining an effective coupling constant consistent with the form from NLO pQCD, the relationship of these constants can be calculated.



Brodsky, Lu, PRD51(95)3652; Deur, PLB650(07)244

Effective α_S vs. Q^2



$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$\alpha_R(Q)$ in terms of $\alpha_{g_1}(Q)$.

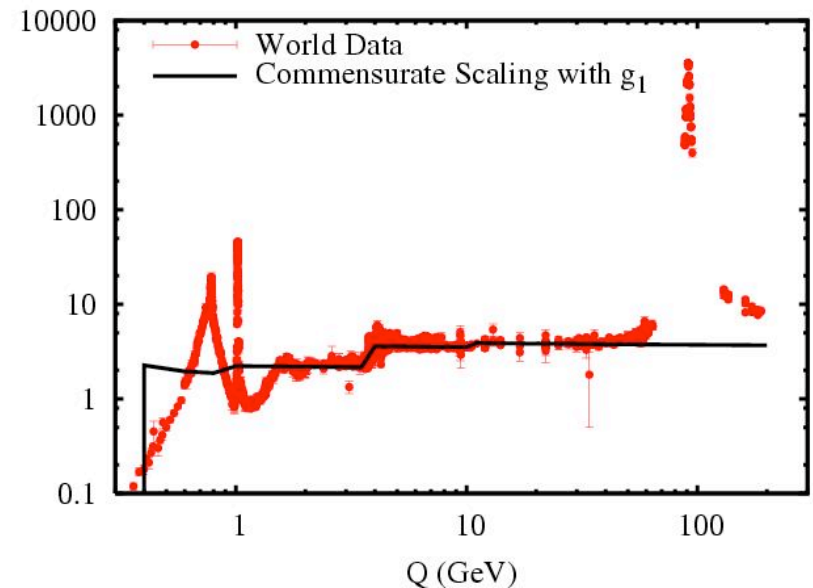
$$R(Q) \equiv 3 \sum_f Q_f^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right]$$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{g_1}(Q^*)}{\pi} + \frac{3}{4} C_F \left(\frac{\alpha_{g_1}(Q^{**})}{\pi} \right)^2 + \left[\frac{9}{16} C_F^2 + \left(\frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F N} \left(\frac{\sum_f Q_f^2}{\sum_f Q_f^2} \right)^2 \right] \left(\frac{\alpha_{g_1}(Q^{**})}{\pi} \right)^3,$$

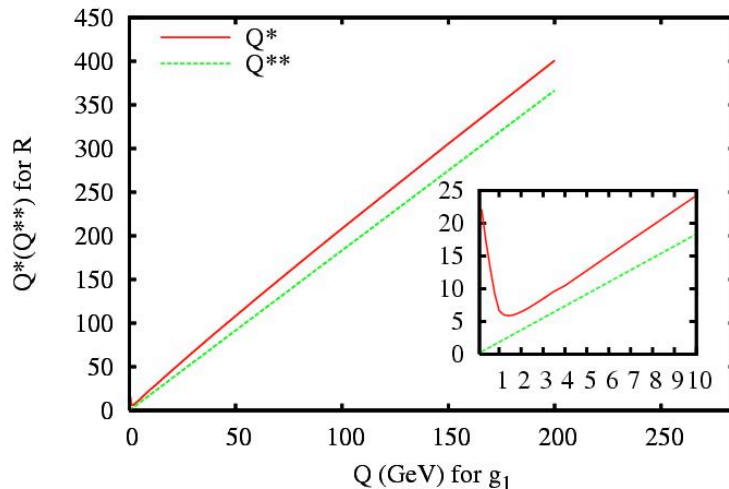
$$\ln(Q^*/Q) = -\frac{7}{4} + 2\zeta_3 + \left(-\frac{11}{96} - \frac{7}{3}\zeta_3 + 2\zeta_3^2 + \frac{\pi^2}{24} \right) \left(\frac{11}{3} C_A - \frac{2}{3} f \right) \frac{\alpha_{g_1}(Q)}{\pi},$$

$$\ln(Q^{**}/Q) = -\frac{233}{216} + \frac{64}{9}\zeta_3 - \frac{20}{3}\zeta_5 + \left(-\frac{13}{54} + \frac{2}{9}\zeta_3 \right) \frac{C_A}{C_F}.$$

$R(e^+e^- \rightarrow \text{more than 2 hadrons})$



Commensurate scales for R in terms of Q for g_1





CONCLUSIONS

Although much is known, there is still much to be learned about nucleon spin that will keep us busy for at least another decade.



- What's still missing:
 - high x : $A_1^{p,d}$ for polarized PDFs (CLAS12)
 - g_2^p (transverse target); SANE (Hall C) covers $Q^2 > 1 \text{ GeV}^2$; nobody's measuring $Q^2 < 1 \text{ GeV}^2$
 - low Q^2 evolution of $g_1^{p,d}$ (EG4)
 - gluon spin: evolution for $1 < Q^2 < 10$ (CLAS12); direct measurements?
 - orbital angular momentum (GPD experiments)
- What's gained:
 - understanding three regions
 - Q^2 near 0 (χ PT)
 - Q^2 from 0.1-10 GeV^2 (TMC, higher twists, resonances, the transition)
 - Q^2 near infinity (pQCD)