

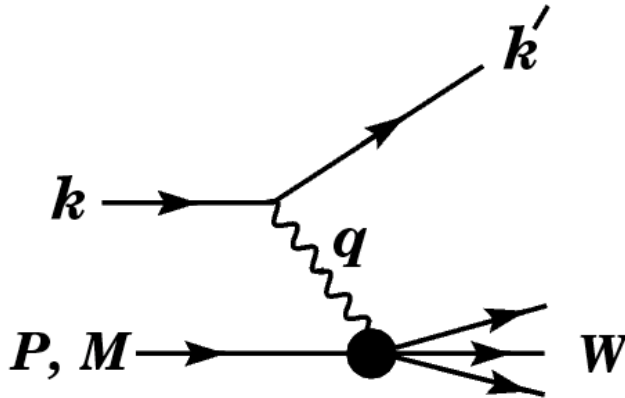
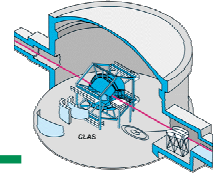
# A Nucleon Observed: Of QCD, Higher Twist, and Hydrogen Hyperfine Splittings

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4 September 2007



# Inelastic Scattering



Unpolarized Cross Section:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{8\pi\alpha^2 y}{Q^4} \left[ \frac{y}{2} F_1 + \frac{\xi}{2xy} F_2 \right]$$

Polarized Cross Section:

$$\frac{d^2\Delta\sigma}{dx dQ^2} = \frac{8\pi\alpha^2 y}{Q^4} \left[ \cos\alpha \left\{ \left( \xi + \frac{y}{2} \right) g_1 - \frac{\gamma y}{2} g_2 \right\} - \sin\alpha \cos\phi \left\{ \frac{y}{2} g_1 + g_2 \right\} \right]$$

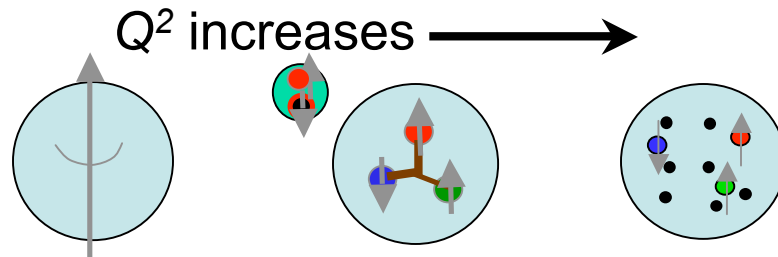
Lorentz invariants:

$$\begin{aligned} \nu &= p \cdot q / M = (E - E')_{\text{lab}} \\ Q^2 &= -q \cdot q = (4EE' \sin^2 \frac{\theta}{2})_{\text{lab}} \\ x &= -q \cdot q / 2p \cdot q = (Q^2 / 2M\nu)_{\text{lab}} \\ y &= p \cdot q / p \cdot k = (\nu / E)_{\text{lab}} \\ W^2 &= (p + q)^2 = (M^2 + 2M\nu - Q^2)_{\text{lab}} \\ s &= (k + p)^2 = (2EM + M^2)_{\text{lab}} \end{aligned}$$

$\alpha$  = polar angle of target spin wrt the beam axis  
 $\phi$  = azimuthal spin angle wrt the scattering plane  
 $\alpha = 0^\circ$  (longitudinal);  $\alpha = 90^\circ, \phi = 0^\circ$  (transverse).  
 $\gamma^2 = 4M^2 x^2 / Q^2 = Q^2 / \nu^2$   
 $\xi = 1 - y - \gamma y^2 / 4$

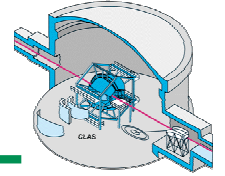
Parton Model:

$$\begin{aligned} F_1(x, Q^2) &= \frac{1}{2} \sum_i e_i^2 (q^\uparrow(x) + q^\downarrow(x) + \bar{q}^\uparrow(x) + \bar{q}^\downarrow(x)) \\ F_2(x, Q^2) &= 2x F_1(x, Q^2) \\ g_1(x, Q^2) &= \frac{1}{2} \sum_i e_i^2 (q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)) \\ g_2(x, Q^2) &= 0 \end{aligned}$$

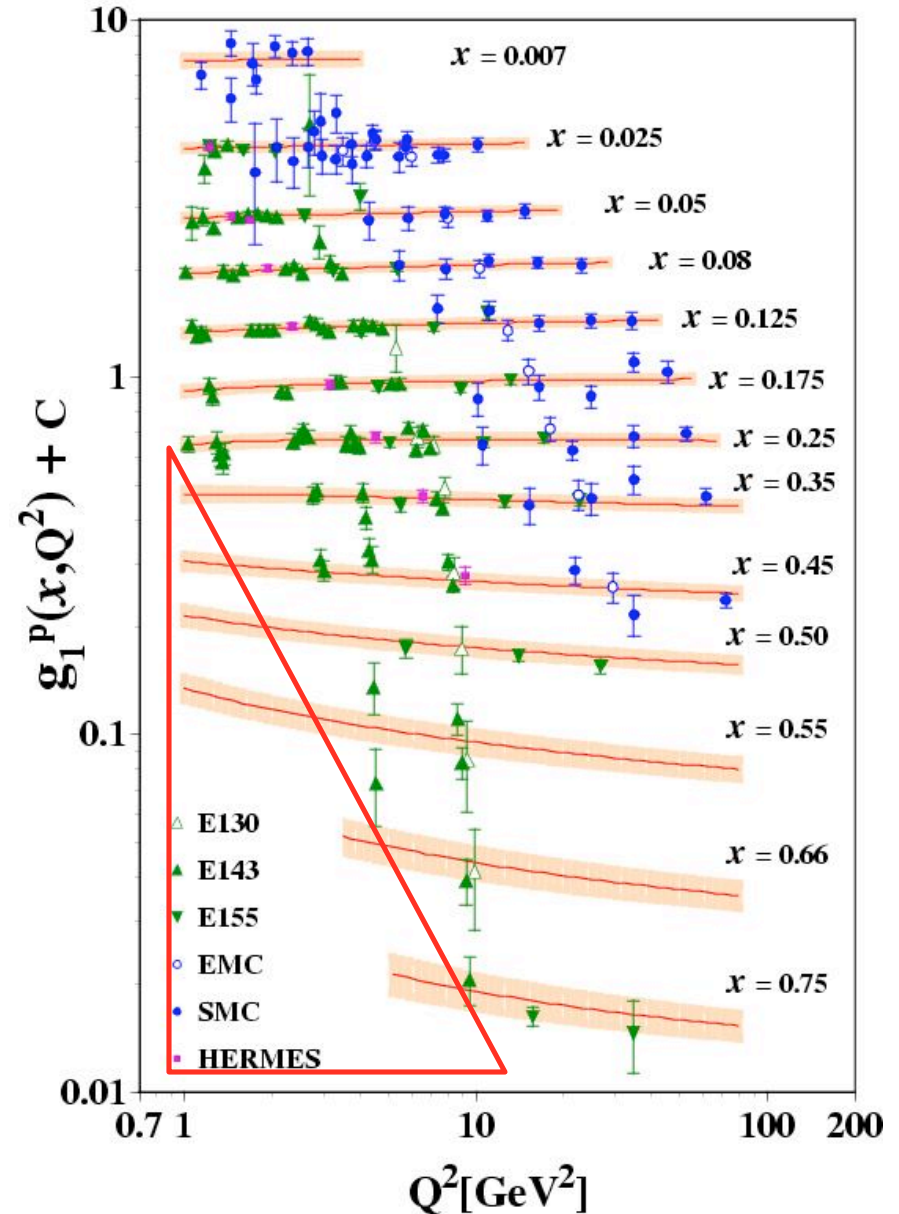




# $g_1(x, Q^2)$

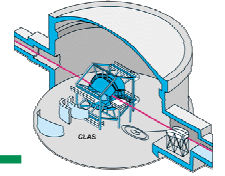


- $g_1$  DIS measured for p, d and  $^3\text{He}$
- CERN/HERA/SLAC
- (AAC: hep-ph/0603213) NLO fits to extract quark and gluon distributions
- Notice the hole at high  $x$  and  $Q^2 < 10 \text{ GeV}^2$
- CLAS12 DIS coverage (red)
- CLAS 6 covers this triangle in the resonance region



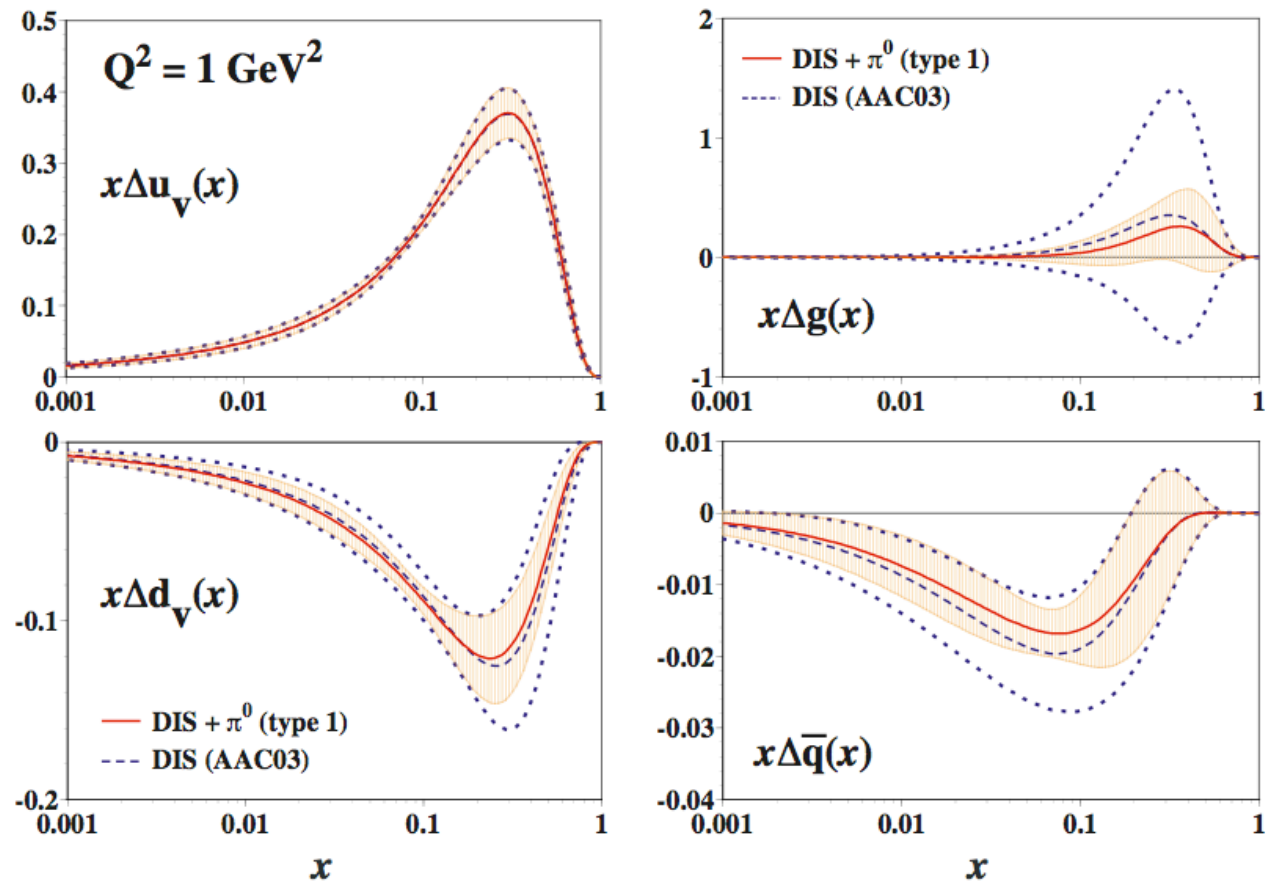


# NLO Fits



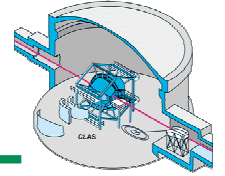
$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

	$\Delta\bar{q}$	$\Delta g$	$\Delta\Sigma$
Type 1	$-0.05 \pm 0.01$	$0.31 \pm 0.32$	$0.27 \pm 0.07$
Type 2	$-0.06 \pm 0.02$	$0.47 \pm 1.08$	$0.25 \pm 0.10$
AAC03	$-0.06 \pm 0.02$	$0.50 \pm 1.27$	$0.21 \pm 0.14$

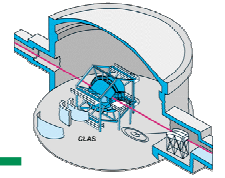




# A Nucleon Observed



- We've measured quite a lot about the proton in the last 50 years since Hofstadter observed its charge distribution  
"Messen ist wissen" -- E. W. von Siemens
- The proton has proved to be quite complex  
"Il y a science des choses simples, et art des choses compliquées." -- Paul Valéry
- There's still a lot more work to be done  
"Great art is never produced for its own sake. It is too difficult to be worth the effort" -- George Bernard Shaw



Our tendency is to go from inclusive to exclusive reactions

$$\boxed{f(x, Q^2)} \quad \boxed{f(x, Q^2, p_{\perp})}$$

Our tendency is to go from low resolution ( $Q^2$ ) to high

$\Sigma$  of constituents      resolved constituents

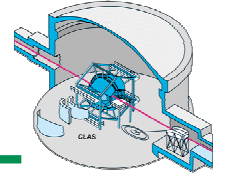
Our tendency is to go from holism to reductionism

Much of his talk is about going in the other direction:

$$\int f(x, Q^2, p_{\perp}) dp_{\perp} \rightarrow f(x, Q^2)$$

$$\int f(x, Q^2) dx \rightarrow f(Q^2)$$

$$\int f(Q^2) dQ^2 \rightarrow \text{hyperfine splitting}$$



## Energy-Weighted Sum Rule

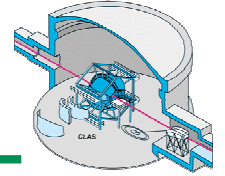
$$S(F) = \sum_a (E_a - E_0) |\langle a | F | 0 \rangle|^2 = \langle 0 | [F, [H, F]] | 0 \rangle$$

## GDH Sum Rule

$$\int_{k_\pi}^{\infty} \frac{dk}{k} \Delta \sigma^{\gamma N}(k) = \frac{2\pi^2 \alpha \kappa^2}{M^2}$$

$$\Delta \bar{\sigma}^{\gamma N} = \sigma_{3/2}^{\gamma N} - \sigma_{1/2}^{\gamma N}$$

Sum over excited states is tied to properties of the ground state



## Gottfried Sum Rule

0.235(26) at  $Q^2=4 \text{ GeV}^2$

$$\Phi_1^{p,n}(Q^2) = \int_0^1 F_1^{p,n}(x, Q^2) dx$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x)$$

$$\Phi_1^p - \Phi_1^n = \frac{1}{6} [u_v - d_v + 2\bar{u} - 2\bar{d}]$$

## Bjorken Sum Rule

0.176(7) at  $Q^2=5 \text{ GeV}^2$

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 g_1^{p,n}(x, Q^2) dx$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x)$$

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} [\Delta u_v - \Delta d_v + 2\Delta\bar{u} - 2\Delta\bar{d}]$$

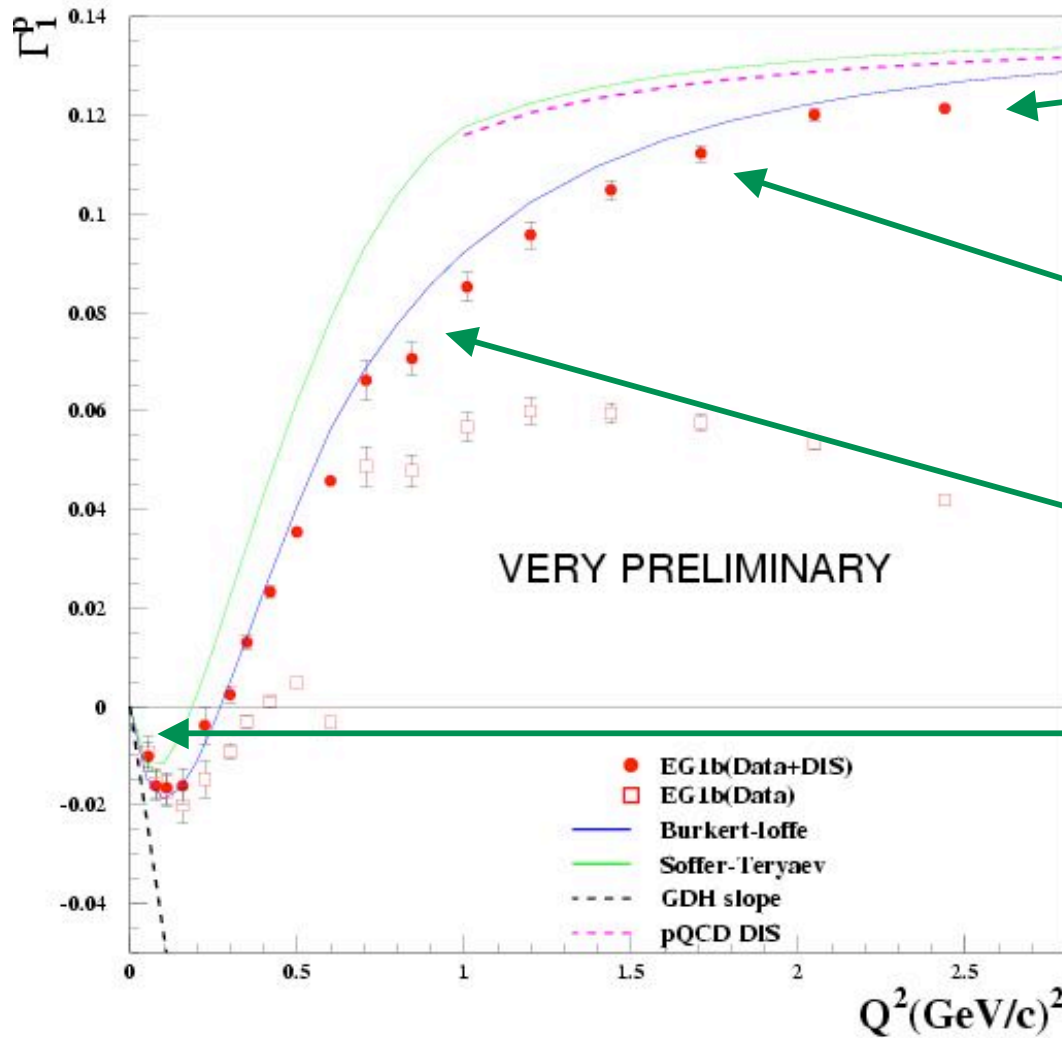
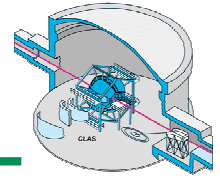
## Complicating Factor

$$\Delta C_{NS}^{\bar{M}S} = 1 - \frac{\alpha_S}{\pi} - 3.583 \left(\frac{\alpha_S}{\pi}\right)^2 - 20.215 \left(\frac{\alpha_S}{\pi}\right)^3 + \dots$$





# Regions of $Q^2$

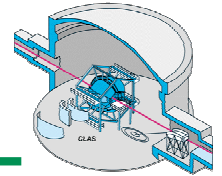


scaling:  $\ln Q^2$

higher twist:  $(1/Q^2)^n$

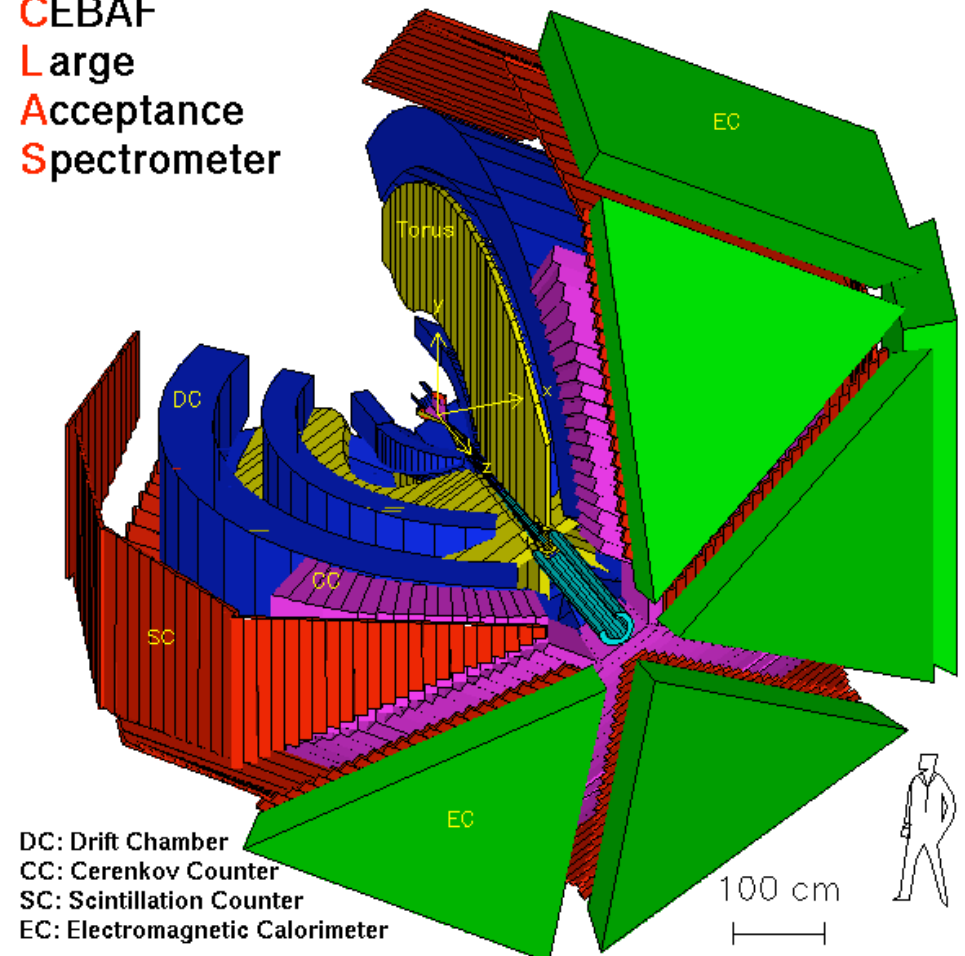
no nice expansion

$\chi_{PT}$ :  $(Q^2)^n$



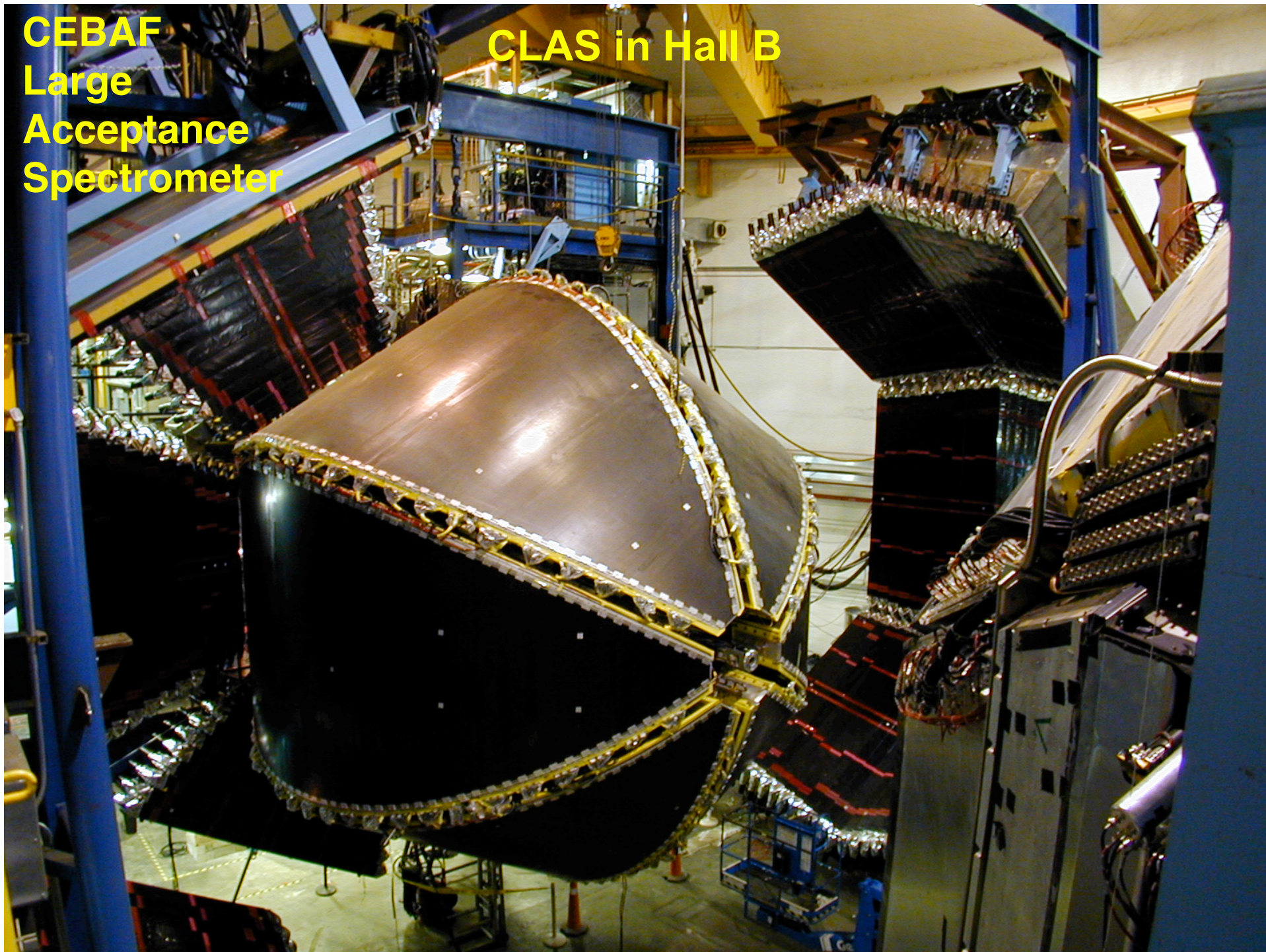
- Long-standing program in Hall-B at JLab to measure longitudinal double spin asymmetries  $A_{||}$  on  $^{15}\text{NH}_3$  and  $^{15}\text{ND}_3$
- EG1:  $0.05 < Q^2 < 3.5 \text{ GeV}^2$   
– data (2001); anal (2007)
- EG4:  $0.01 < Q^2 < 1 \text{ GeV}^2$   
– data (2006); anal (2008)
- EG12:  $0.5 < Q^2 < 7 \text{ GeV}^2$   
– data (2012?); anal (2014)

CEBAF  
Large  
Acceptance  
Spectrometer



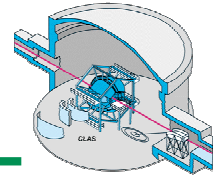
**CEBAF  
Large  
Acceptance  
Spectrometer**

**CLAS in Hall B**

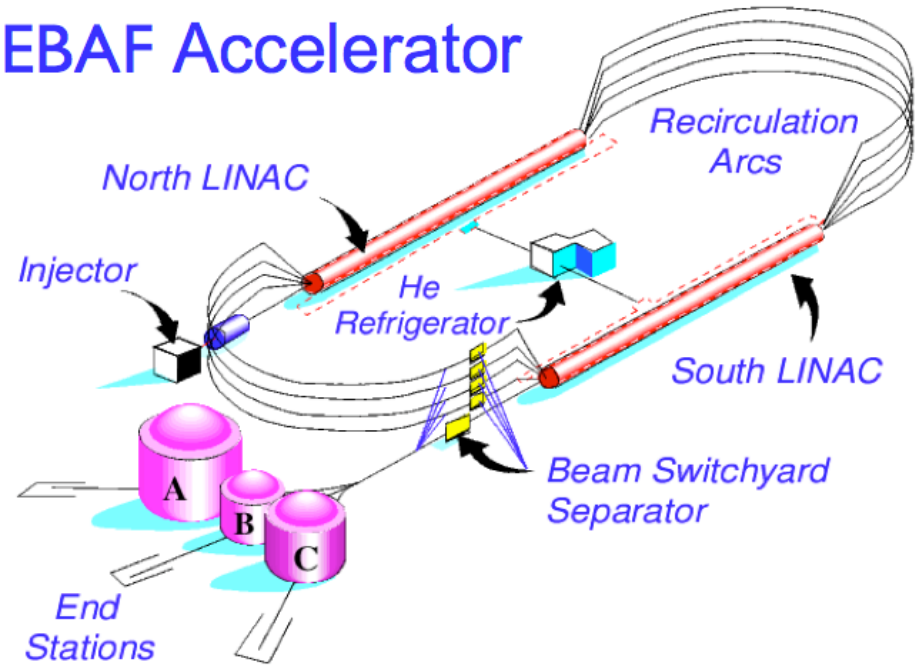




# CEBAF Accelerator



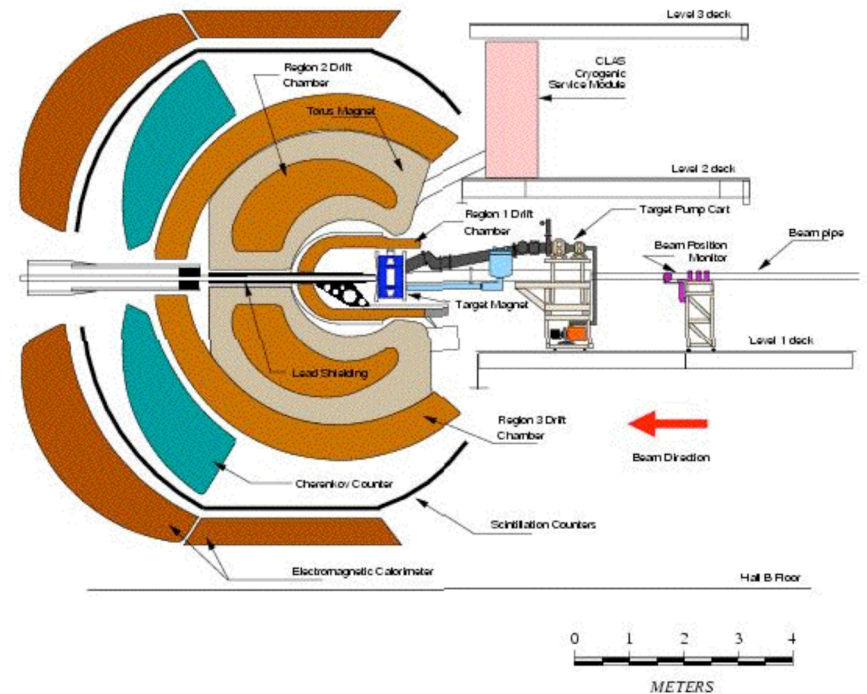
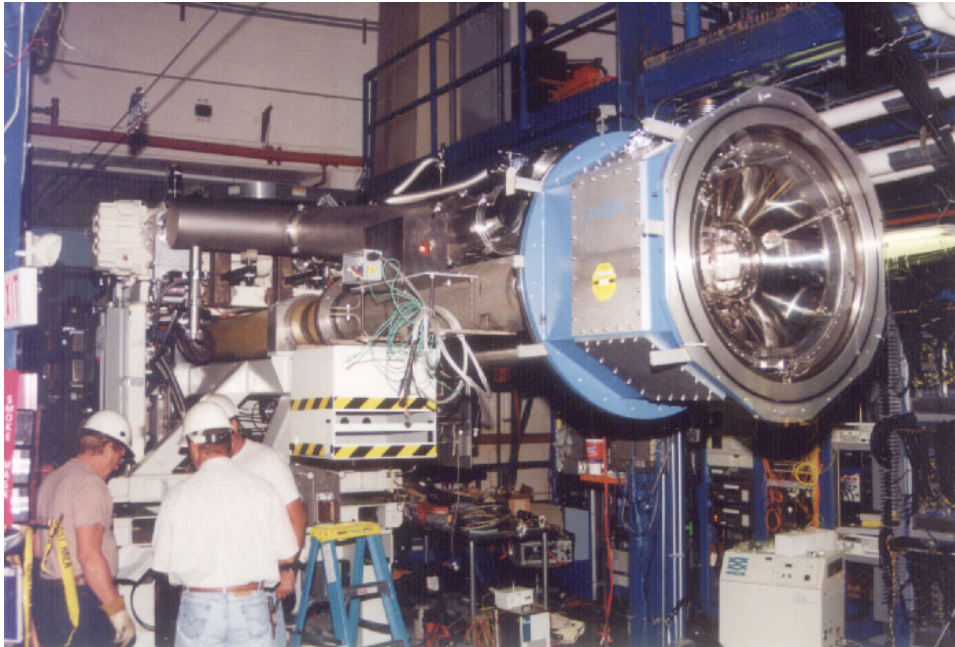
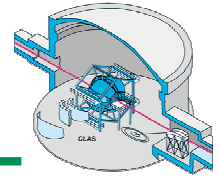
## The CEBAF Accelerator



- Electron beams up to 5.7 GeV with >80% longitudinal polarization
- Beam currents of 1-50 nA in Hall B



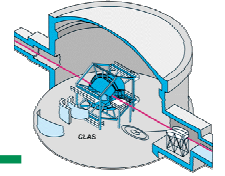
# Polarized Target



- Dynamic nuclear polarization of  $\text{NH}_3$  and  $\text{ND}_3$
- Polarizations of 70-80% for p and 20-30% for d
- Luminosity  $10^{35} \text{ cm}^{-2}\text{s}^{-1}$



# Formalism



$$A_{\parallel} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}}$$

$$A_{\parallel} = D(A_1 + \eta A_2)$$

We can extract  $A_1$  using a model for  $A_2$  (small), or  $g_1$  using a model for  $g_2$  (small)

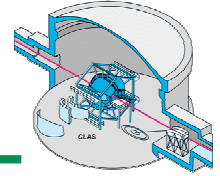
We can extract  $A_1$  and  $A_2$  from  $A_{\parallel}$  at multiple values of  $\eta(E_{\text{beam}})$

$$A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T}$$

$$= \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{F_1(x, Q^2)}$$

$$A_2 = \frac{2\sigma_{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T}$$

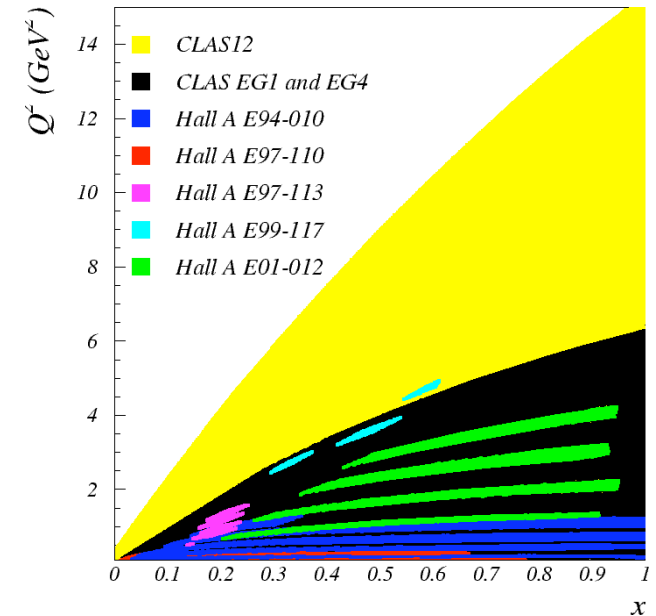
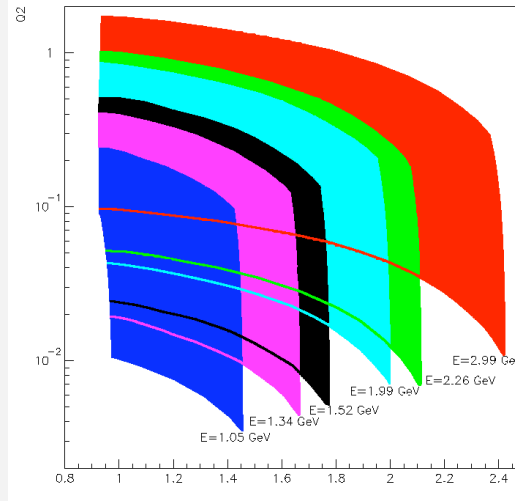
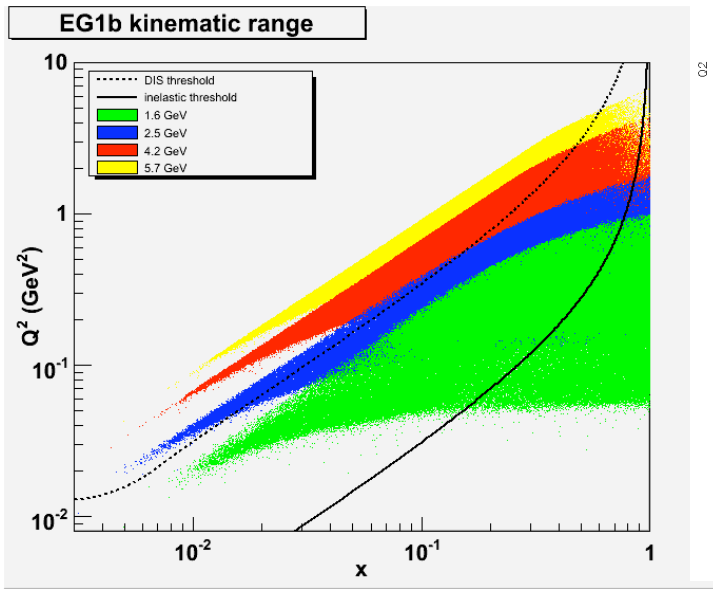
$$= \frac{\gamma[g_1(x, Q^2) + g_2(x, Q^2)]}{F_1(x, Q^2)}$$



## EG1

## EG4

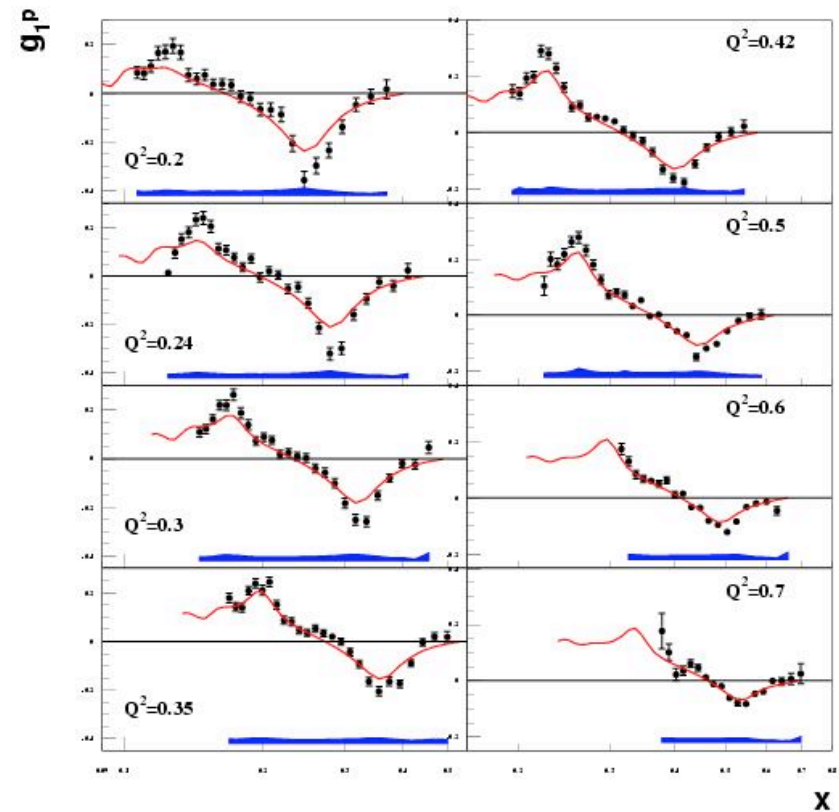
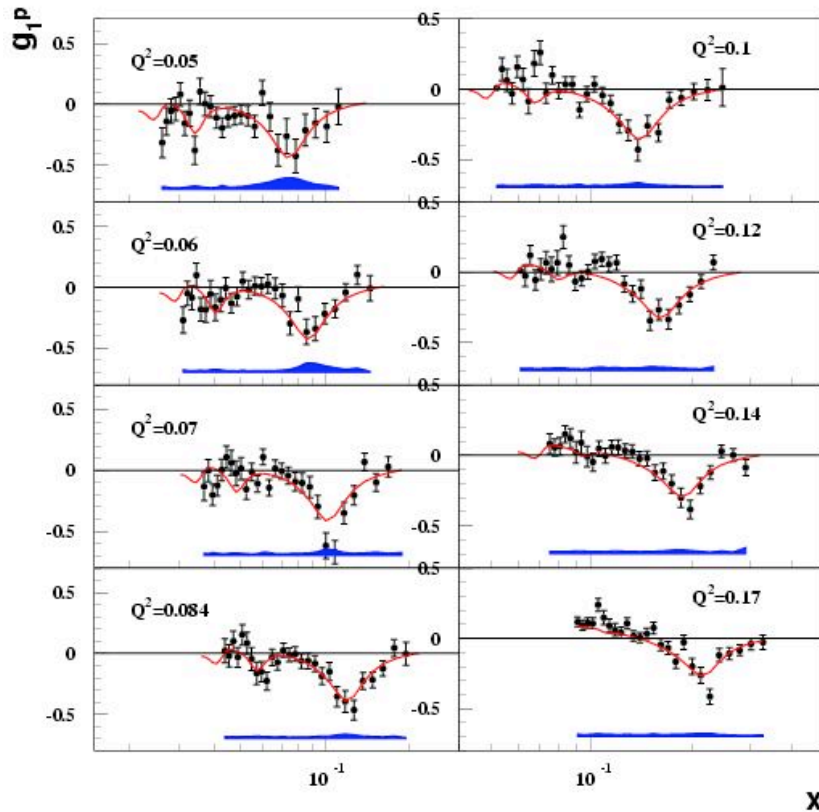
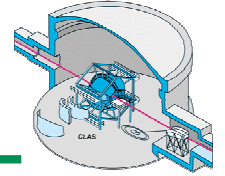
## EG12



- Overlapping colors correspond to different beam energies
- CLAS measures a large range in  $x$  at each fixed  $Q^2$
- Different  $E_{\text{beam}}$  for fixed  $(x, Q^2)$  allows separation of  $A_1$  &  $A_2$

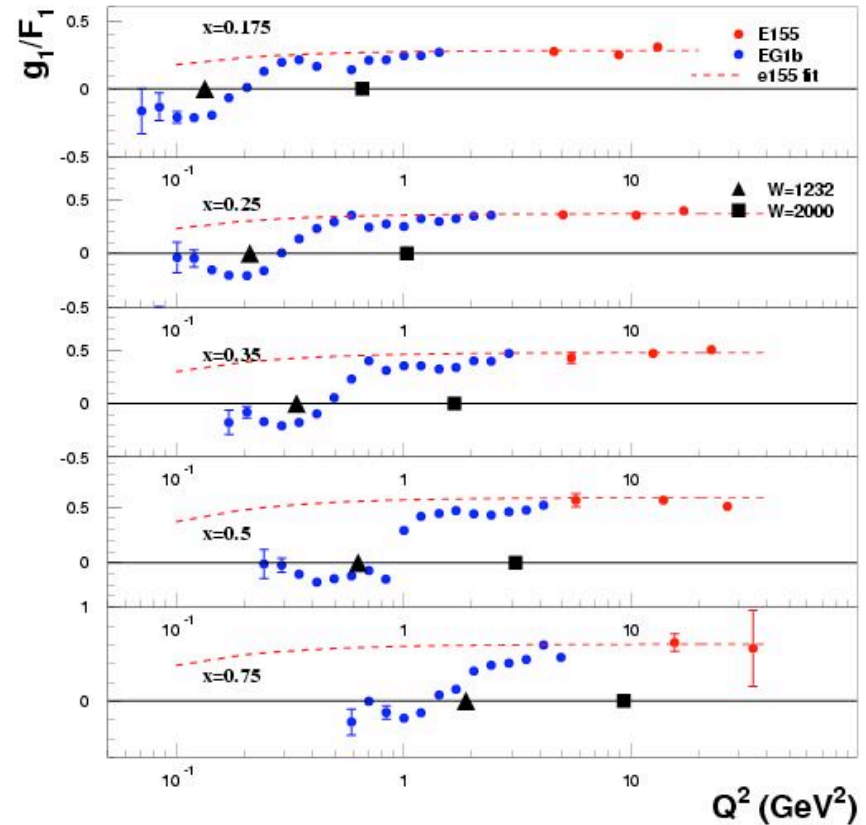
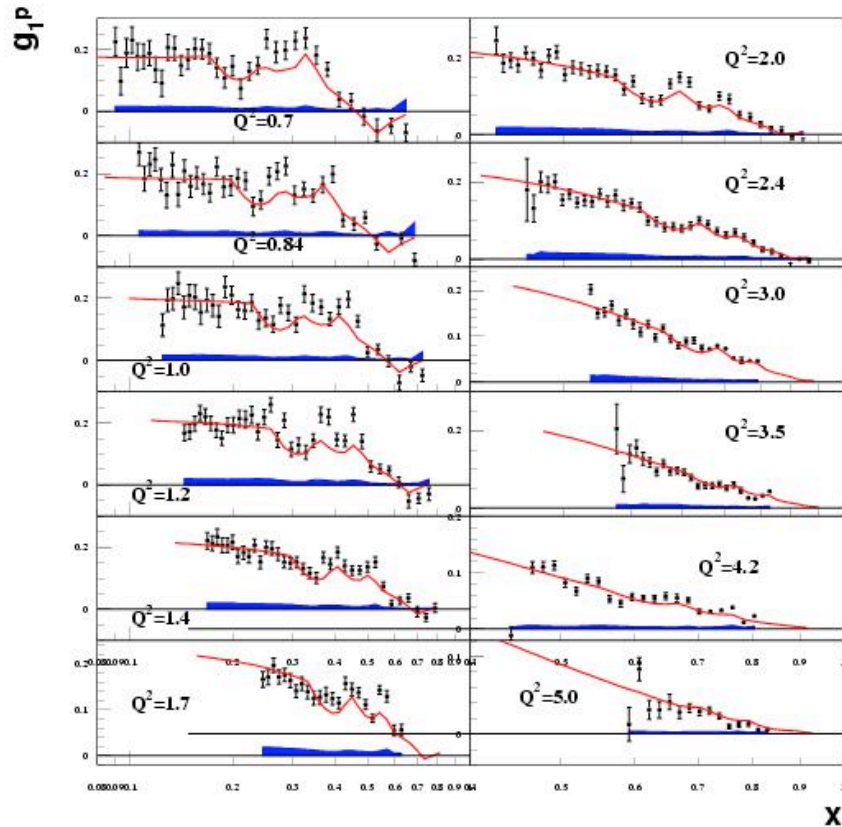
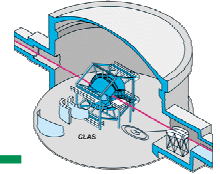


# EG1 $g_1^p(Q^2 < 0.7)$

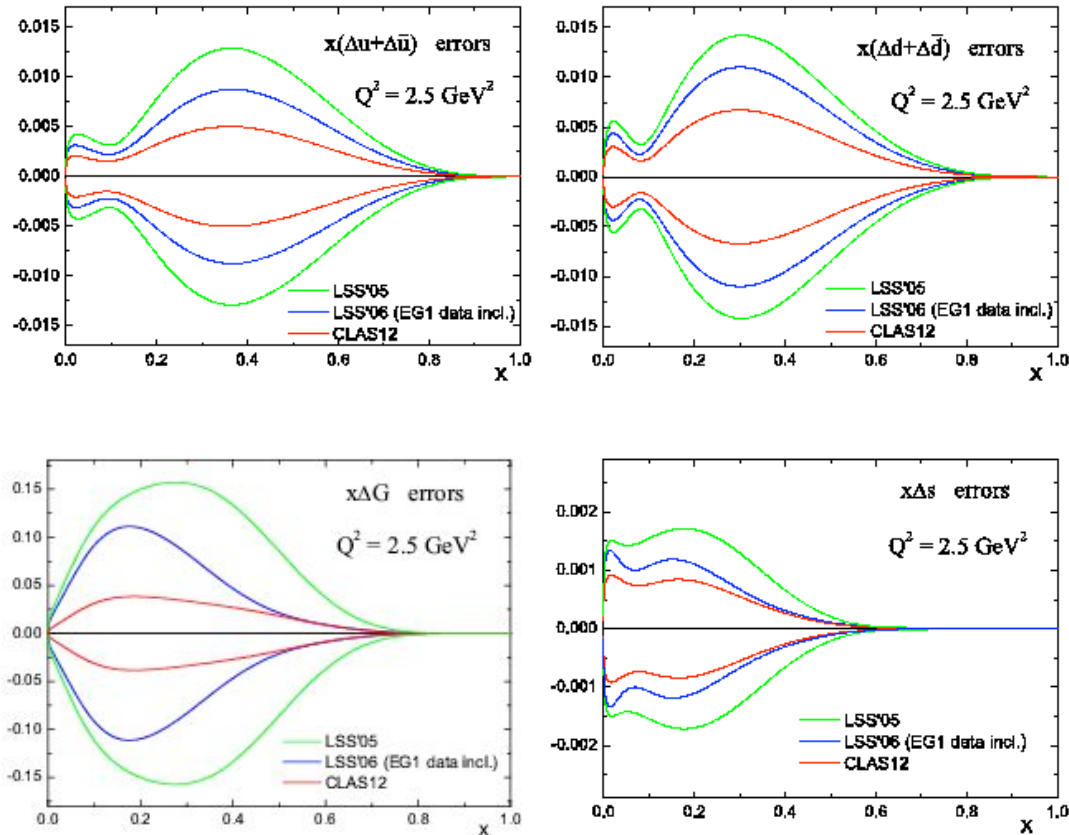
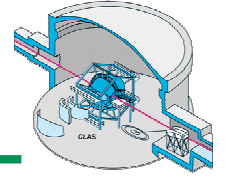


- At low  $Q^2$  the  $\Delta$  resonance drives  $g_1$  negative
- Extensive  $x$ -range at fixed  $Q^2$  allows integration over  $x$
- Red curve is the EG1 model used for radiative corrections





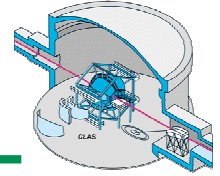
- At higher  $Q^2$ ,  $g_1$  becomes positive everywhere
- $g_1/F_1$  falls far below the DIS extrapolation at low  $Q^2$
- Red curve is the EG1 model (dashed: DIS extrapolation)



- Error envelopes for PDFs from LSS05 global analysis (green)
- CLAS EG1 data significantly improve errors on  $\Delta u$ ,  $\Delta d$ ,  $\Delta x$  and  $\Delta G$  (blue)
- CLAS EG12 (12 GeV upgrade) will especially improve  $\Delta G$  (red)

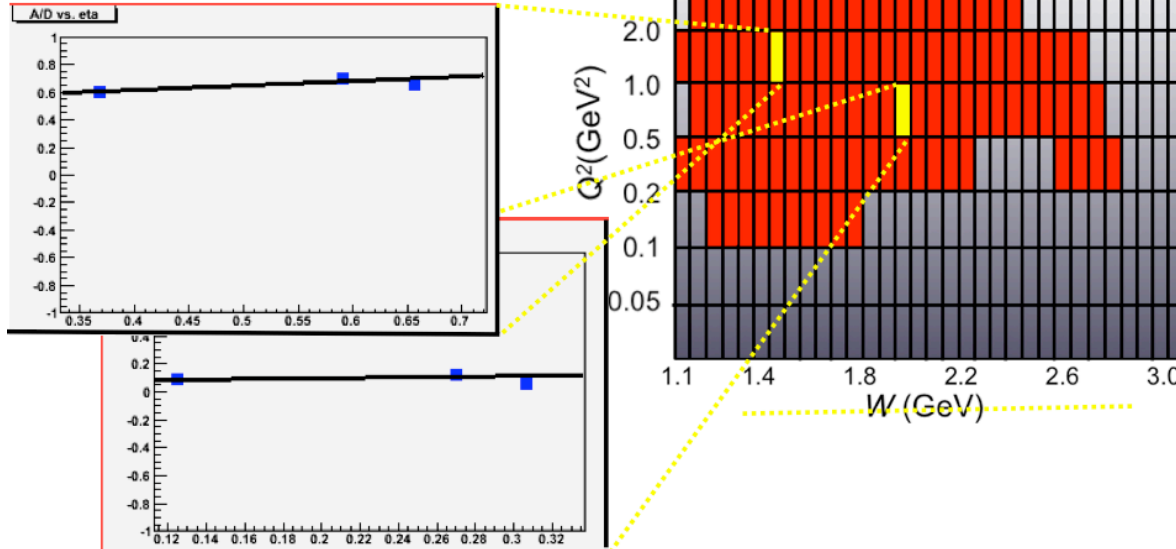


# EG1 Extraction of $A_2$

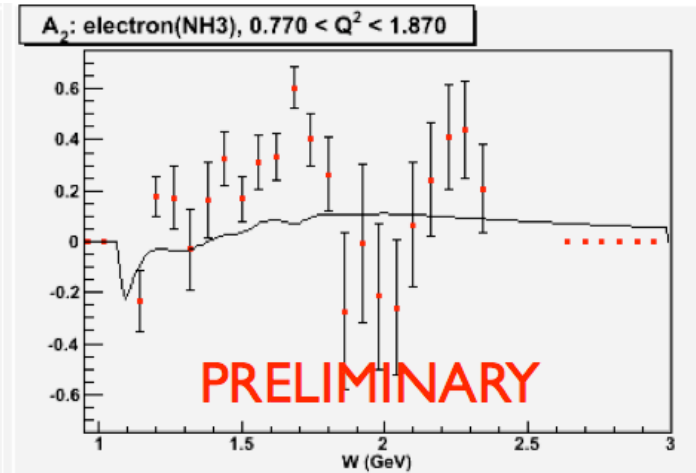
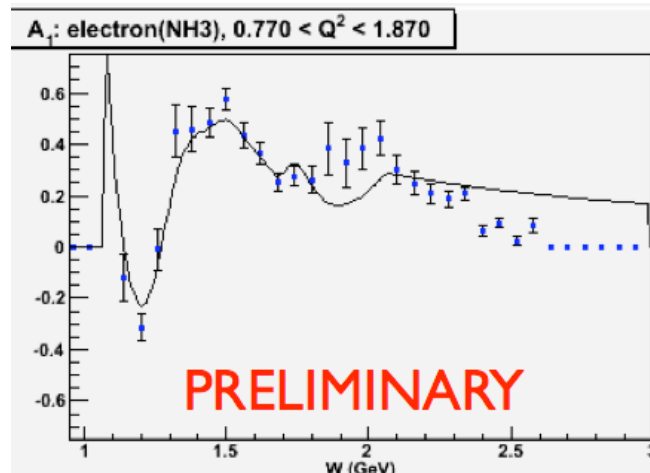


$$A_{\parallel} = D(A_1 + \eta A_2)$$

$$\eta = \frac{\epsilon \sqrt{Q^2/E}}{1 - \epsilon E'/E} \quad D = \frac{1 - \epsilon E'/E}{1 + \epsilon R}$$

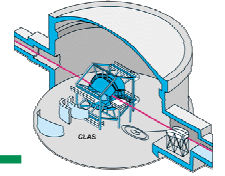


- Analysis is in progress to obtain both  $A_1$  and  $A_2$  from the EG1 data
- Intercept gives  $A_1$
- Slope gives  $A_2$
- $A_2$  is larger than EG1 model (MAID, AO) as is Hall C RSS experiment



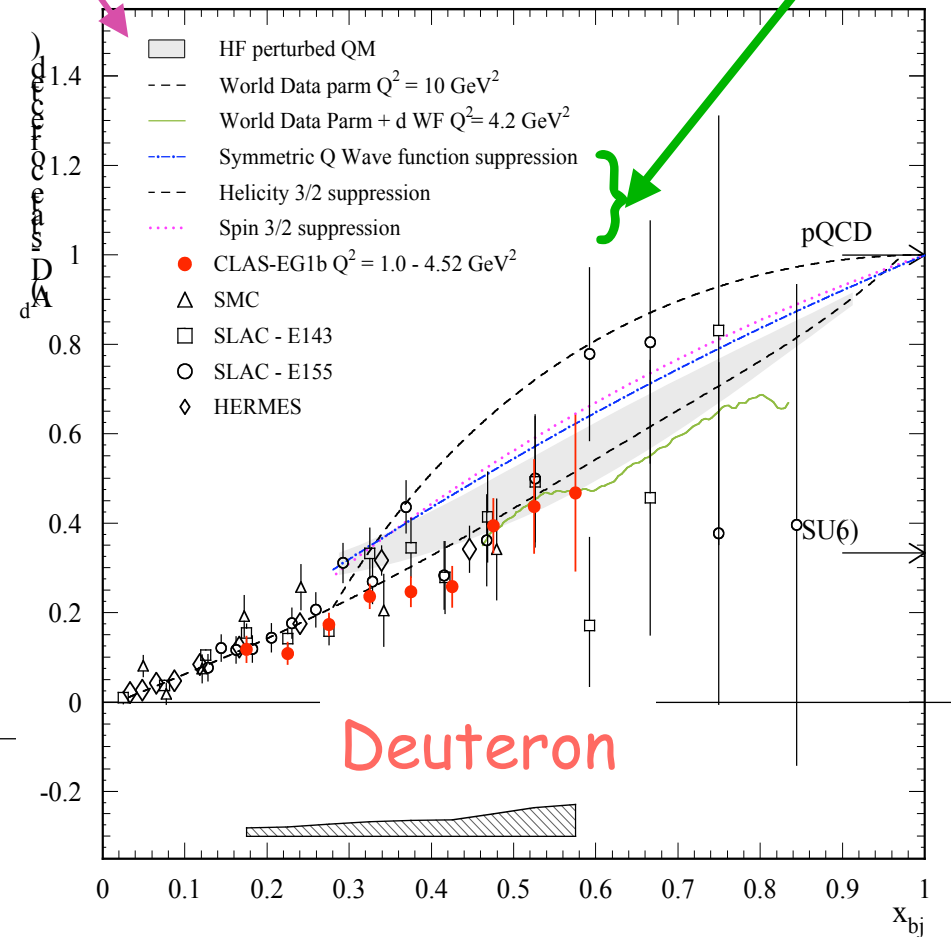
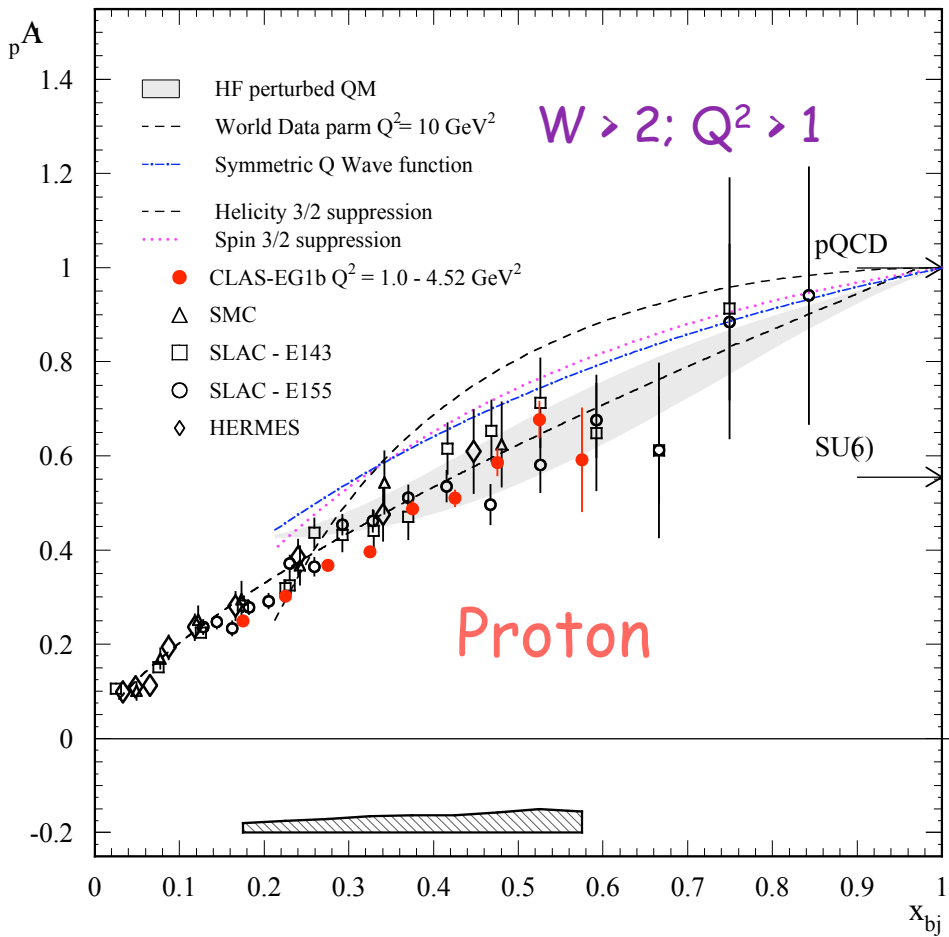


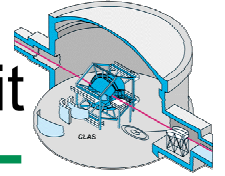
# A<sub>1</sub> Data from EG1



Isgur, PRD **59**, 034013 (2003)

Close and Melnitchouk, PRC  
**68**, 035210 (2003)

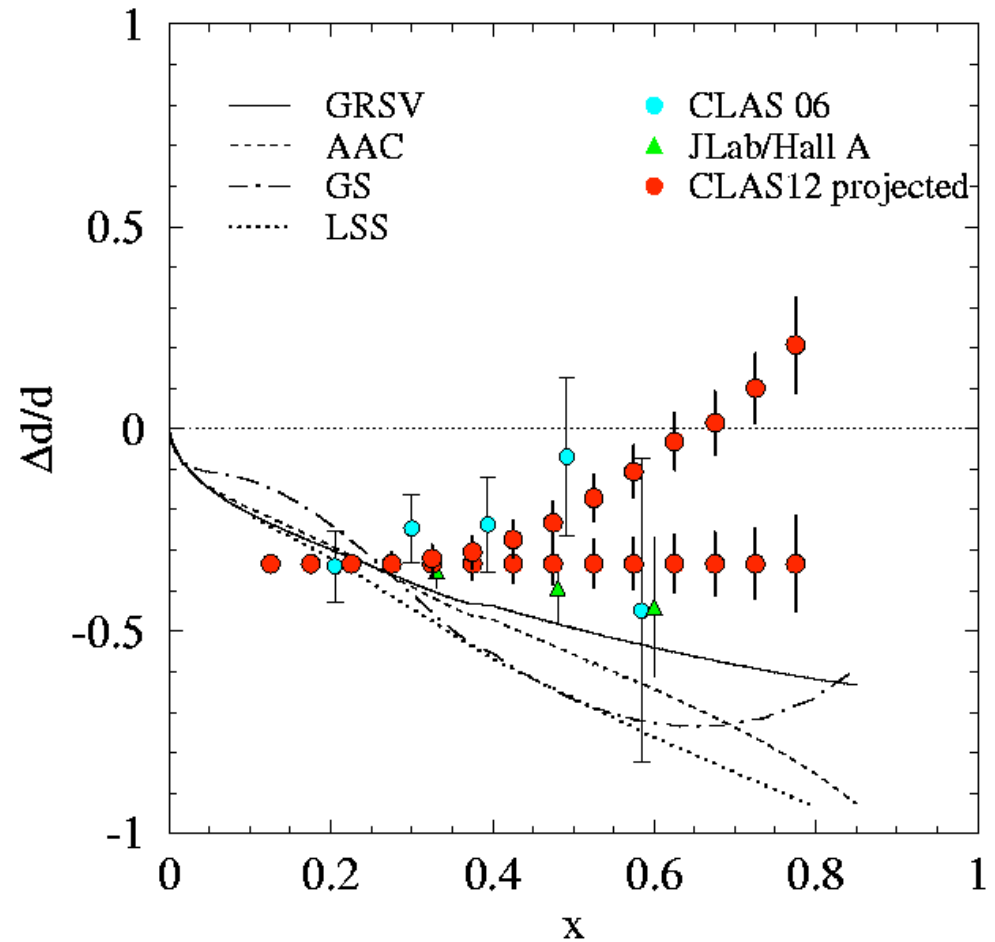
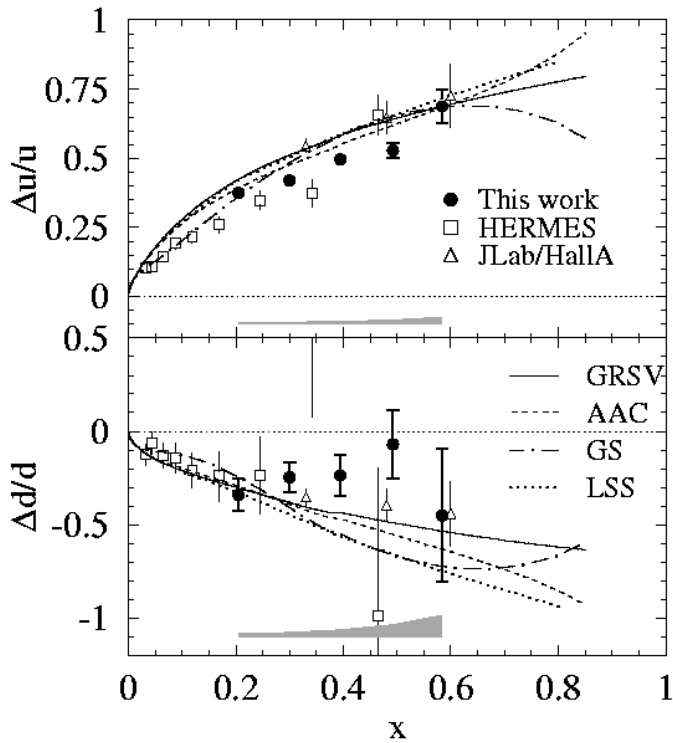




$$A_1(x, Q^2) = \frac{\sum e_i^2 \Delta q_i(x, Q^2)}{\sum e_i^2 q_i(x, Q^2)}$$

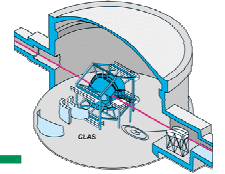
Simulated Data for EG12  
Extracted from  $A_1^p$ ,  $A_1^d$  and  $d/u$

## EG1 Existing Data

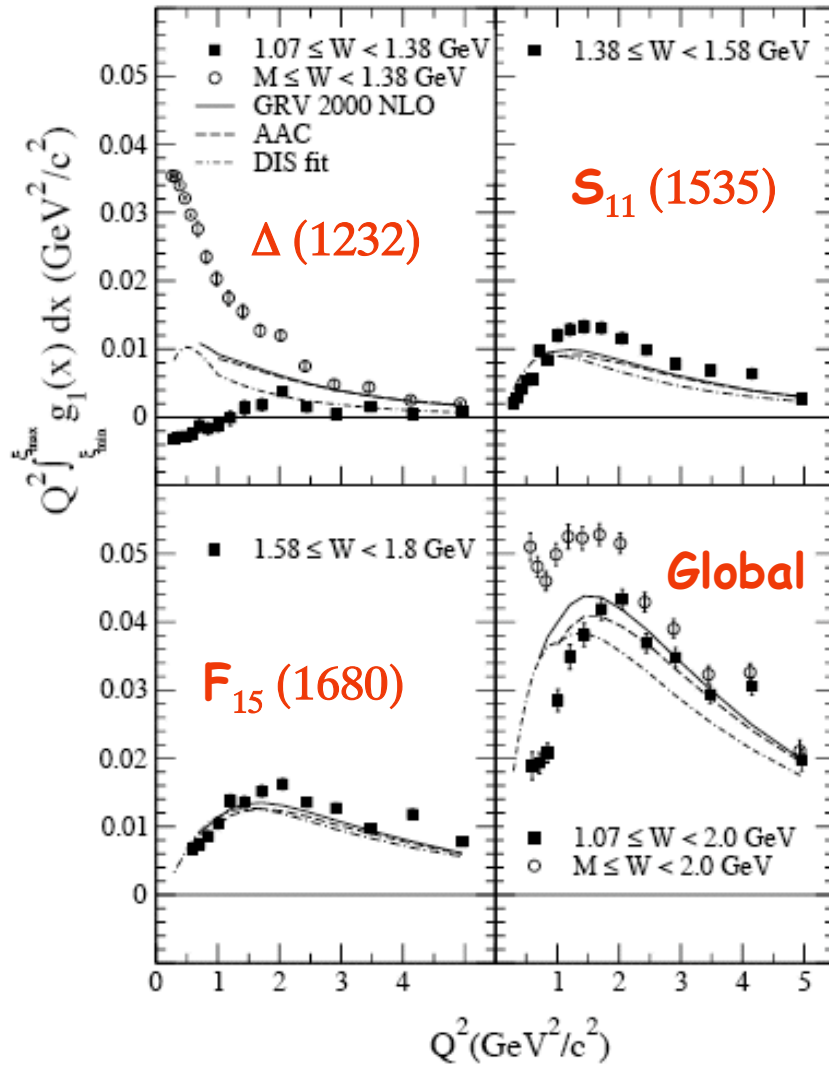




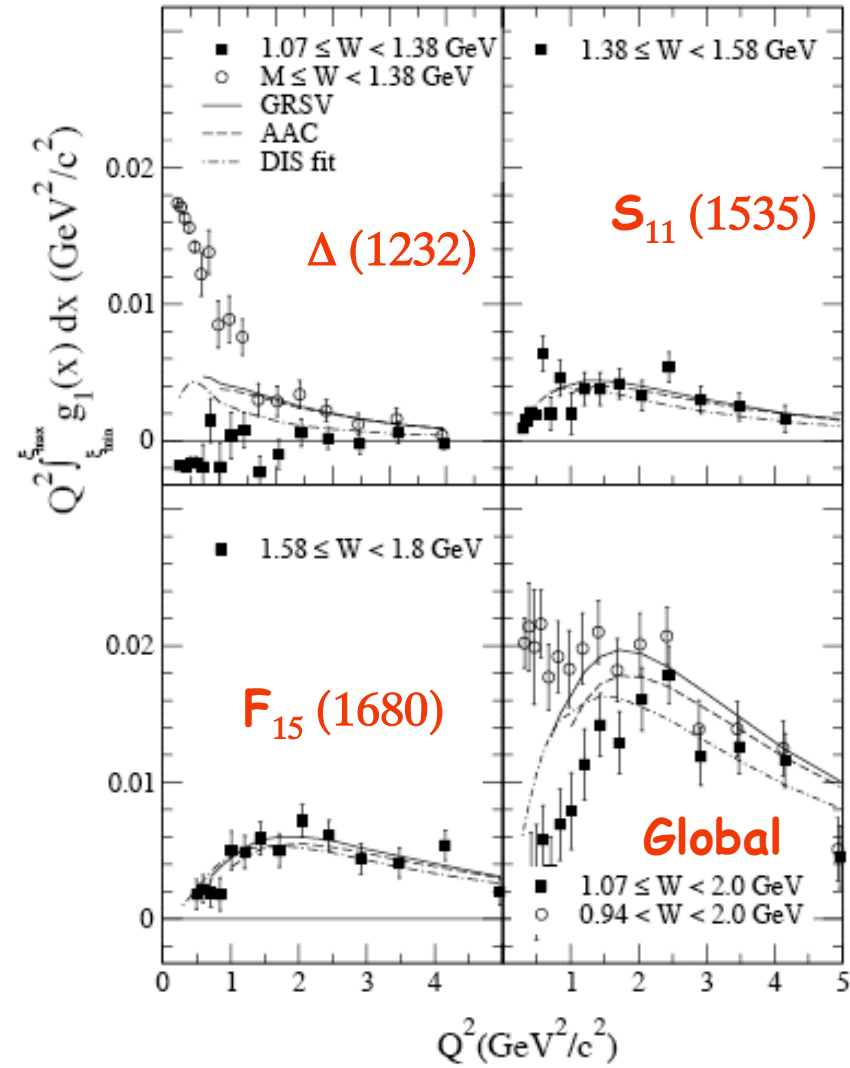
# Duality



## Proton

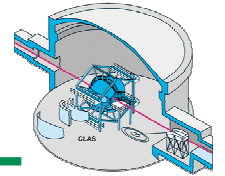


## Deuteron

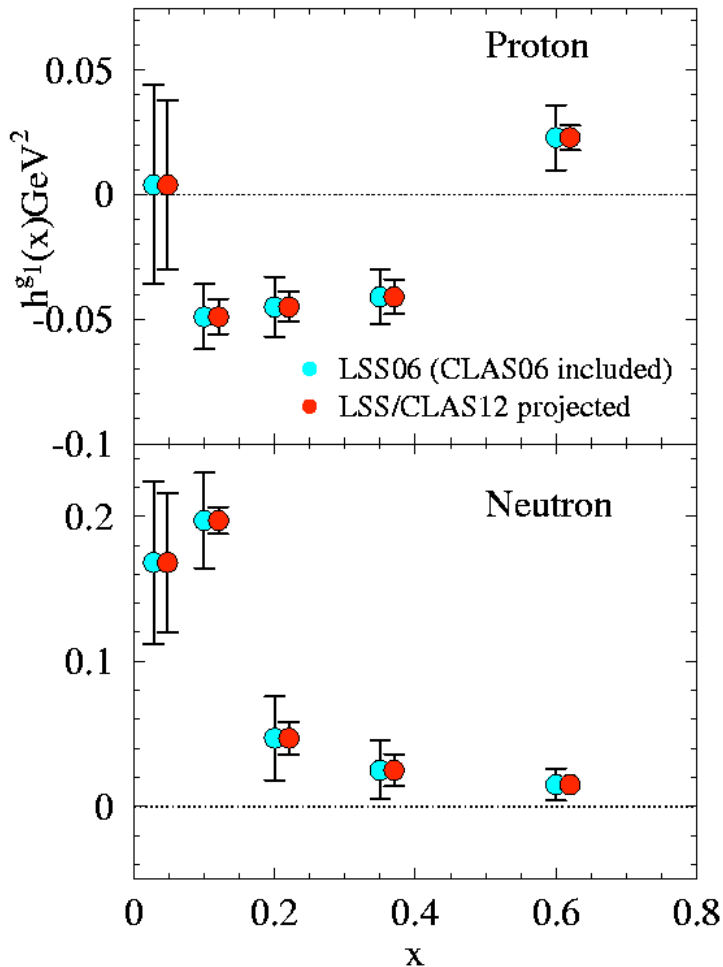




# Higher Twist from $g_1$



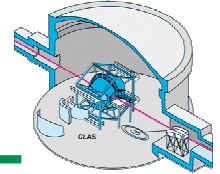
$$\left[ \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right]_{\text{exp}} \quad F_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + h^{g_1}(x)/Q^2$$



$$1 < Q^2 < 5 \text{ GeV}^2, \quad 2 < W < 3.5 \text{ GeV}$$

$$\int_0^1 dx h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)$$

- $F_1$  from NMC fit to  $F_2$  and 1998 SLAC fit to R
- $g_1$  (leading twist) from NLO fit at high  $Q^2$
- $h$  from fit to all data, especially CLAS in the pre-asymptotic region
- $d_2$ : twist-3,  $f_2$ : twist-4



$$\Gamma_1^{(n)} = \int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2}, \quad n=0,2,4,\dots,$$

$$\Gamma_2^{(n)} = \int_0^1 x^n g_2(x, Q^2) dx = \frac{1}{2} \frac{n}{n+1} (d_n - a_n), \quad n=2,4,\dots,$$

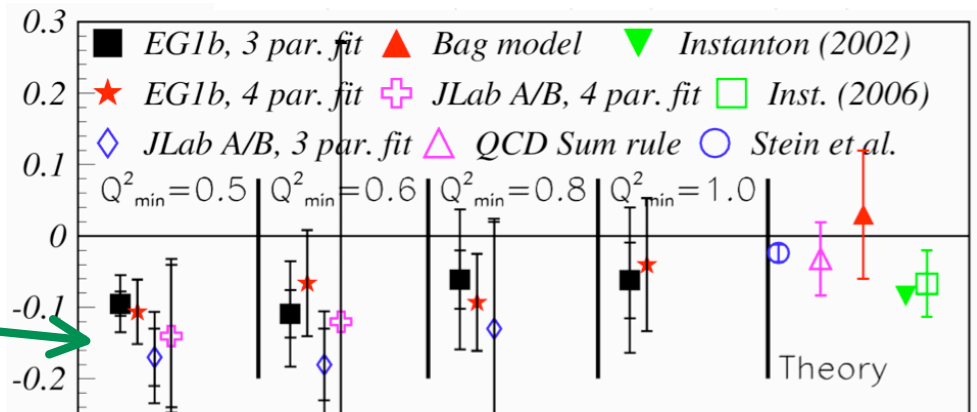
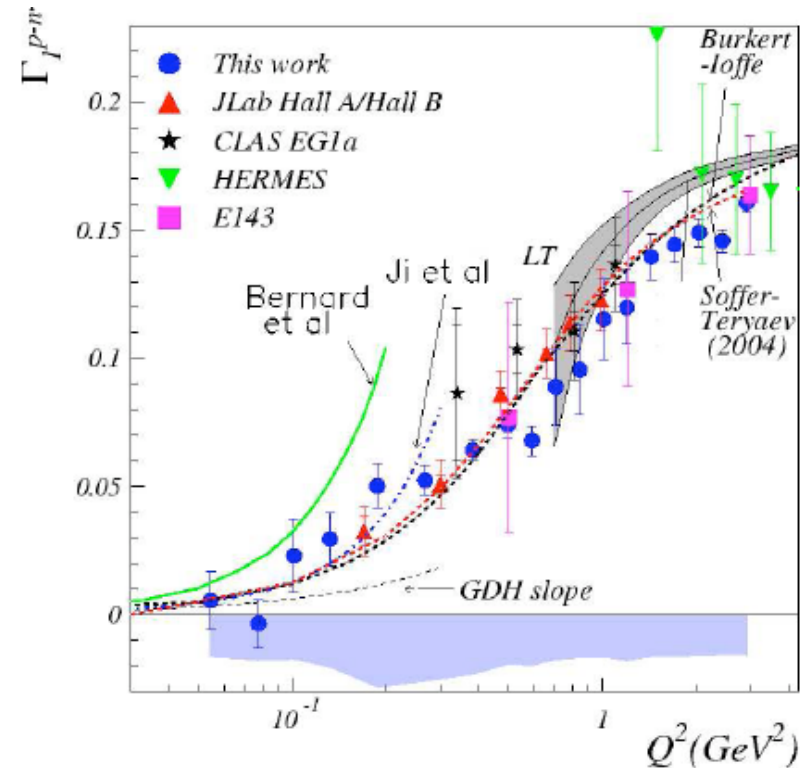
**Bjorken Sum Rule:**

$$\Gamma_1^{p-n} = \frac{g_A}{6} \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 \right] + \frac{\mu_4^{p-n}}{Q^2} + \dots$$

$$\mu_4^{p-n} = \frac{M^2}{9} (a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n})$$

$$d_2^{p-n} = \int_0^1 dx x^2 (2g_1^{p-n} + 3g_2^{p-n})$$

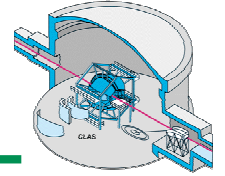
Fit  $\Gamma_1^{p-n}$  to powers of  $1/Q^2$  and extract  $f_2^{p-n}$



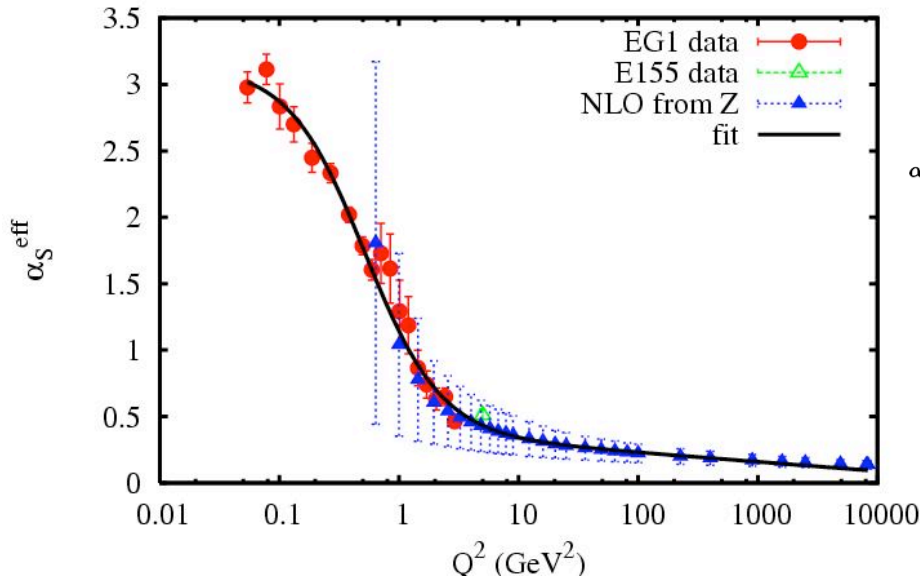




# Commensurate Scaling



Effective  $\alpha_S$  vs.  $Q^2$



Brodsky, Lu, PRD51(95)3652

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$R(Q) \equiv 3 \sum_f Q_f^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right]$$

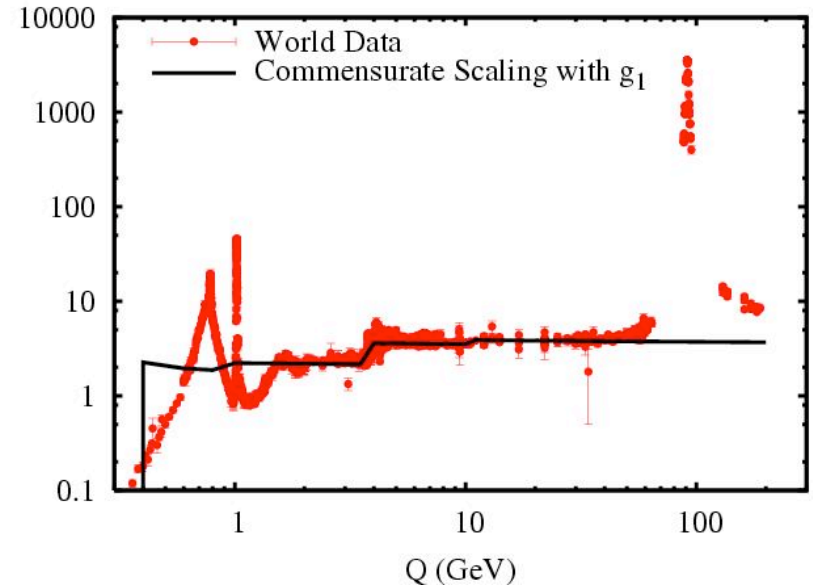
$\alpha_R(Q)$  in terms of  $\alpha_{g_1}(Q)$ .

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{g_1}(Q^*)}{\pi} + \frac{3}{4} C_F \left( \frac{\alpha_{g_1}(Q^{**})}{\pi} \right)^2 + \left[ \frac{9}{16} C_F^2 + \left( \frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F N} \left( \frac{\sum_f Q_f^2}{\sum_f Q_f^2} \right)^2 \right] \left( \frac{\alpha_{g_1}(Q^{**})}{\pi} \right)^3,$$

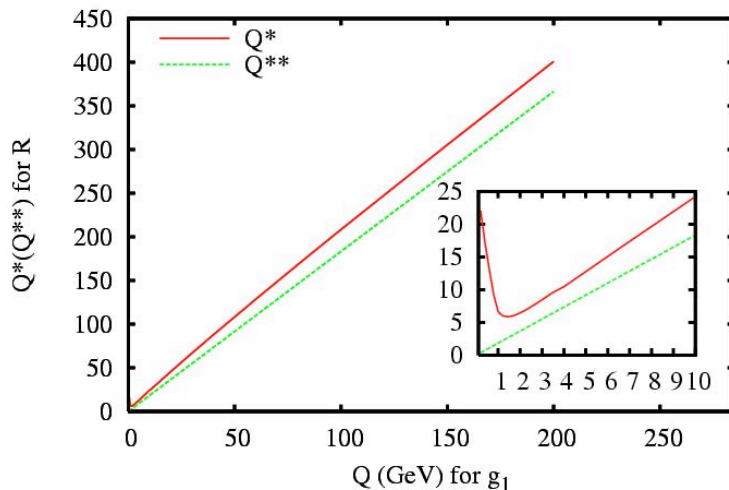
$$\ln(Q^*/Q) = -\frac{7}{4} + 2\zeta_3 + \left( -\frac{11}{96} - \frac{7}{3}\zeta_3 + 2\zeta_3^2 + \frac{\pi^2}{24} \right) \left( \frac{11}{3} C_A - \frac{2}{3} f \right) \frac{\alpha_{g_1}(Q)}{\pi},$$

$$\ln(Q^{**}/Q) = -\frac{233}{216} + \frac{64}{9}\zeta_3 - \frac{20}{3}\zeta_5 + \left( -\frac{13}{54} + \frac{2}{9}\zeta_3 \right) \frac{C_A}{C_F}.$$

$R(e^+e^- \rightarrow \text{more than 2 hadrons})$

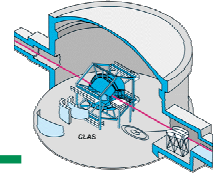


Commensurate scales for R in terms of Q for  $g_1$





# Moments $\Gamma_1^{p,d}$



$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x, Q^2) dx \quad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

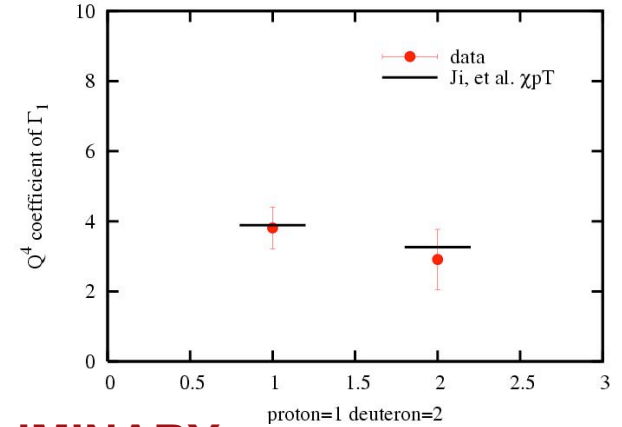
$$\Gamma_1^d(Q^2) = (1 - 1.5\omega_D) \{ \Gamma_1^p(Q^2) + \Gamma_1^n(Q^2) \}$$

$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^z}{8M^2} Q^2 + 3.89Q^4 + \dots$$

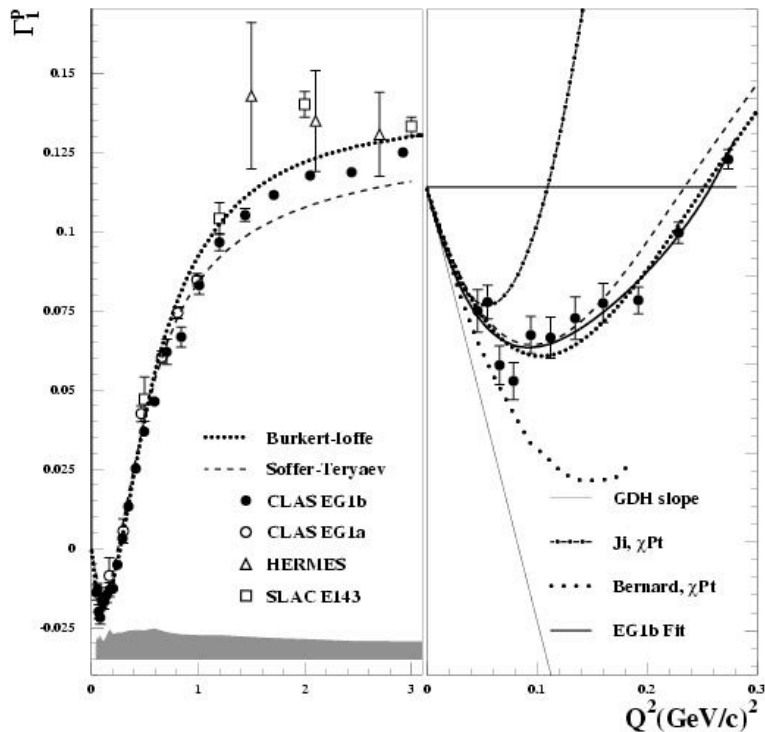
$$\Gamma_1^n(Q^2) = -\frac{\kappa_n^z}{8M^2} Q^2 + 3.15Q^4 + \dots$$

low  $Q^2$  fit

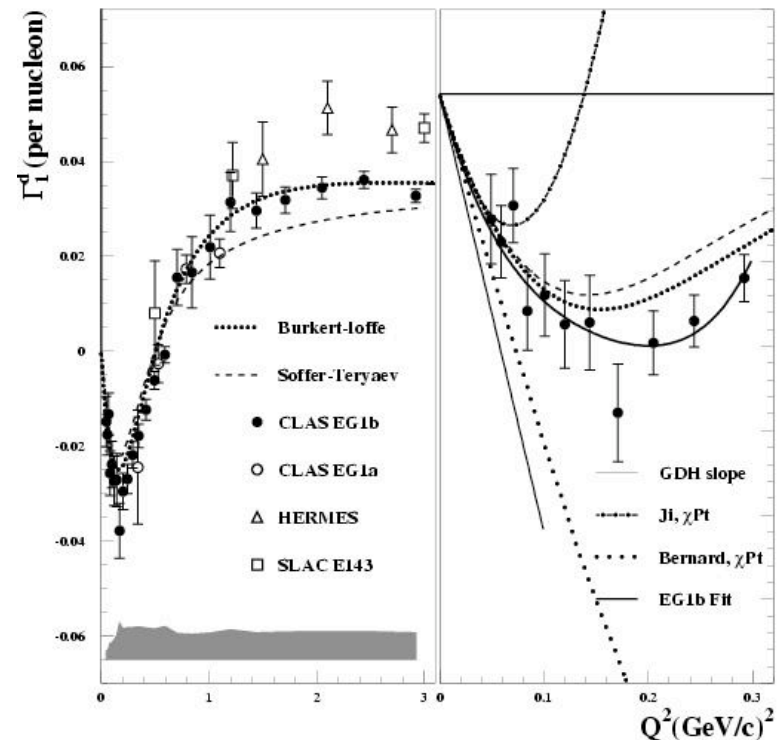
GDH +  $\chi pT$



**PRELIMINARY**

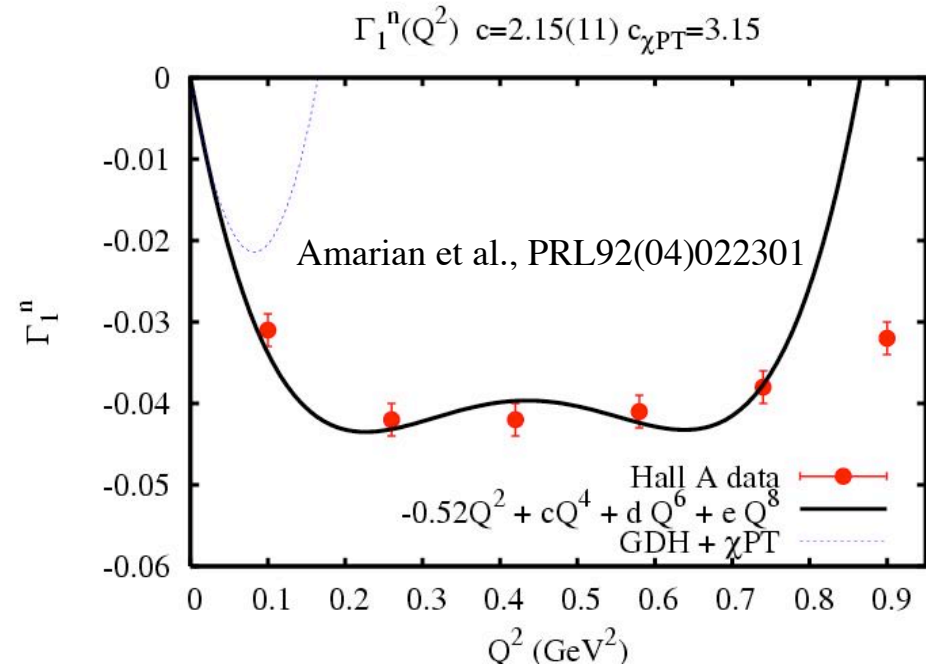
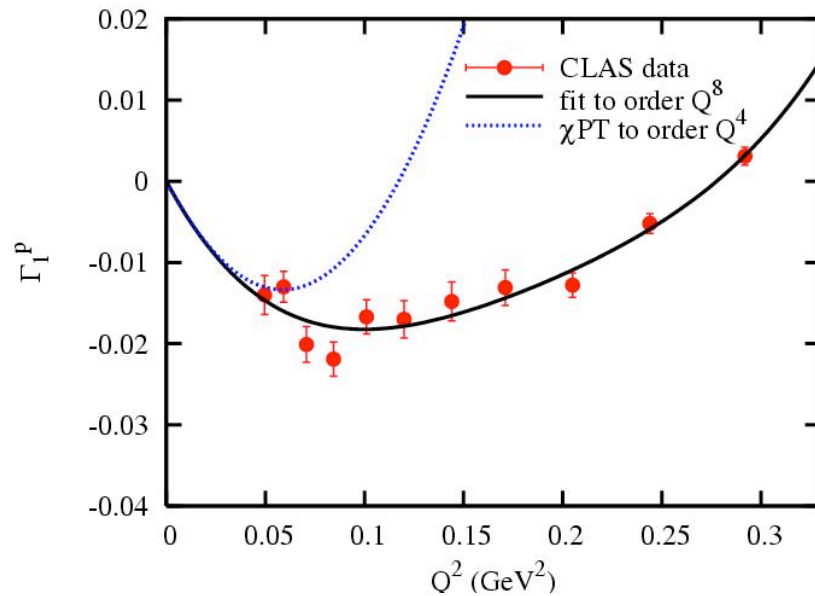
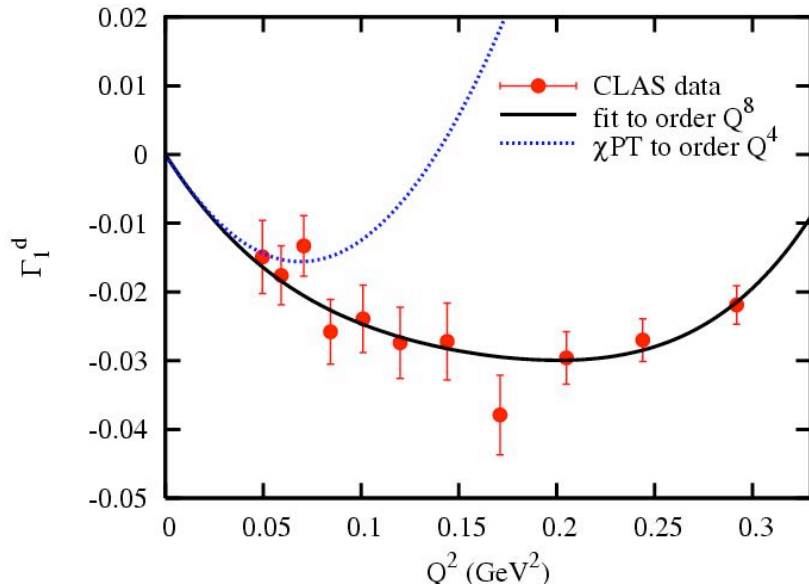
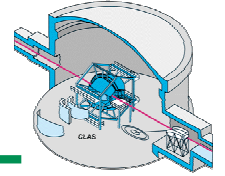


**PRELIMINARY**





# Low $Q^2$ Fits of $\Gamma_1$



$\chi$ PT

X. D. Ji, C. W. Kao and J. Osborne, Phys. Lett. B **472**, 1 (2000) [arXiv:hep-ph/9910256].

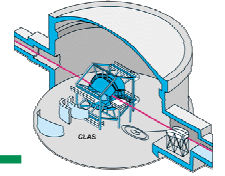
fit to  $aQ^2 + bQ^4 + cQ^6 + dQ^8$

a fixed by GDH

b compared to  $\chi$ PT



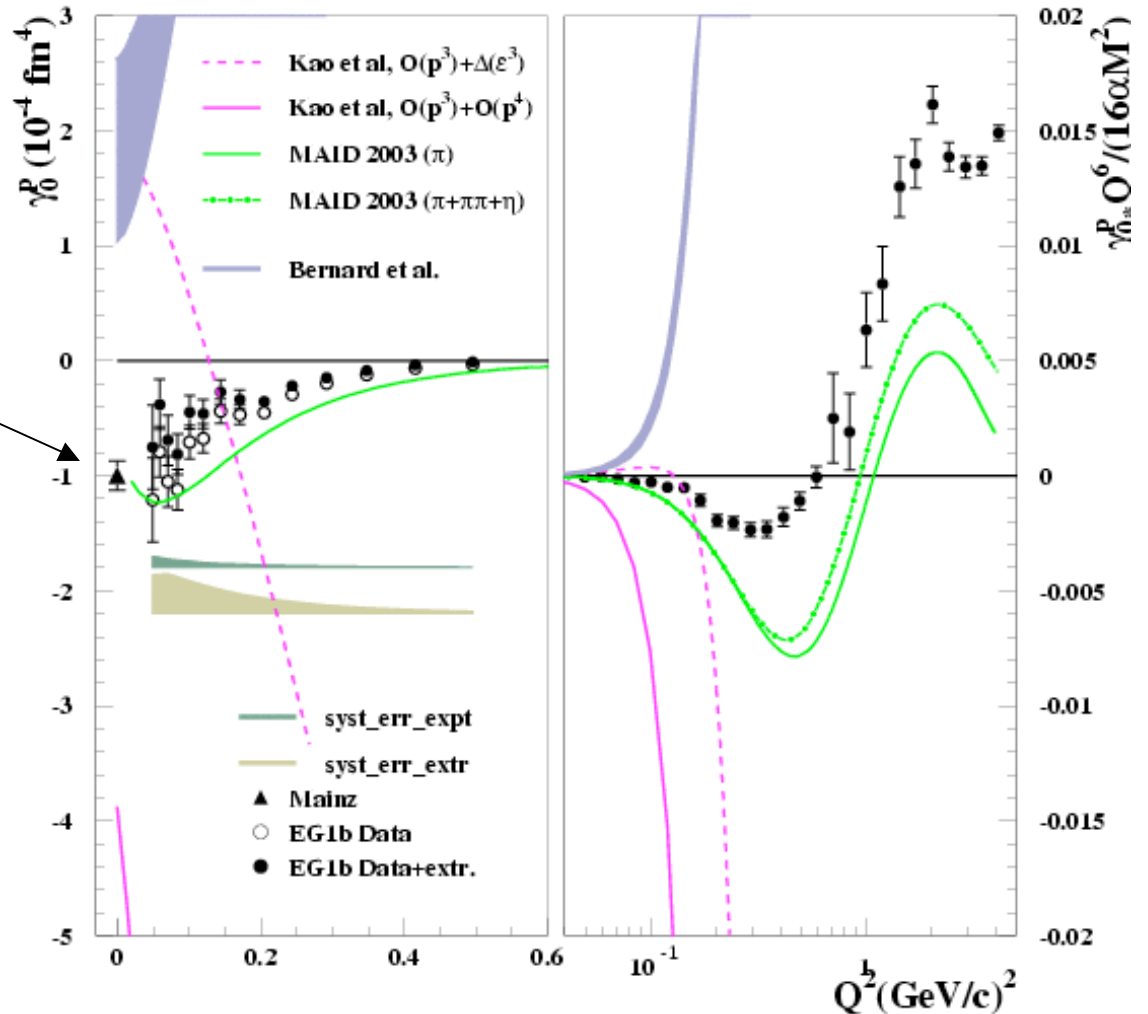
# Forward Spin Polarizability



$$\gamma_0(Q^2) = \frac{4e^2 M^2}{\pi Q^6} \int_0^{x_0} dx x^2 \{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)\}$$

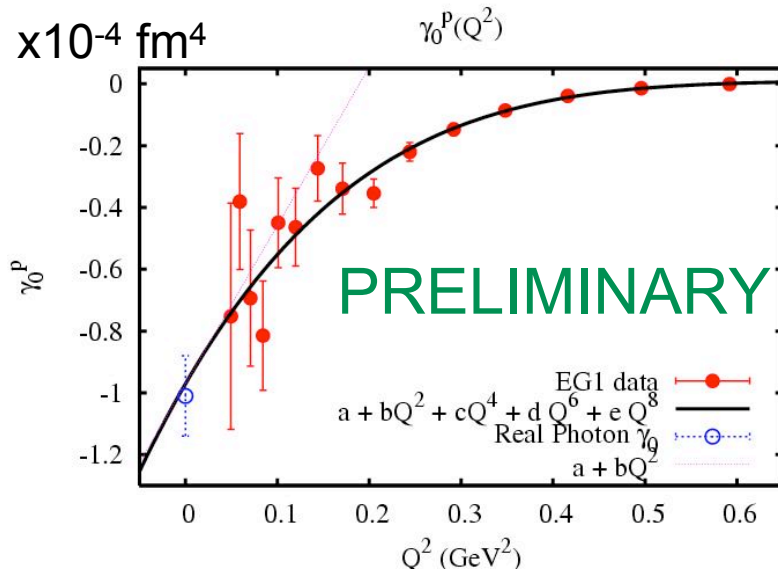
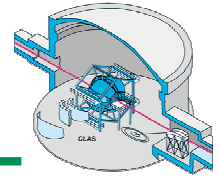
**PRELIMINARY**

real photon point





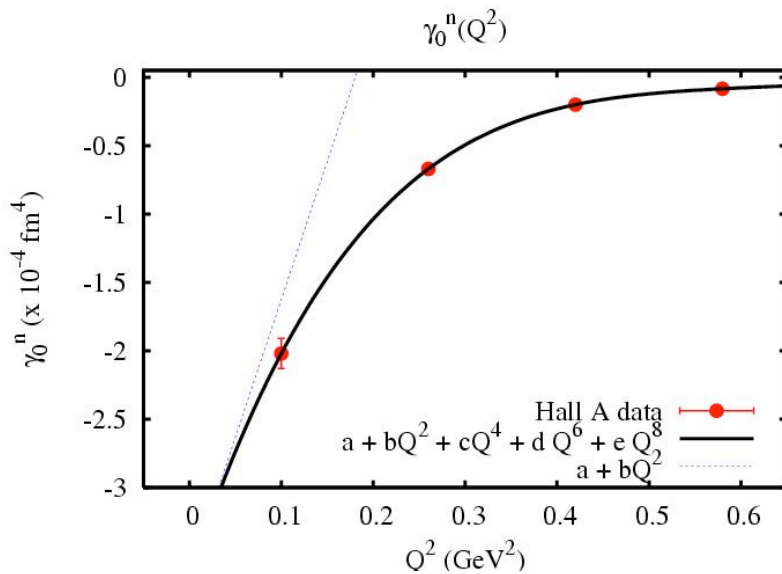
# $\gamma_0$ Fits at Low $Q^2$



$$a = -0.97(11)$$

$$b = 5.13(94)$$

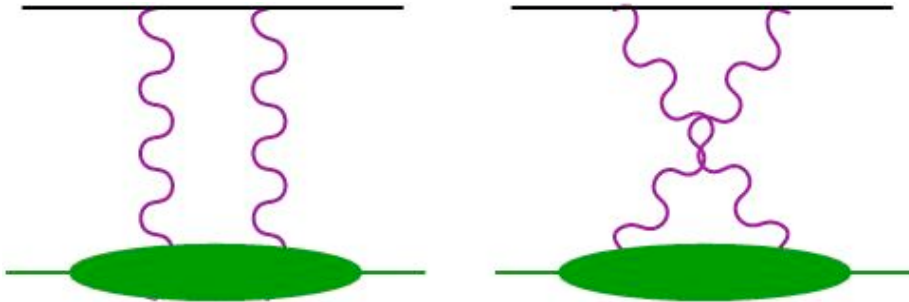
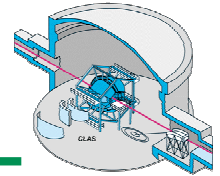
Prok et al., CLAS EG1



$$a = -3.643(1)$$

$$b = 20.180(8)$$

Amarian et al., PRL93(04)152301

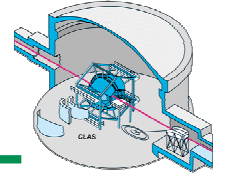


Proton Feynman diagrams for the proton polarizability term in the hydrogen hyperfine splitting.

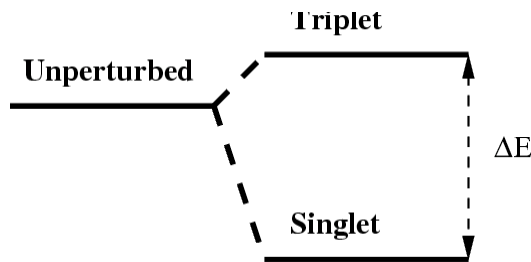
- see Brodsky, Carlson, Hiller and Hwang, PRL94(05)022001,169902(E).

- $$E_{\text{hfs}}(e^-p) = E_F^p (1 + \Delta_{\text{QED}} + \Delta_R^p + \Delta_S + \Delta_{\text{hvp}}^p + \Delta_{\mu\text{vp}}^p + \Delta_{\text{weak}}^p) = 1.4204057517667(9)\text{GHz}$$

- $\Delta_{\text{QED}} = 1136.09(14)$  ppm ( $\frac{\alpha}{2\pi} + \dots$ )
- $\Delta_R^p = 5.86(15)$  ppm (recoil)
- $\Delta_{\text{hvp}}^p = 0.01$  ppm (hadronic vacuum polarization)
- $\Delta_{\mu\text{vp}}^p = 0.07$  ppm (muonic vacuum polarization)
- $\Delta_{\text{weak}}^p = 0.06$  ppm (weak interaction)
- $\Delta_S = -38.62(16)$  ppm (nucleon structure; deduced)
- $\Delta_S$  is the largest uncertainty in theoretical calculation of  $E_{\text{hfs}}(e^-p)$



$$E_{\text{HFS}}(e^- p) = 1.4204057517667(9) \text{ GHz} = (1 + \Delta_{\text{QED}} + \Delta_R^p + \Delta_S) E_F^p$$



$$\Delta_S = \Delta_Z + \Delta_{\text{pol}} \quad \delta_Z^{\text{rad}} = \frac{\alpha}{3\pi} \left[ 2 \ln \frac{\Lambda^2}{m^2} - \frac{4111}{420} \right]$$

$$\text{Zemach: } \Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\text{rad}})$$

$$\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[ G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$$

$$\Delta_S = -38.62(16) \text{ ppm} \quad \Delta_Z = -41.0(5) \text{ ppm} \quad \Delta_{\text{pol}} = 2.38(58) \text{ ppm}$$

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{2\pi(1+\kappa)M} (\Delta_1 + \Delta_2) = (0.2264798 \text{ ppm}) (\Delta_1 + \Delta_2)$$

$$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + \frac{8m_p^2}{Q^2} B_1(Q^2) \right\}$$

$$\Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2).$$

$$\tau = \nu^2 / Q^2$$

$$B_1 = \int_0^{x_{\text{th}}} dx \beta(\tau) g_1(x, Q^2),$$

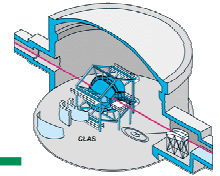
$$B_2 = \int_0^{x_{\text{th}}} dx \beta_2(\tau) g_2(x, Q^2),$$

$$\beta(\tau) = \frac{4}{9} \left( -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)} \right)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)},$$



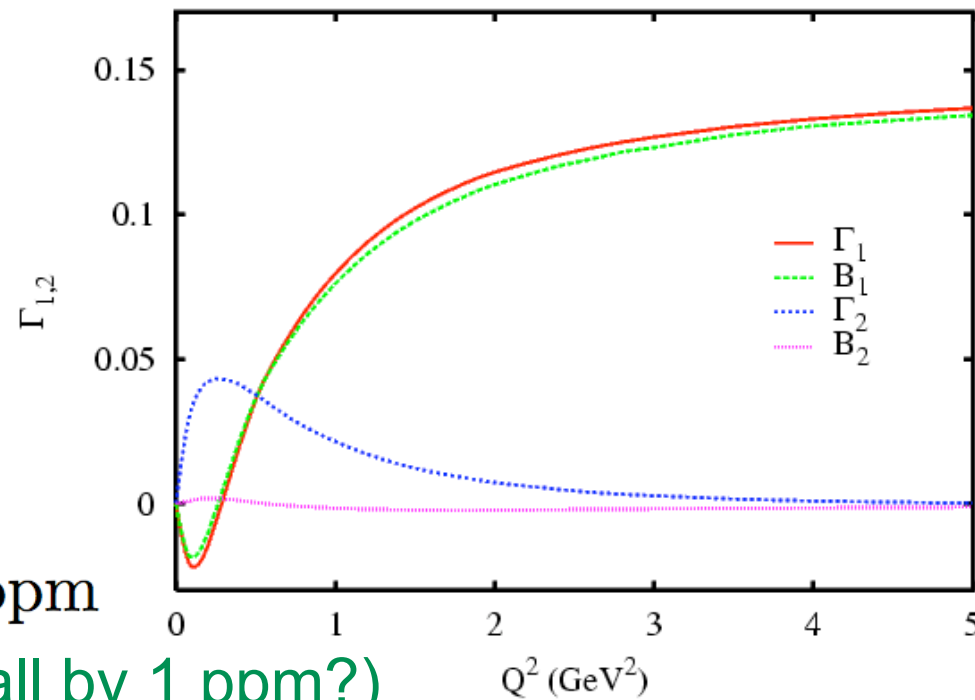
# $\Delta_{1,2}$ from $g_{1,2}$



Comparisons between  $\Gamma_1 = \int g_1 dx$  and  $B_1 = \int \beta_1 g_1 dx$   
and between  $\Gamma_2 = \int g_2 dx$  and  $B_2 = \int \beta_2 g_2 dx$

PRL96,163001

- $B_1 \approx \Gamma_1$
- $B_2 \approx 0$
- Experimentally, errors on  $\Gamma_1$  are understood; we exploit this fact.
- $\Gamma_2 = \int g_2 dx \neq 0$  at low  $Q^2$ .



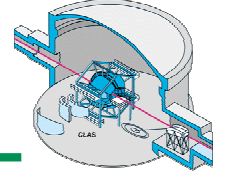
$\Delta_{\text{pol}} = (1.3 \pm 0.3) \text{ ppm}$   
(from EG1: too small by 1 ppm?)

Nucleon structure is the largest uncertainty in calculating HFS.  
Better  $g_1, g_2, G_M, G_E$  data at low  $Q^2$  required to resolve discrepancy.





# Moments at Low $Q^2$



$$\Gamma_{1,2}^{(N)}(Q^2) = \int_0^{x_{th}} x^N g_{1,2}(x, Q^2) dx$$

$$\Gamma_{1,2}^{(N)} \sim (Q^2)^{N+1}$$

$$\Gamma_1^{(2)} \rightarrow \gamma_0 Q^6 / (16\alpha m_p^2)$$

$$\gamma_0(Q^2) = \frac{16\alpha m_p^2}{Q^6} \int_0^{x_{th}} x^2 \left( g_1 - \frac{4m_p^2 x^2}{Q^2} g_2 \right) dx$$

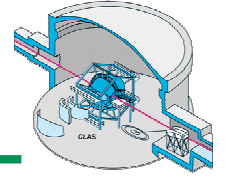
$$\Gamma_1^{(0)} = -\kappa_p^2 Q^2 / (8m_p^2) + c_1 Q^4 + \dots$$

$$B_1 = \Gamma_1^{(0)} - 10m_p^2 \Gamma_1^{(2)} / (9Q^2) + \dots$$

$$\Delta_1[0, Q_1^2] = \left\{ -\frac{3}{4} r_P^2 \kappa_p^2 + 18m_p^2 c_1 - \frac{5m_p^2}{4\alpha} \gamma_0 \right\} Q_1^2$$

$$\Delta_2[0, Q_1^2] = 3m_p^2 Q_1^2 (\gamma_0 - \delta_{LT}) / 2\alpha$$

$$\begin{aligned} \delta_{LT}(Q^2) &= \left( \frac{1}{2\pi^2} \right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu \\ &= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1(x, Q^2) + g_2(x, Q^2)] dx \end{aligned}$$



$$\Delta_2[0, Q_1^2] = 3m_p^2 Q_1^2 (\gamma_0 - \delta_{LT}) / 2\alpha$$

$$\Delta_1[0, Q_1^2] = \left\{ -\frac{3}{4} r_P^2 \kappa_p^2 + 18m_p^2 c_1 - \frac{5m_p^2}{4\alpha} \gamma_0 \right\} Q_1^2$$

$\gamma_0 = -1.01 \times 10^{-4} \text{ fm}^4$  (photons)  
 $r_P = 0.878(15) \text{ fm}$  (Kelly)  
 $c_1 = 2.95\text{-}3.89$  (fits/ $\chi$ PT)  
 $\delta_{LT} = 1.35 \times 10^{-4} \text{ fm}^4$  (MAID)

$$g_2 = g_2^{WW} \rightarrow \Gamma_2^{(N)} = -N \Gamma_1^{(N)} / (N+1)$$

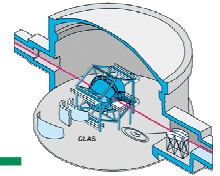
$$\Delta_2[0, 0.05] =$$

- 0.40(05) [ $g_2^{WW}$ ]
- 1.4 [MAID]
- 0.24 [EG1 Model]

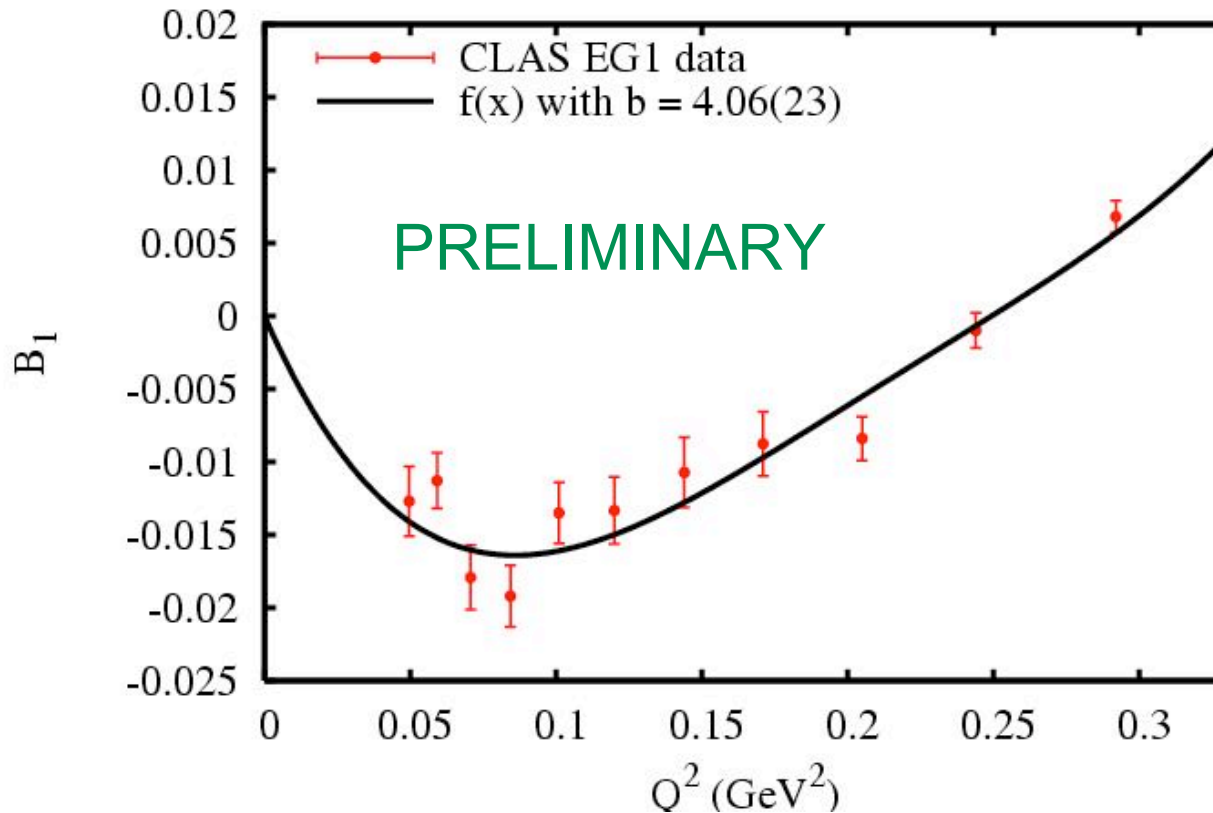
term	$Q^2$ (GeV <sup>2</sup> )	from	Kelly's $F_2$
$\Delta_1$	[0,0.05]	$F_2$ and $g_1$	$0.45 \pm 0.30$
	[0.05,20]	$F_2$	$7.01 \pm 0.22$
		$g_1$	$-1.10 \pm 0.55$
	[20, $\infty$ ]	$F_2$	0.00
		$g_1$	$0.12 \pm 0.01$
total $\Delta_1$			$6.48 \pm 0.89$
$\Delta_2$	[0,0.05]	$g_2$	$-0.24 \pm 0.24$
	[0.05,20]	$g_2$	$-0.33 \pm 0.33$
	[20, $\infty$ ]	$g_2$	0.00
total $\Delta_2$			$-0.57 \pm 0.57$
$\Delta_1 + \Delta_2$			$5.91 \pm 1.06$
$\Delta_{\text{pol}}$			$1.34 \pm 0.24 \text{ ppm}$



# B<sub>1</sub> from CLAS



$$B_1(Q^2) \text{ fit to } -0.4564Q^2 + bQ^4 + cQ^6 + dQ^8$$

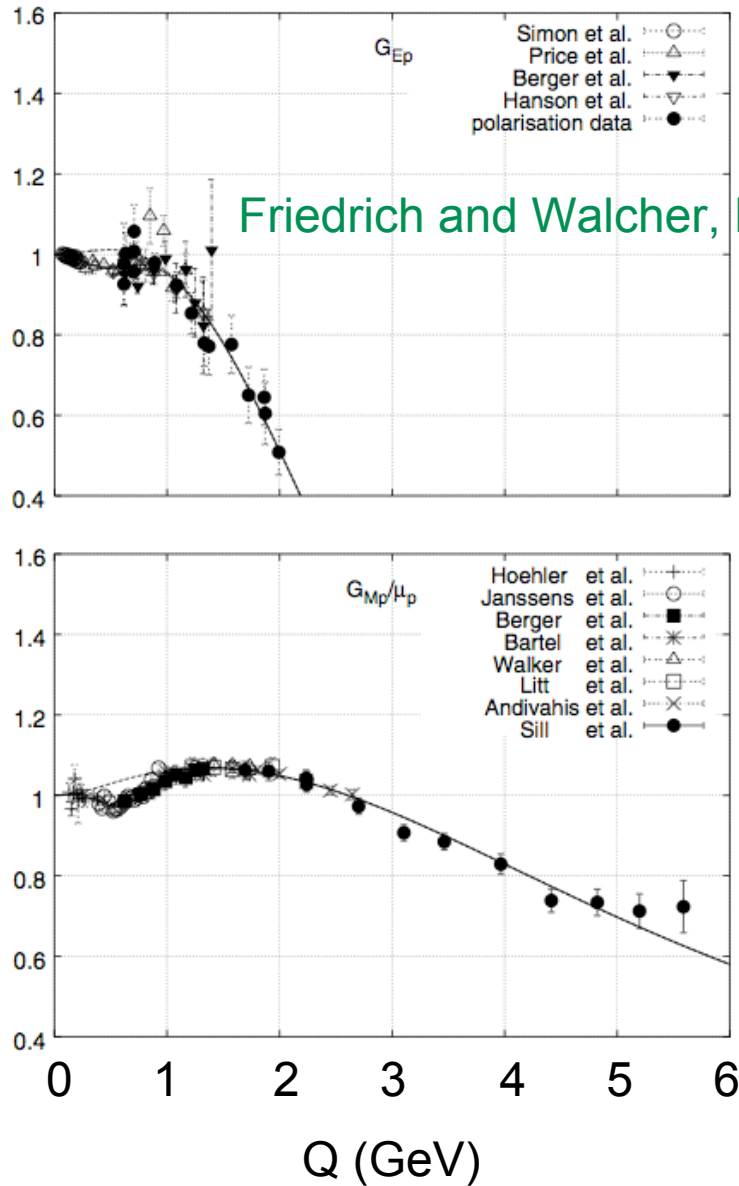
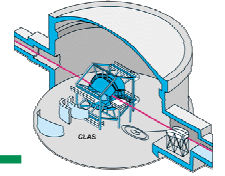


Various estimates change  $\Delta_{pol}$  up or down within the quoted errors. New data at low  $Q^2$  are needed to improve this.

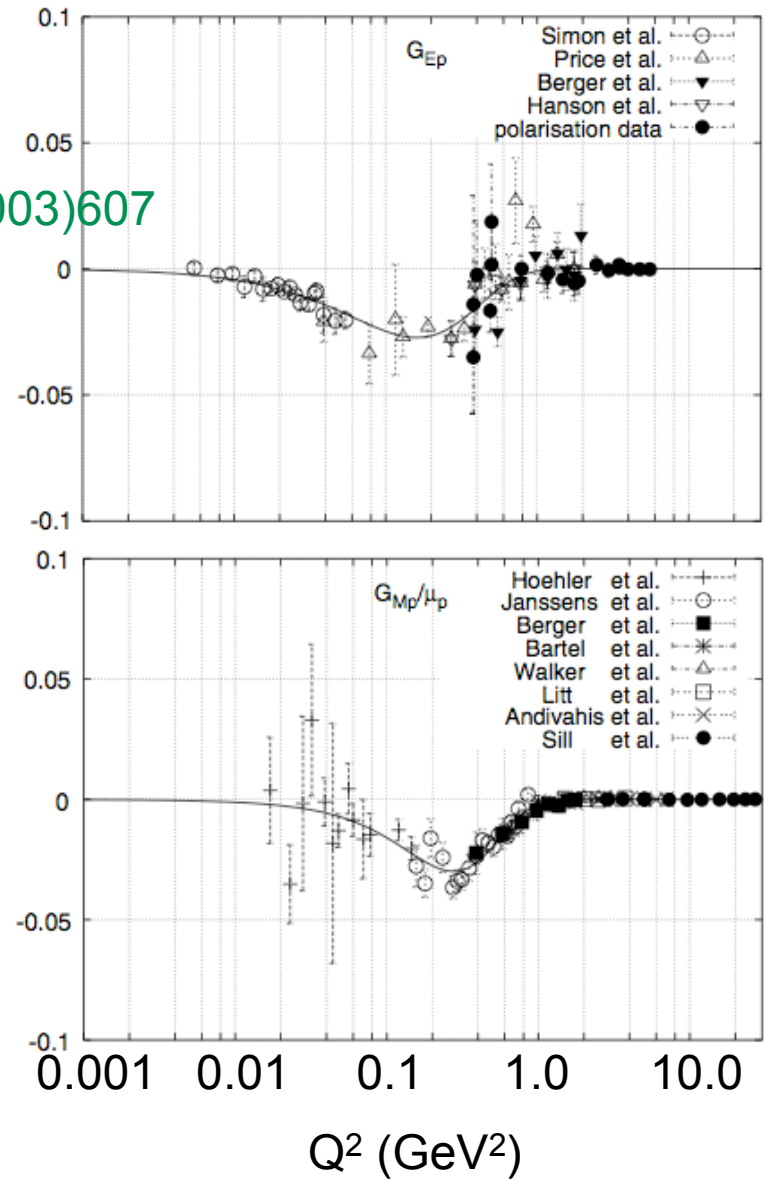
$$\Delta_1[0,0.05] = [-0.75r_p^2\kappa_p^2 + 18m_p^2b]0.05 = 0.32$$



# $G_{Ep}$ & $G_{Mp}$

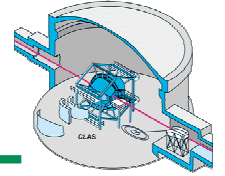


Friedrich and Walcher, EPJA17(2003)607

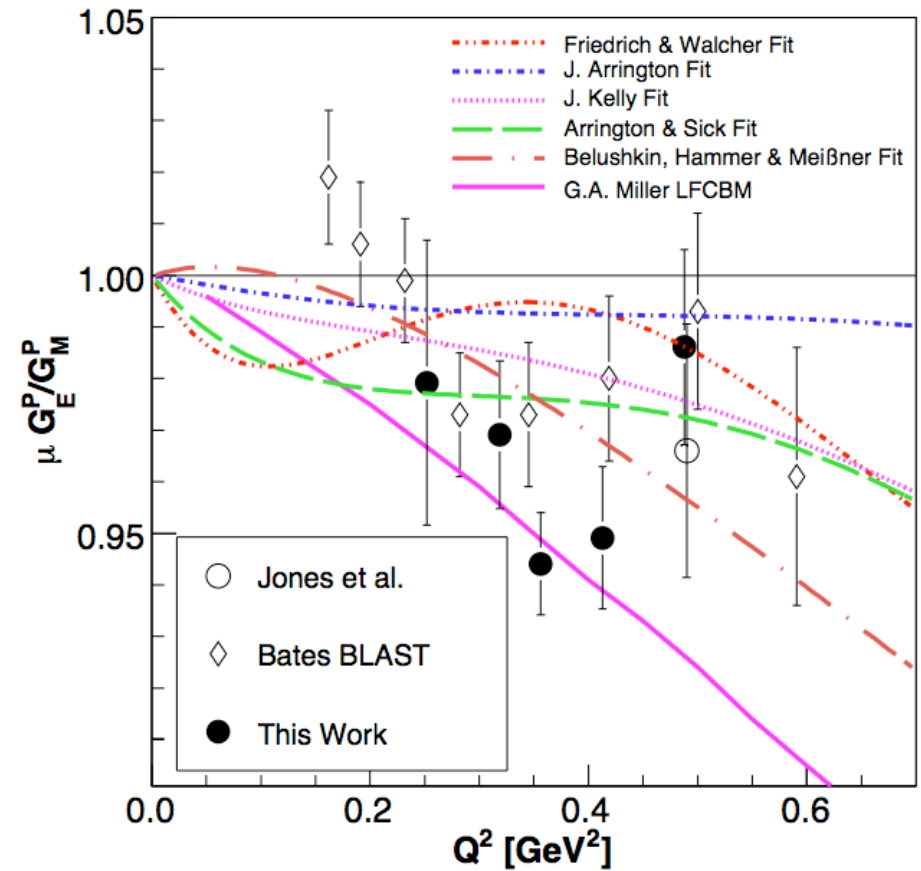
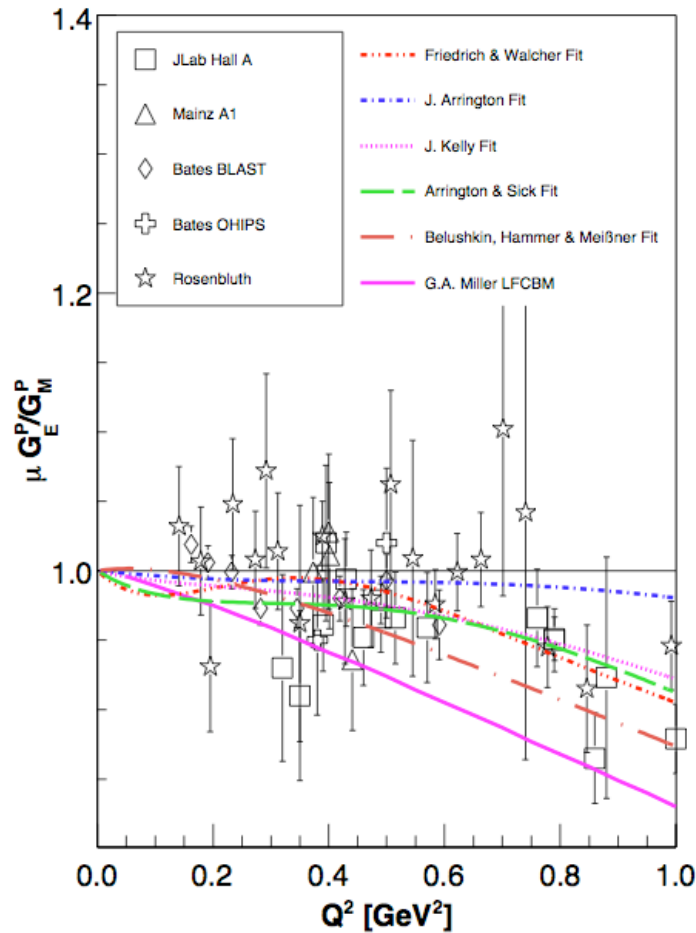




$$\mu G_{Ep}/G_{Mp}$$



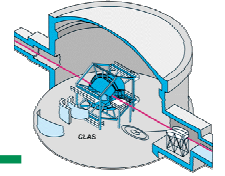
G. Ron et al., nucl-ex 0706.0128  
Hall A



The diversity of fits reflects an inaccurate knowledge of the form factors at low  $Q^2$



# Zemach Radius



$$\text{Zemach: } \Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\text{rad}})$$

$$\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[ G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$$

Reference	$r_Z$ (fm)	$\Delta_Z$ (ppm)	$\Delta_S - \Delta_Z - \Delta_{\text{pol}}$ (ppm)
Kelly	1.069(13)	-41.01	1.11
Sick	1.086(12)	-41.67	1.77
Friedrich	1.048	-40.20	0.30
Dipole	1.025	-39.32	-0.58

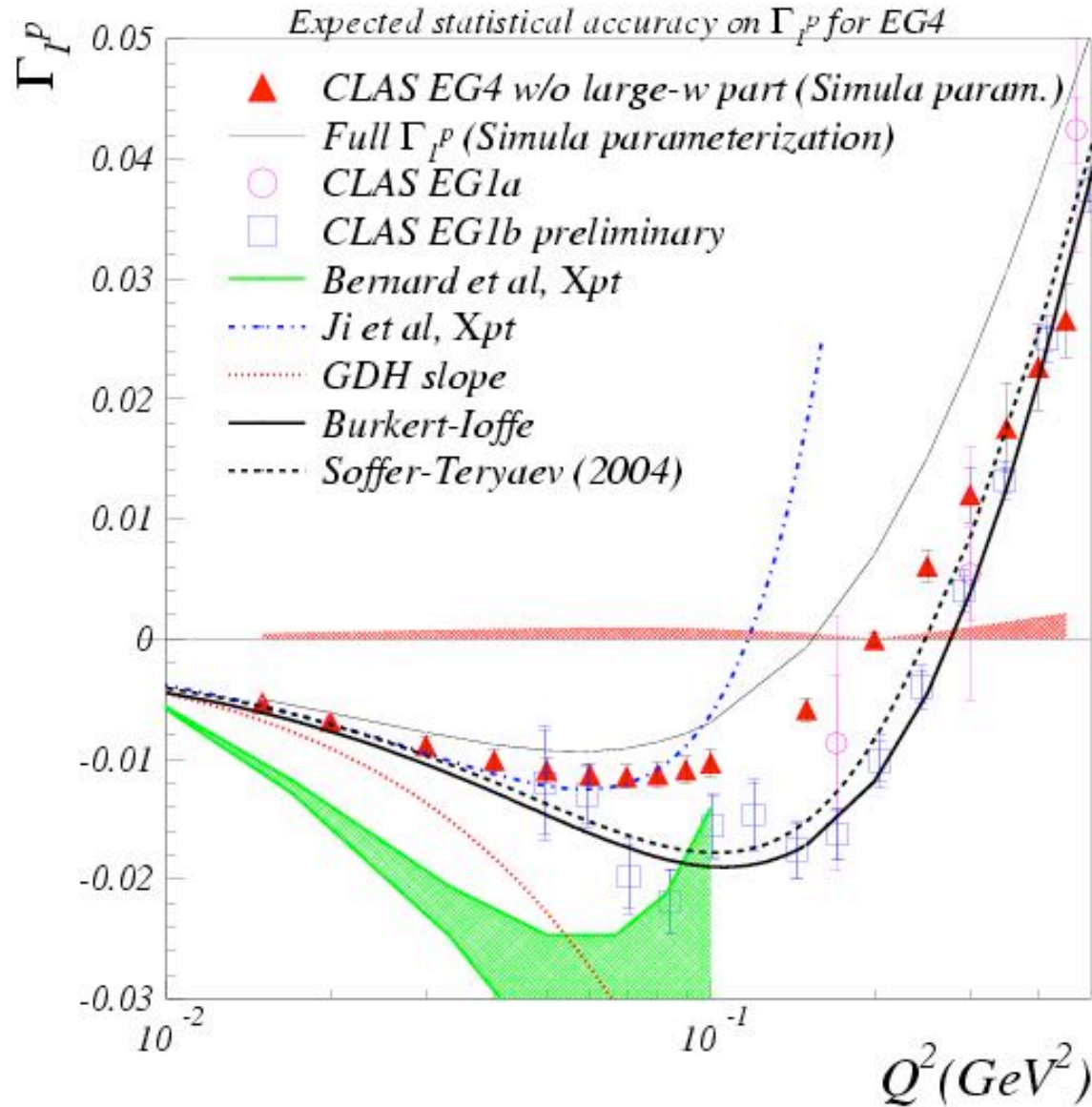
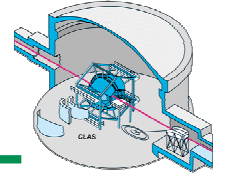
Quoted errors on S,Z and pol are 0.16, 0.49, and 0.24 ppm respectively.

Quoted error on S-Z-pol is 0.57 ppm.

Largest uncertainty in hyperfine splitting comes from low  $Q^2$  form factors!

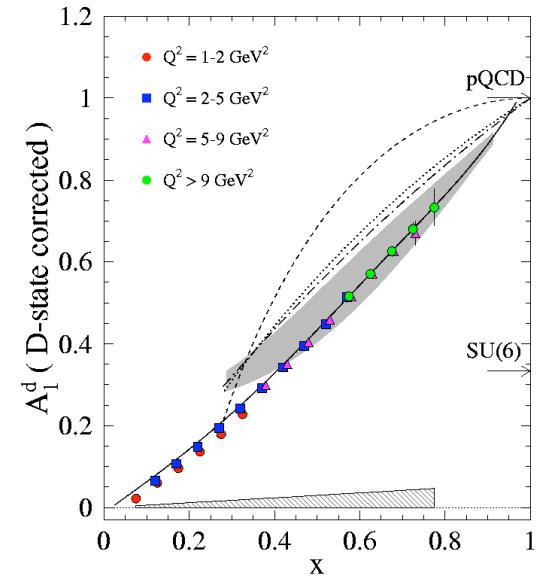
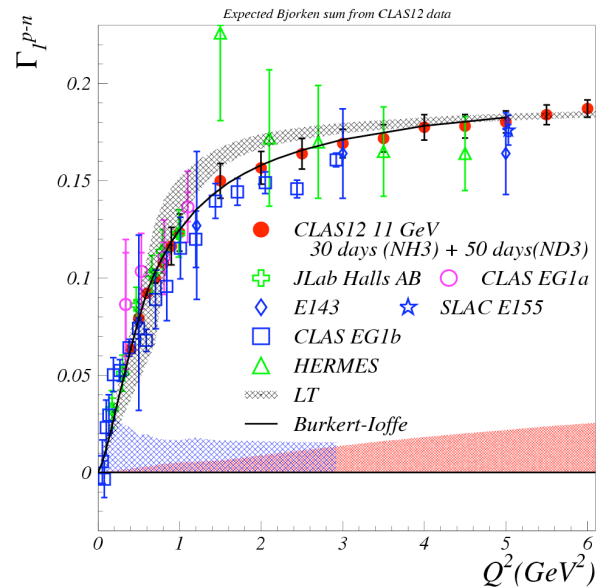
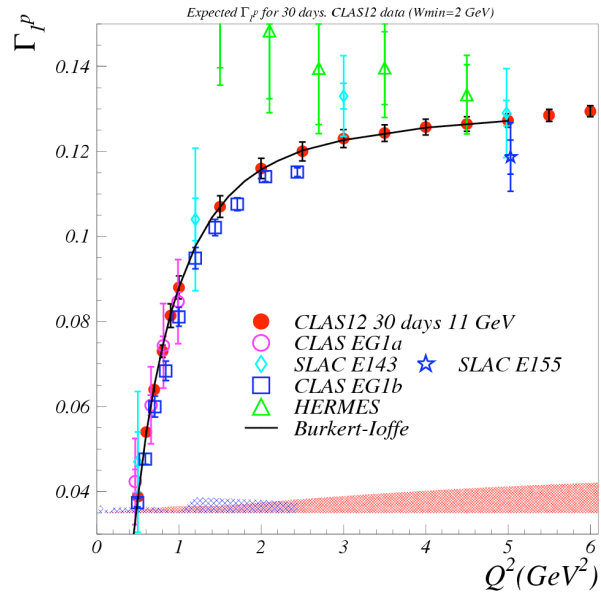
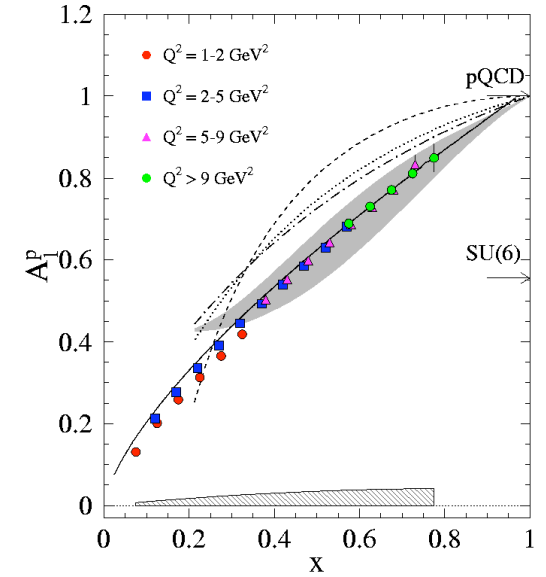
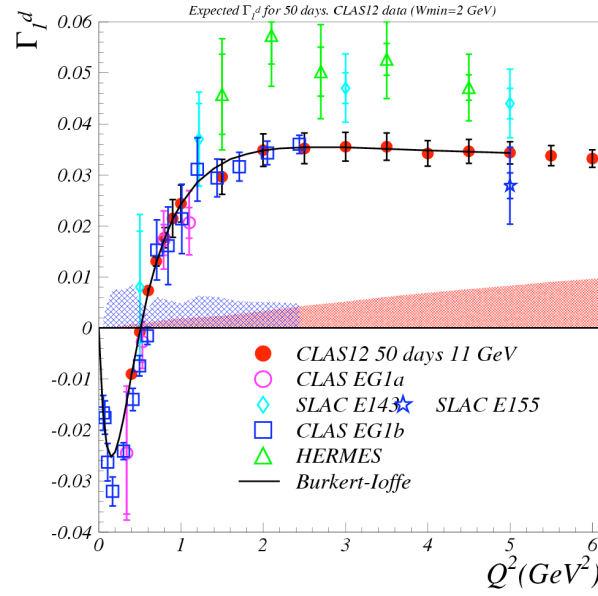
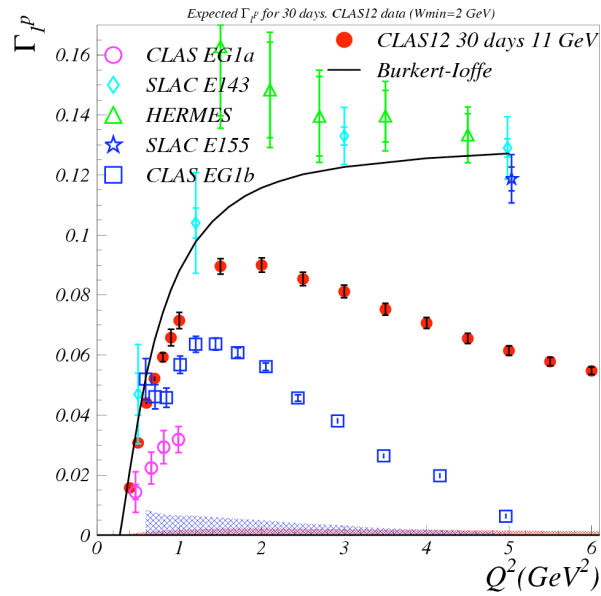
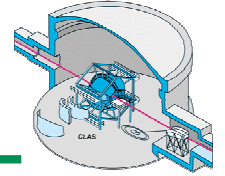


# EG4 Expectations

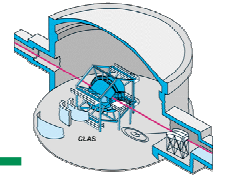




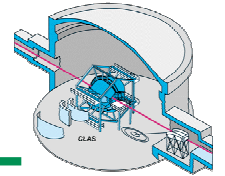
# EG12 Expectations







- CLAS, past, present and future, provides high-quality  $A_{||}$  data over a large and continuous range in  $x$  and  $Q^2$  that
- significantly improve global PDF fits to  $\Delta u$ ,  $\Delta d$ ,  $\Delta s$  and  $\Delta G$
  - precisely determine higher twists
  - rigorously probe duality over a wide  $Q^2$  range
  - quantitatively test  $\chi$ PT calculations at low  $Q^2$
  - accurately yield the polarizability correction to hydrogen hyperfine splittings



- Our patchwork understanding of the nucleon is not elegant and still incomplete

“If you are out to describe the truth, leave elegance to the tailor” -- A. Einstein

- After 50 years the nucleon still captures our interest

“La nature, pour nous hommes, et plus en profondeur qu’en surface.” -  
- Paul Cézanne

- The nucleon, boring on the surface, has a rich internal life

“o sweet spontaneous  
earth how often  
has the naughty thumb  
of science prodded  
thy  
beauty  
thou answerest  
them only with  
spring” -- e e cummings