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Spin Structure of the Proton and Deuteron

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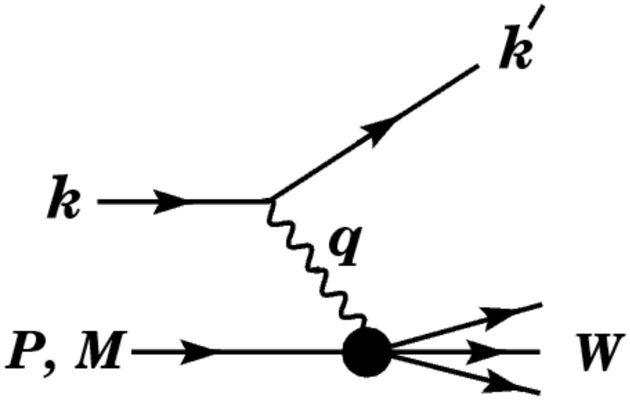
Spin Structure at Long Distances

Jefferson Lab

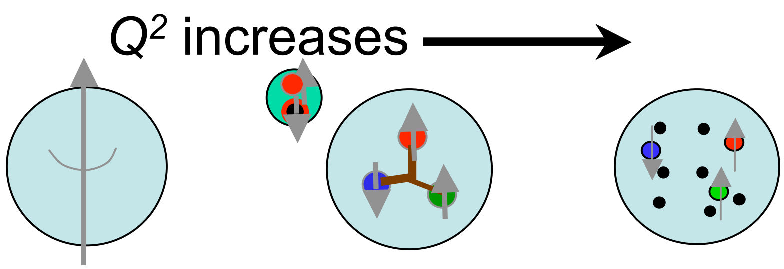
12 March 2009

Inelastic Scattering



Lorentz invariants:

$$\nu = p\cdot q/M = (E-E')_{\rm lab}$$
$$Q^2 = -q\cdot q = (4EE'\sin^2\tfrac{\theta}{2})_{\rm lab}$$
$$x = -q\cdot q/2p\cdot q = (Q^2/2M\nu)_{\rm lab}$$
$$y = p\cdot q/p\cdot k = (\nu/E)_{\rm lab}$$
$$W^2 = (p+q)^2 = (M^2+2M\nu-Q^2)_{\rm lab}$$
$$s = (k+p)^2 = (2EM+M^2)_{\rm lab}$$



Unpolarized Cross Section:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{8\pi\alpha^2 y}{Q^4}\left[\frac{y}{2}F_1 + \frac{\xi}{2xy}F_2\right]$$

Polarized Cross Section:

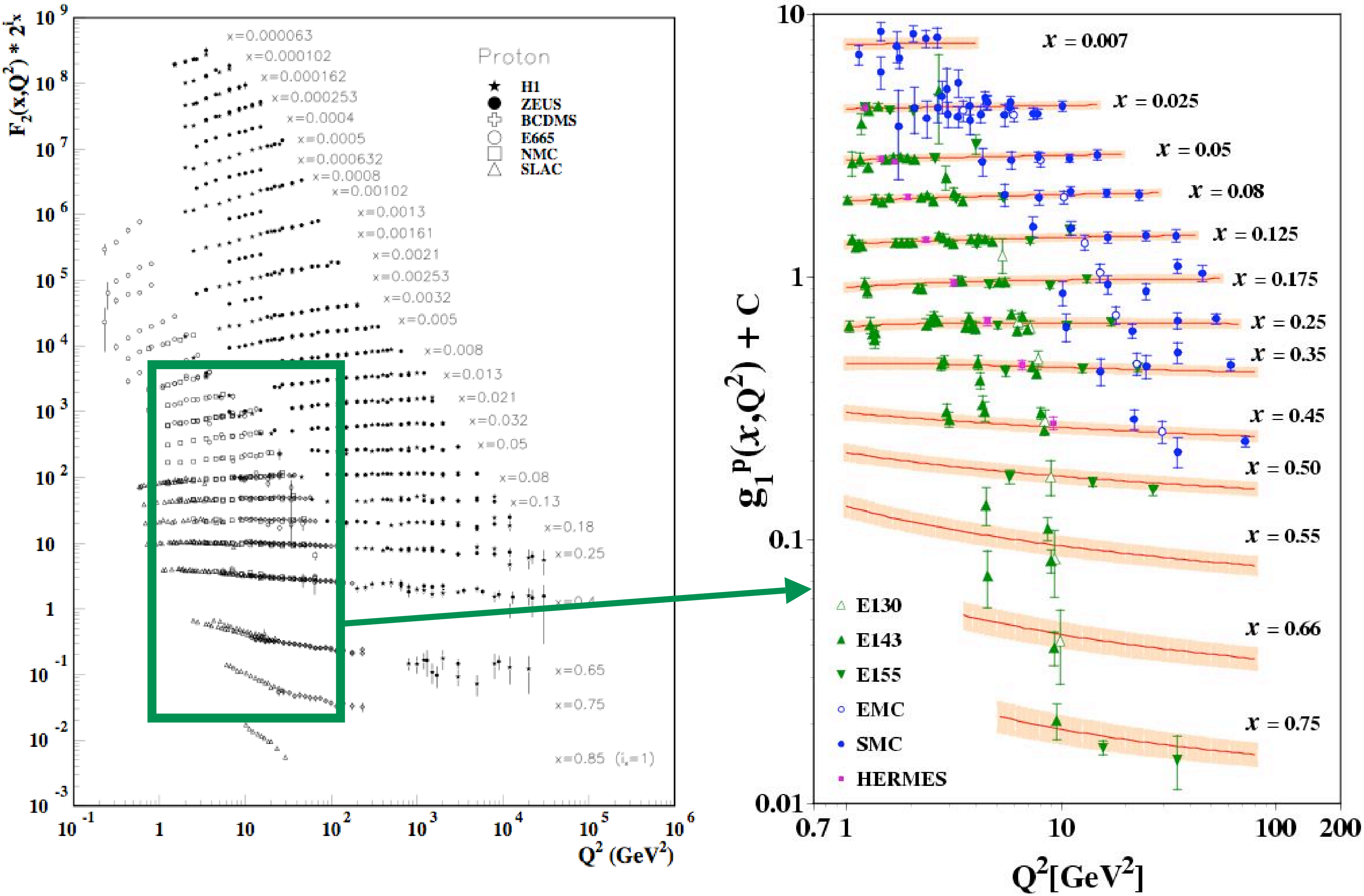
$$\frac{d^2\Delta\sigma}{dxdQ^2} = \frac{8\pi\alpha^2 y}{Q^4}[\cos\alpha\{(\xi+\frac{y}{2})g_1 - \frac{\gamma y}{2}g_2\} - \sin\alpha\cos\phi\{\frac{y}{2}g_1 + g_2\}]$$

α = polar angle of target spin wrt the beam axis
ϕ = azimuthal spin angle wrt the scattering plane
$\alpha = 0°$ (longitudinal); $\alpha = 90°, \phi = 0°$ (transverse).
$\gamma^2 = 4M^2x^2/Q^2 = Q^2/\nu^2$
$\xi = 1 - y - \gamma y^2/4$

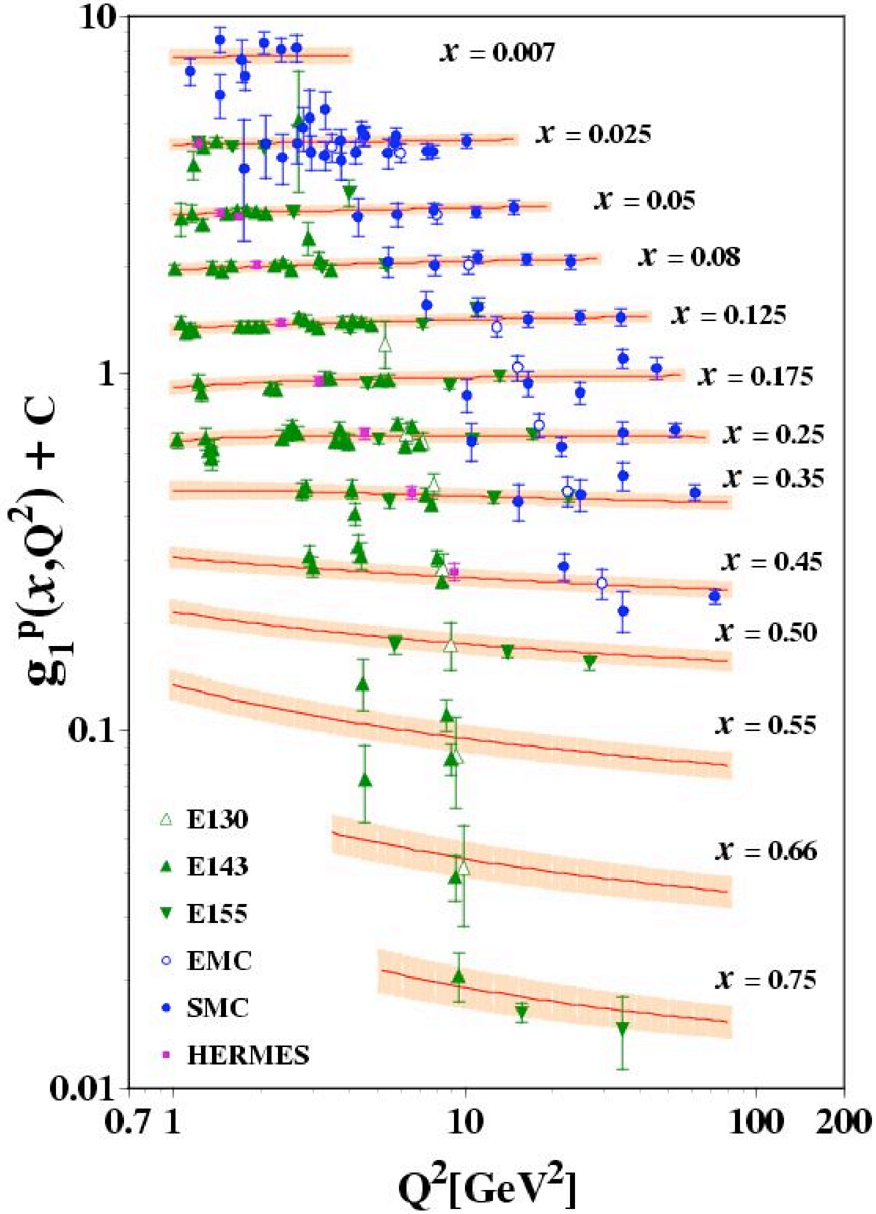
Parton Model:

$$F_1(x,Q^2) = \tfrac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) + q^\downarrow(x) + \bar{q}^\uparrow(x) + \bar{q}^\downarrow(x))$$
$$F_2(x,Q^2) = 2xF_1(x,Q^2)$$
$$g_1(x,Q^2) = \tfrac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x))$$

$F_2^p(x,Q^2)$ and $g_1^p(x,Q^2)$

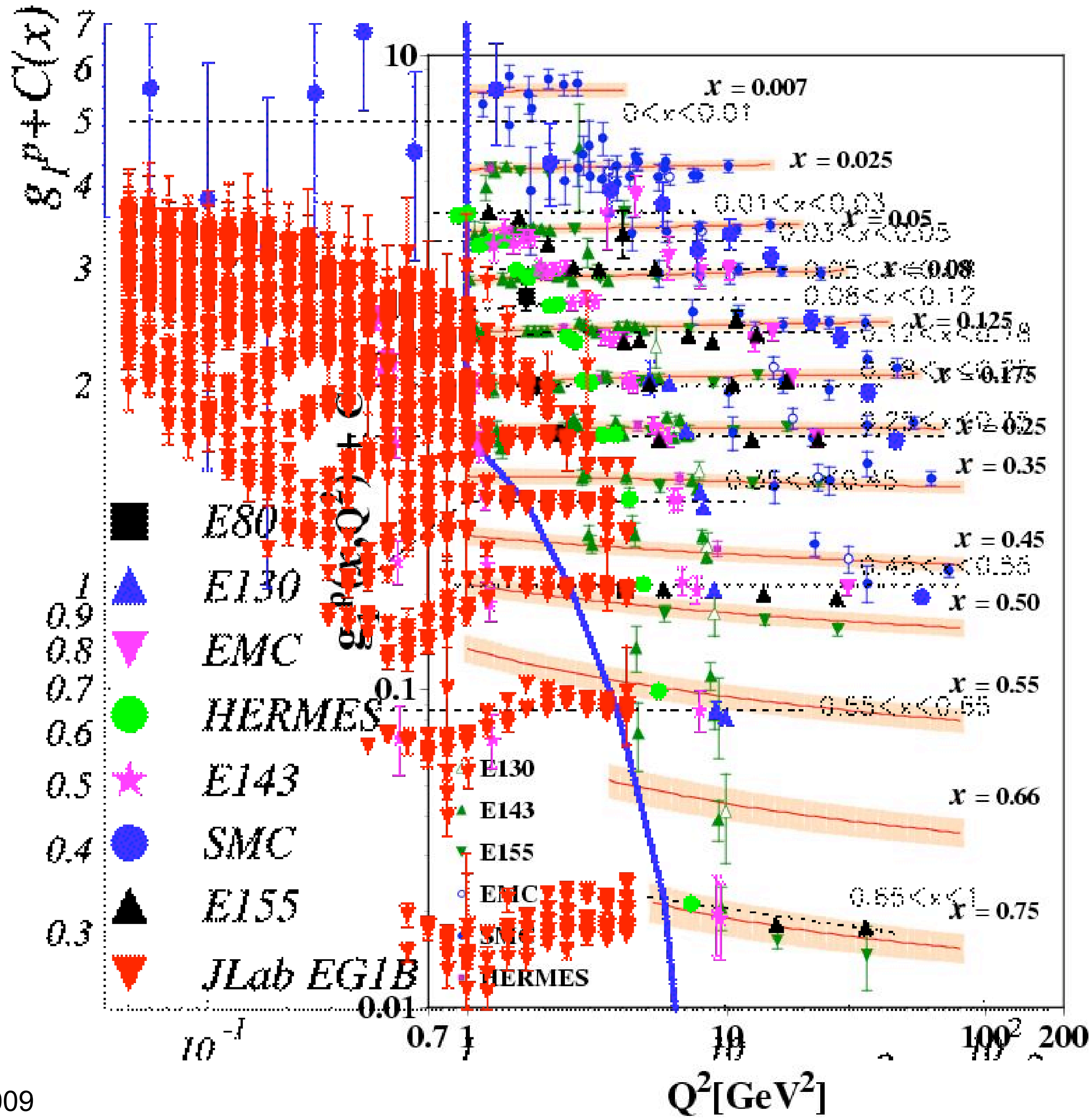


$g_1^p(x,Q^2)$



x = 0.007
x = 0.025
x = 0.05
x = 0.08
x = 0.125
x = 0.175
x = 0.25
x = 0.35
x = 0.45
x = 0.50
x = 0.55
x = 0.66
x = 0.75
E130
E143
E155
EMC
SMC
HERMES
g1p(x,Q2) + C
Q2[GeV2]
10
1
0.1
0.01
0.7
1
10
100
200

$g_1^p(x,Q^2)$



$g_1^p + C(x)$
E80
E130
EMC
HERMES
E143
SMC
E155
JLab EG1B
$g_1^p(x,Q^2) + C$
x = 0.007
x = 0.025
x = 0.05
x = 0.08
x = 0.125
x = 0.175
x = 0.25
x = 0.35
x = 0.45
x = 0.50
x = 0.55
x = 0.66
x = 0.75
0<x<0.01
0.01<x<0.03
0.03<x<0.05
0.08<x<0.12
0.55<x<0.65
0.65<x<1
E130
E143
E155
EMC
SMC
HERMES
$Q^2[GeV^2]$

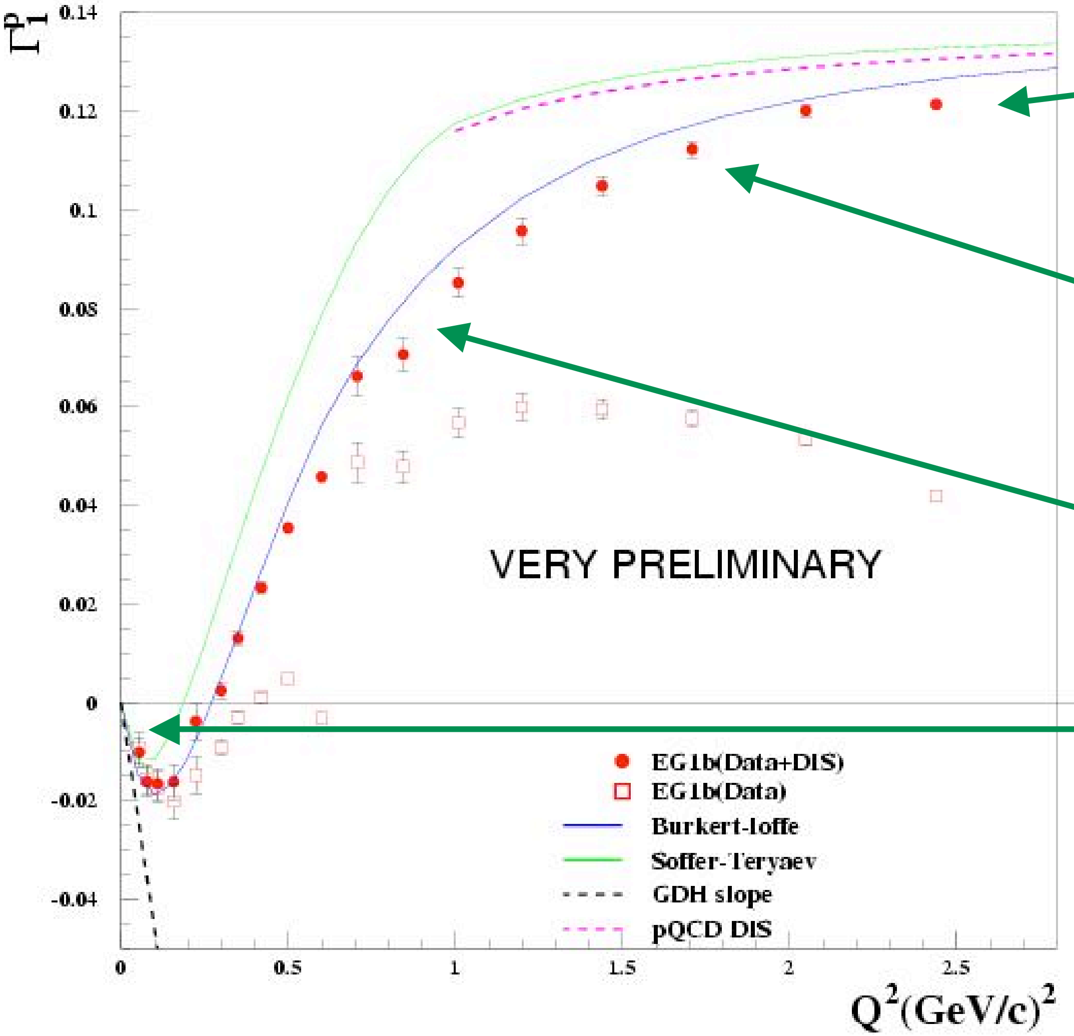
The Q^2 Map

- Infinite Q^2 — Parton Model, PDF(x)
- Large Q^2 — pQCD, PDF(x,log(Q^2))
- Medium Q^2 — Higher twist, target mass correct.
- Low Q^2 — Resonances (complexity)
- Tiny Q^2 — Chiral perturbation theory
- Zero Q^2 — Real photons
- Complexity, as measured by γ_0, δ_{LT}, d_2 and $\Gamma_{1,}$ disappears rapidly at high and low Q^2

Regions of Q^2

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x, Q^2)dx$$

CLAS EG1 Data



scaling: lnQ^2

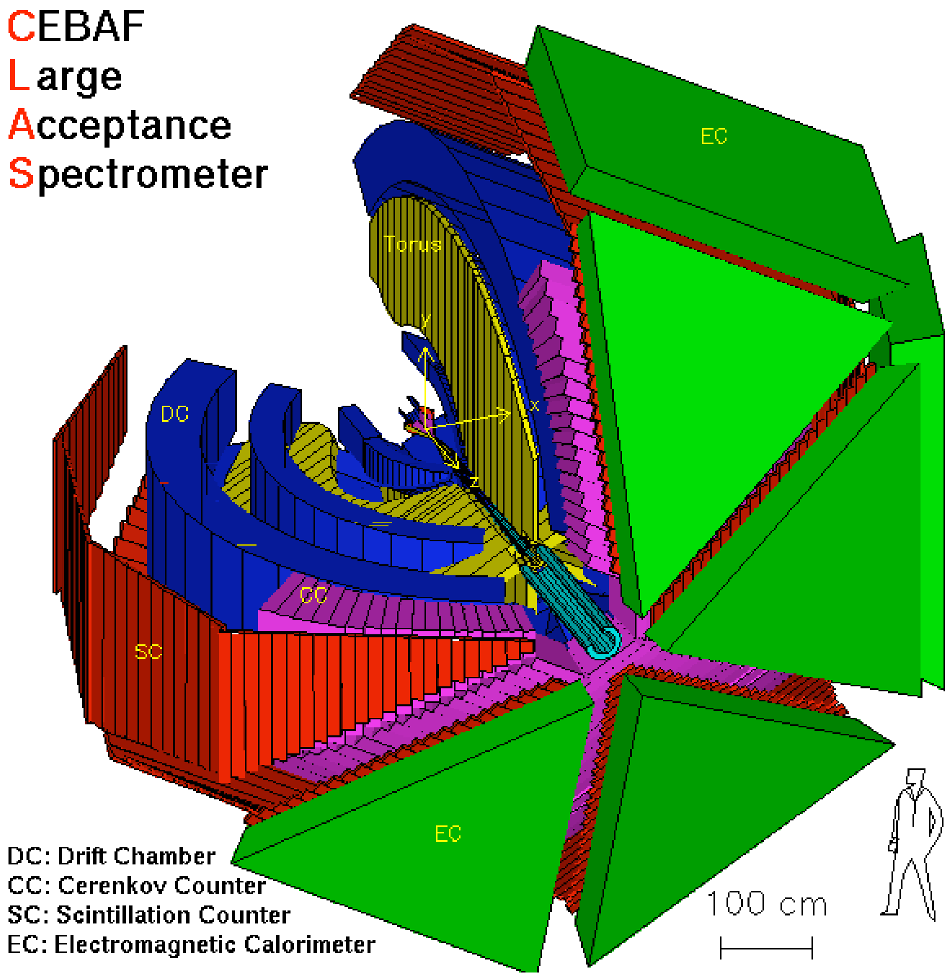
higher twist: $(1/Q^2)^n$

no nice expansion

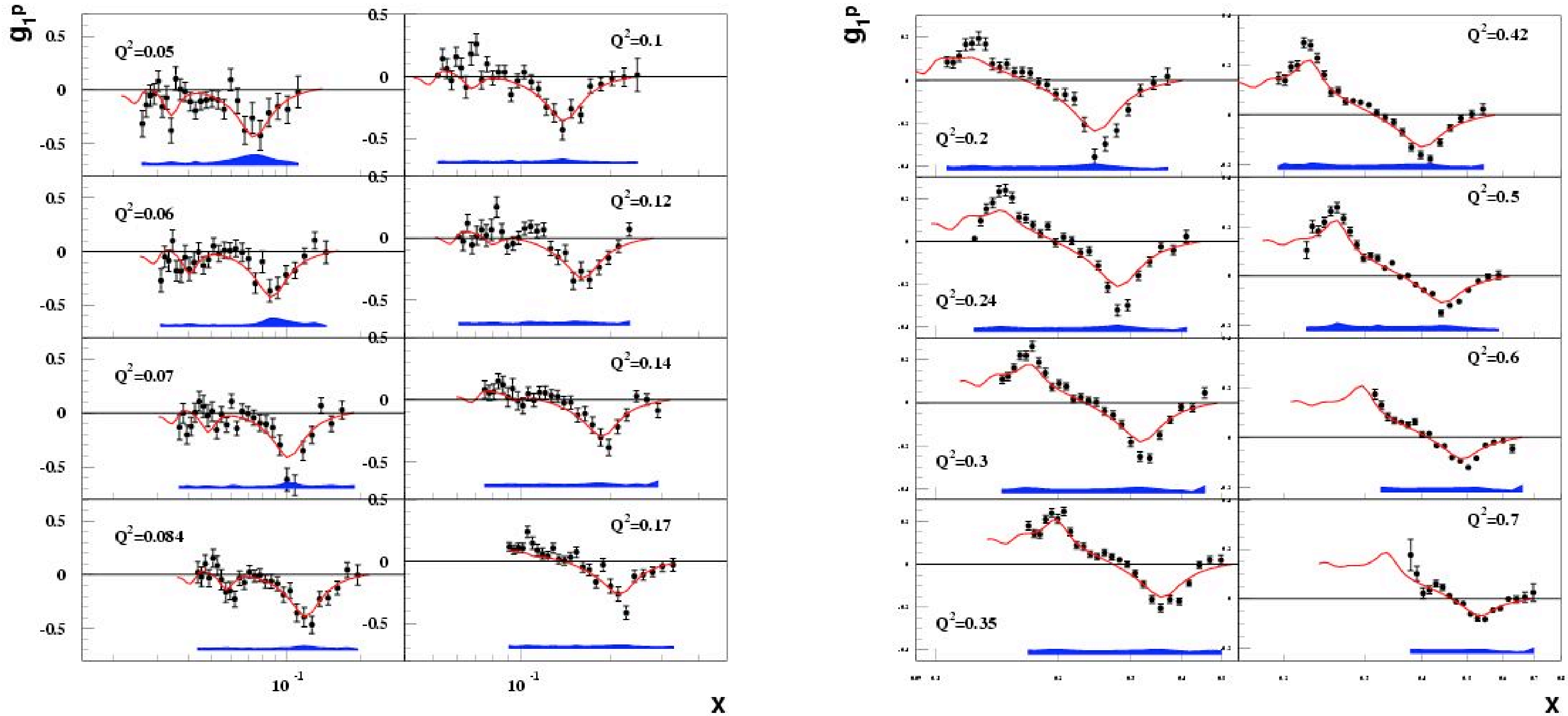
χPT: $(Q^2)^n$

Spin Structure with CLAS

- Long-standing program in Hall-B at JLab to measure longitudinal double spin asymmetries $A_{||}$ on $^{15}NH_3$ and $^{15}ND_3$
- EG1: $0.05<Q^2<3.5$ GeV^2
  - data (2001); anal (2008)
- EG4: $0.01<Q^2<1$ GeV^2
  - data (2006); anal (2009)
- EG12: $0.5<Q^2<7$ GeV^2
  - data (2012?); anal (2014)

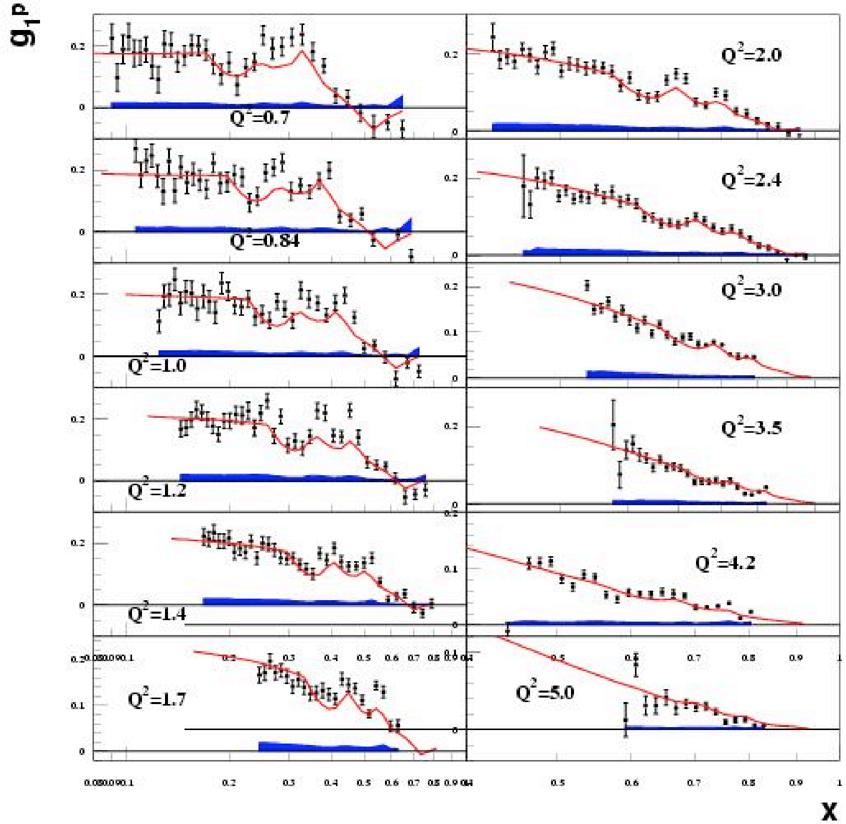


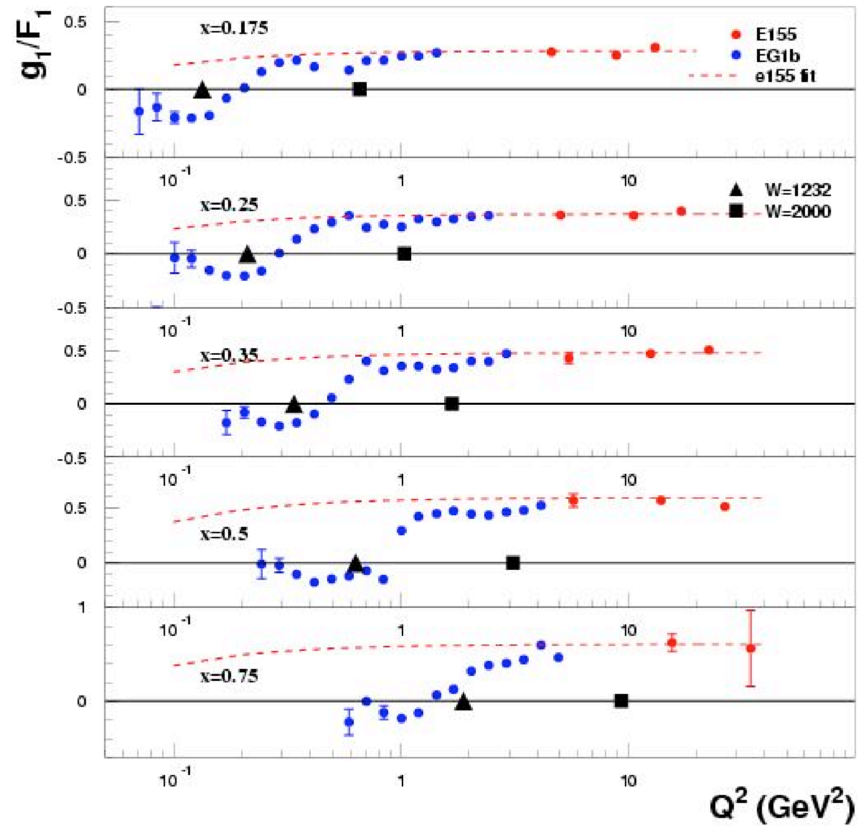
CLAS EG1 g_1^p (Q^2<0.7)



- At low Q^2 the Δ resonance drives g_1 negative
- Extensive x-range at fixed Q^2 allows integration over x
- Red curve is the EG1 model used for radiative corrections

CLAS EG1 g_1^p (Q^2>0.7) & g_1/F_1





- At higher Q^2, g_1 becomes positive everywhere
- g_1/F_1 falls far below the DIS extrapolation at low Q^2
- Red curve is the EG1 model (dashed: DIS extrapolation)

A_1 Data from EG1

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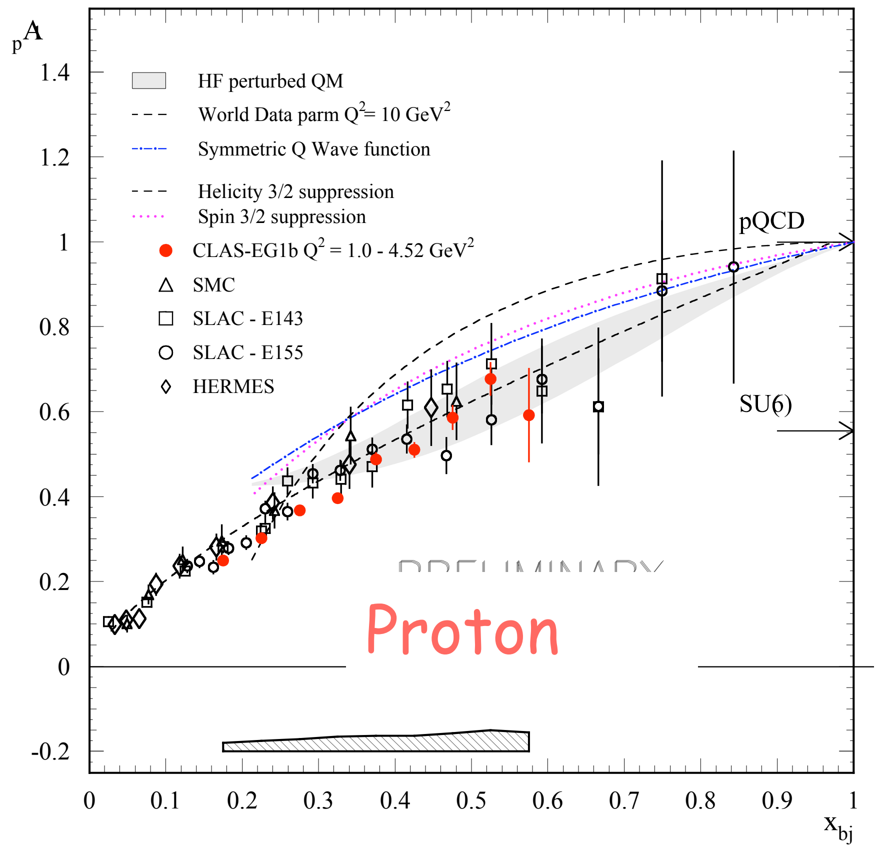
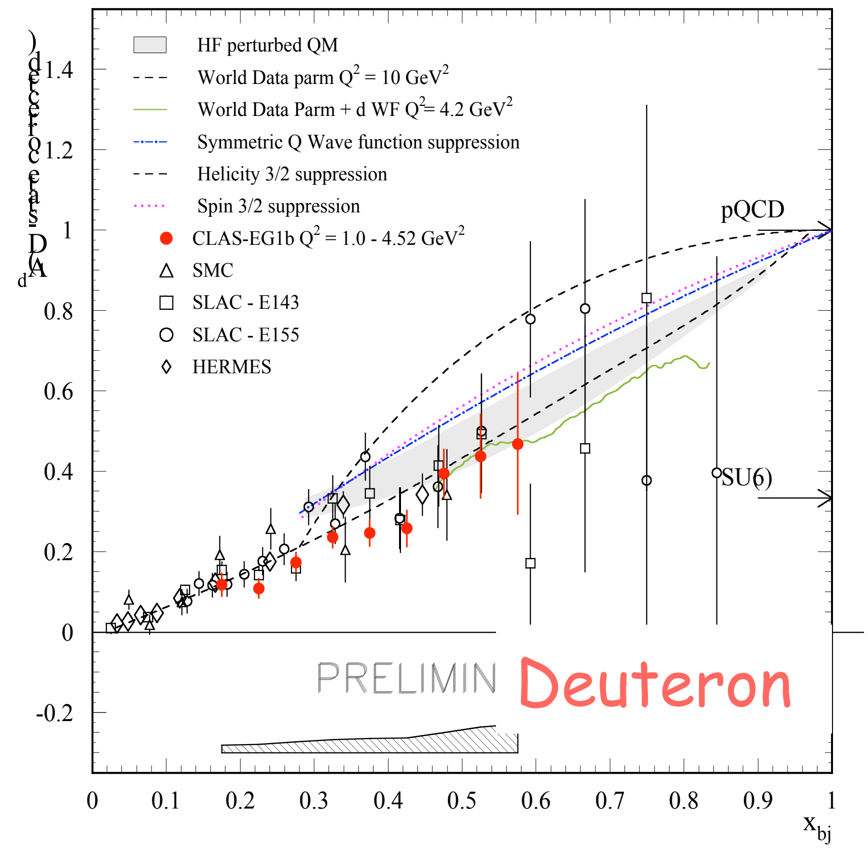
$\sim g_1/F_1$

Close and Melnitchouk, PRC **68**, 035210 (2003)

W > 2; Q^2 > 1

Isgur, PRD **59**, 034013 (2003)





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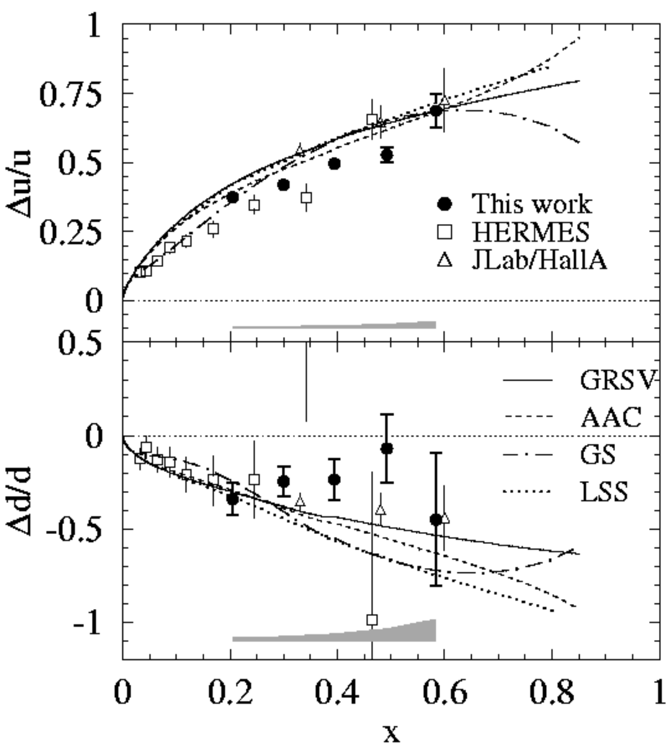
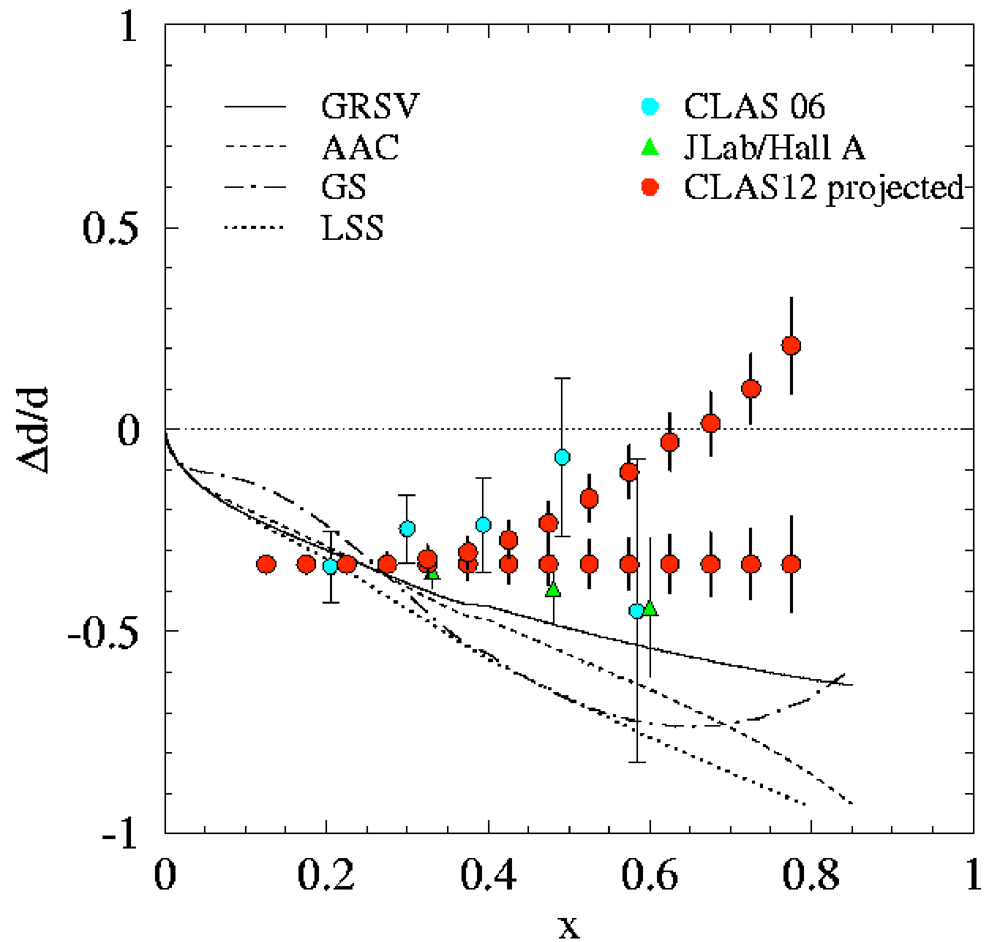
Quark polarization in the valence limit

$$A_1(x,Q^2) = \frac{\sum e_i^2 \Delta q_i(x,Q^2)}{\sum e_i^2 q_i(x,Q^2)}$$

CLAS EG1 Data



Simulated Data for EG12 Extracted from A_1^p, A_1^d and d/u



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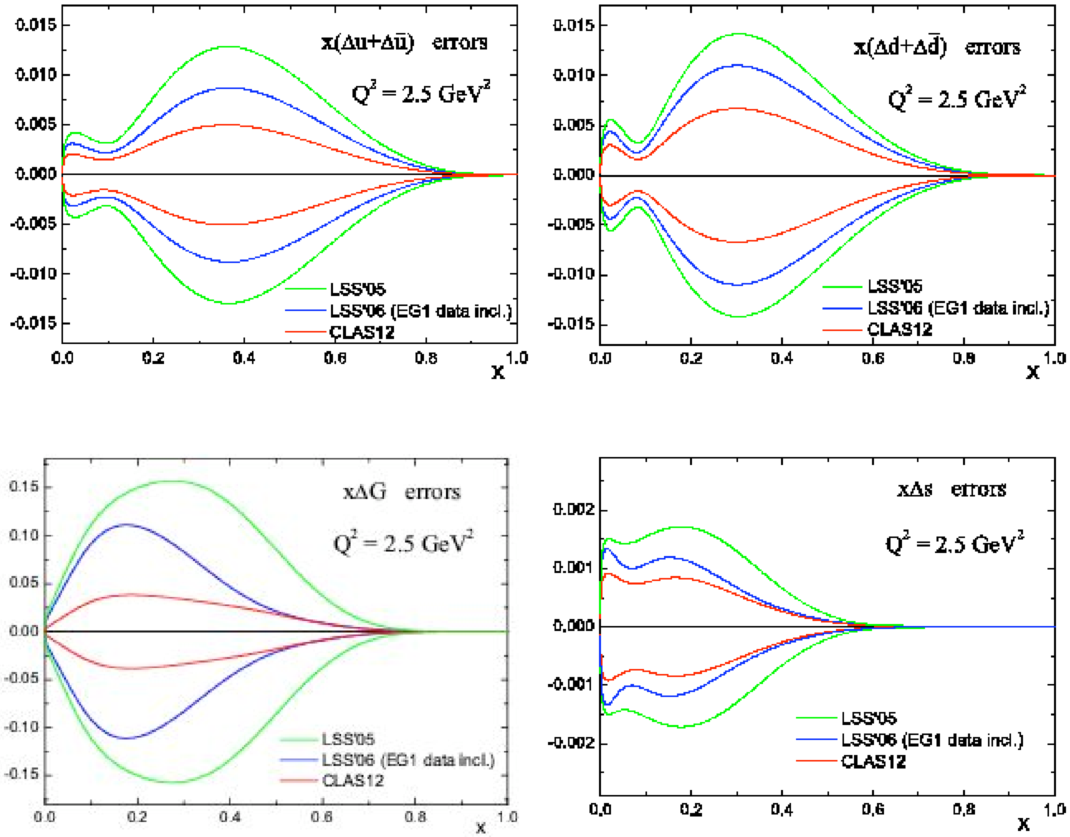
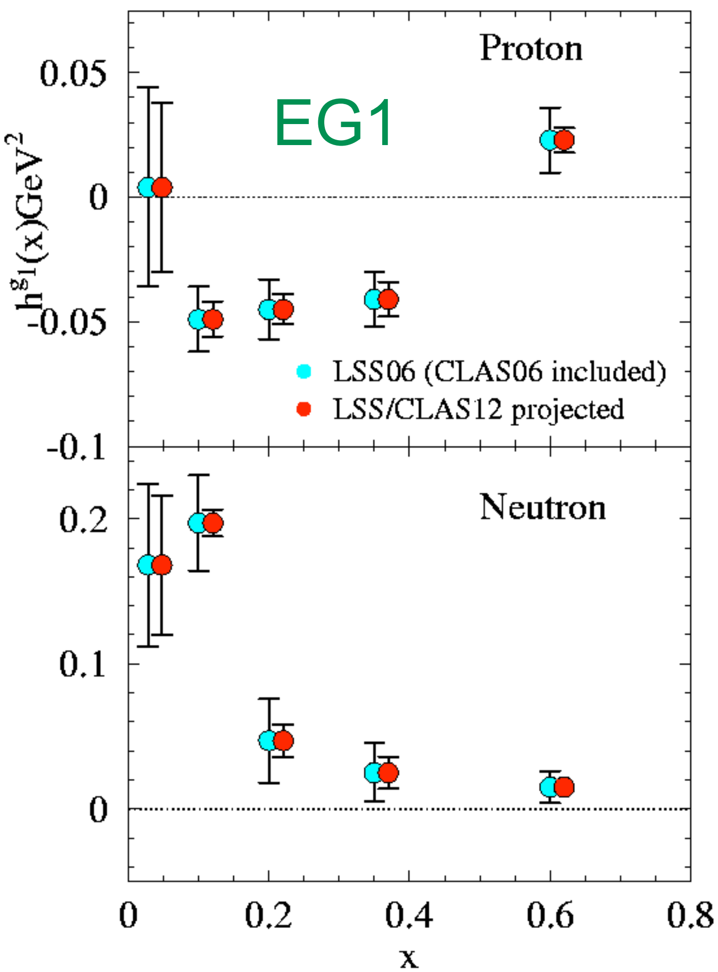
PDFs and CLAS



- Error envelopes for PDFs from LSS05 global analysis (green)
- CLAS EG1 data significantly improve errors on Δu, Δd, Δx and ΔG (blue)
- CLAS EG12 (12 GeV upgrade) will especially improve ΔG (red)

Higher Twist from g_1 in CLAS

$$\left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\exp} F_1(x,Q^2)_{\exp} = g_1(x,Q^2)_{\exp} = g_1(x,Q^2)_{LT} + h^{g_1}(x)/Q^2$$



$$1 < Q^2 < 5 \text{ GeV}^2, \quad 2 < W < 3.5 \text{ GeV}$$

$$\int_0^1 dx h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)$$

- F_1 from NMC fit to F_2 and 1998 SLAC fit to R
- g_1 (leading twist) from NLO fit at high Q^2
- h from fit to all data, especially CLAS in the pre-asymptotic region
- d_2: twist-3, f_2: twist-4

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Bjorken Sum & Higher Twist

$$\Gamma_1^{(n)} = \int_0^1 x^n g_1(x,Q^2)dx = \frac{a_n}{2}, \quad n=0,2,4,\ldots,$$

$$\Gamma_2^{(n)} = \int_0^1 x^n g_2(x,Q^2)dx = \frac{1}{2}\frac{n}{n+1}(d_n - a_n), \quad n=2,4,\ldots,$$

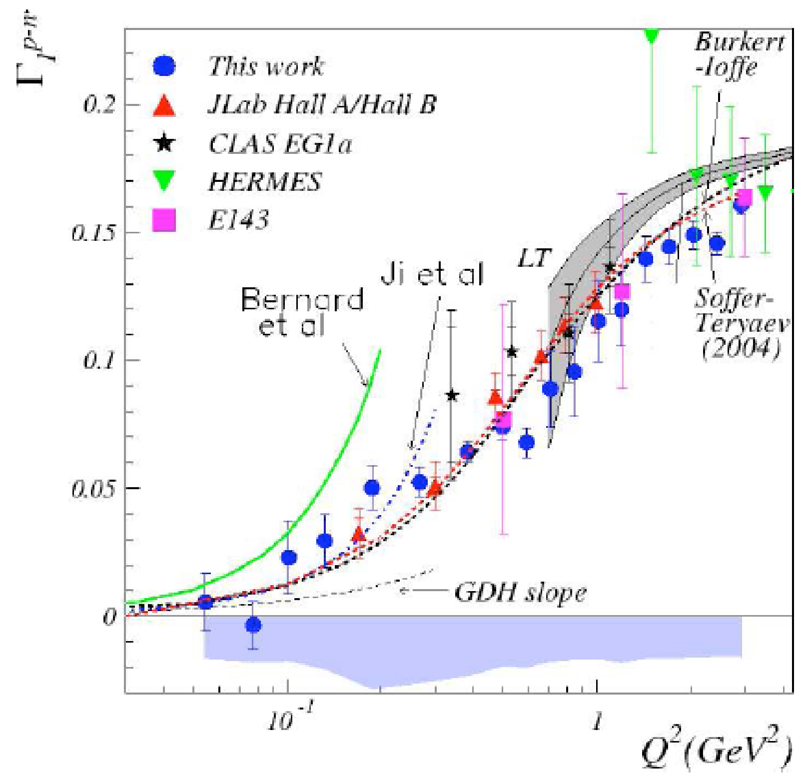
Bjorken Sum Rule:

CLAS: Deur

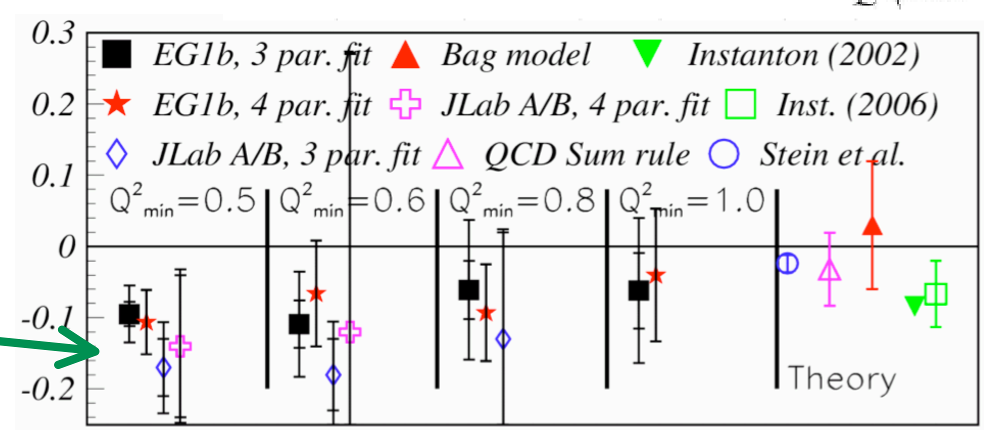
$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3\right] + \frac{\mu_4^{p-n}}{Q^2} + \ldots$$

$$\mu_4^{p-n} = \frac{M^2}{9}\left(a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n}\right)$$

$$d_2^{p-n} = \int_0^1 dx\, x^2 \left(2g_1^{p-n} + 3g_2^{p-n}\right)$$



Fit Γ_1^{p-n} to powers of $1/Q^2$ and extract f_2^{p-n}



Nachtmann Moments

$$M_1(Q^2) = \int_0^1 dx \, \frac{\xi^2}{x^2} \left\{ g_1(x, Q^2) \left(\frac{x}{\xi} - \frac{1}{9} \frac{M^2 x \xi}{Q^2} \right) - g_2(x, Q^2) \frac{4}{3} \frac{M^2 x^2}{Q^2} \right\},$$

$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

$$M_1(Q^2) = \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + \cdots$$

$$\mu_4(Q^2) = 4 f_2(Q^2)/9M^2$$

$$f_2 = 0.039 \pm 0.022(\text{stat}) \pm^{0.000}_{0.018}(\text{sys}) \pm 0.030(\text{low } x) \ \pm^{0.007}_{0.011}(\alpha_s),$$

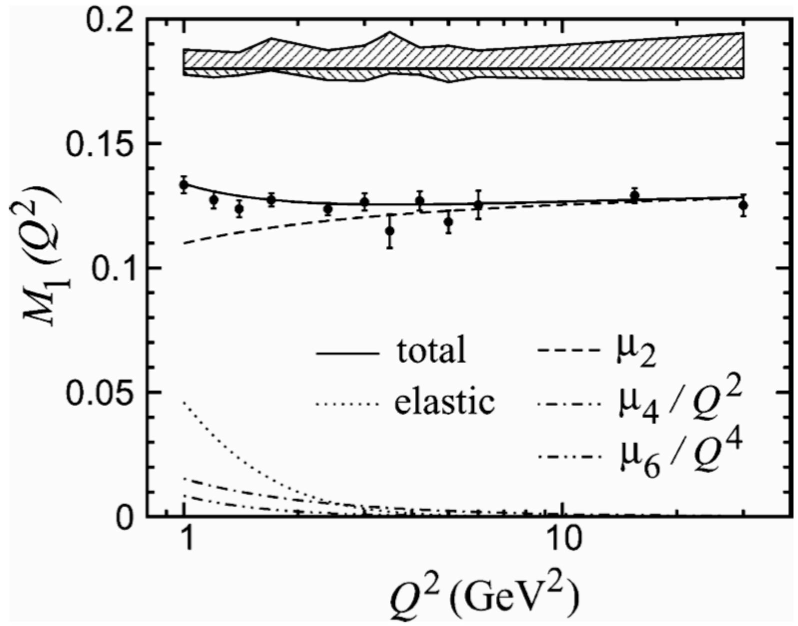
$$d_2(Q^2) = \int_0^1 dx \, x^2 [2 g_1(x, Q^2) + 3 g_2(x, Q^2)]$$

$$\chi_E = \frac{2}{3}(2d_2 + f_2)$$

$$\chi_B = \frac{1}{3}(4d_2 - f_2)$$

CLAS, Osipenko

PLB609(05)259



$$\chi_E = 0.026 \pm 0.015(\text{stat}) \pm^{0.021}_{0.024}(\text{sys}),$$

$$\chi_B = -0.013 \mp 0.007(\text{stat}) \mp^{0.010}_{0.012}(\text{sys})$$

Color Polarizability

- Osipenko, CLAS, proton, PLB609(05)249
  - f_2 = 0.039(39) χ_E = 0.026(27) χ_B = -0.013(13)
- E94-010, Hall A, neutron
  - f_2 = 0.034(43) χ_E = 0.033(29) χ_B = -0.001(16)
- Deur, CLAS, Bjorken (p-n)
  - f_2 = -0.101(74) χ_E = -0.077(50) χ_B = 0.024(28)
- More accurate determinations are needed.

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RSS g_2^p

Wandzura-Wilczek

$$g_2^{WW} = -g_1 + \int_x^1 \frac{g_1}{y} dy$$

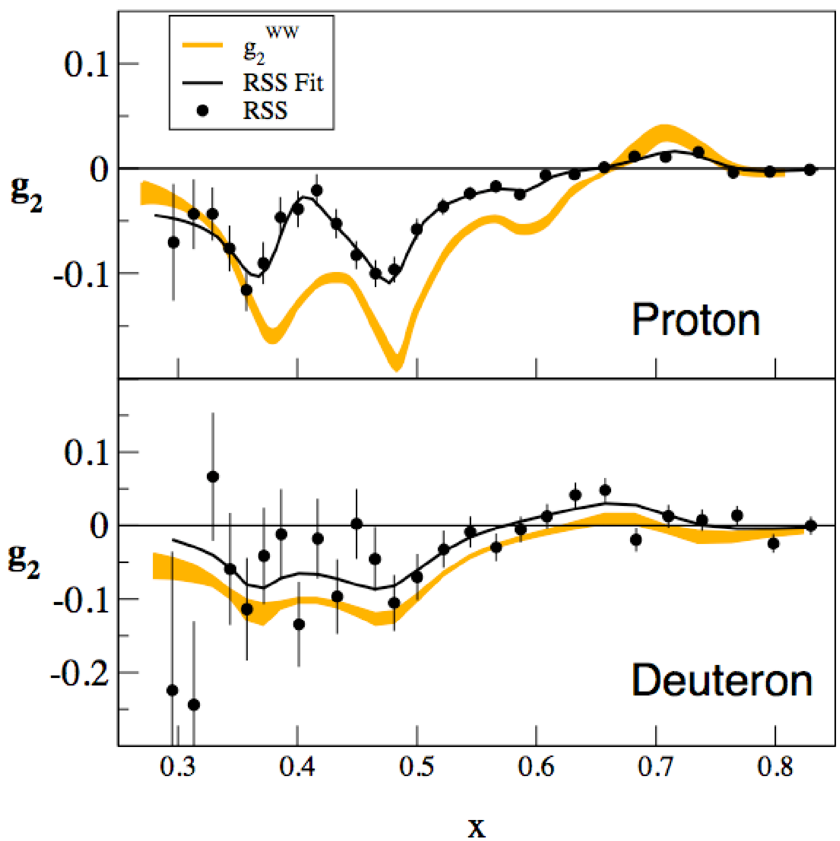
Q²=1.28 GeV²

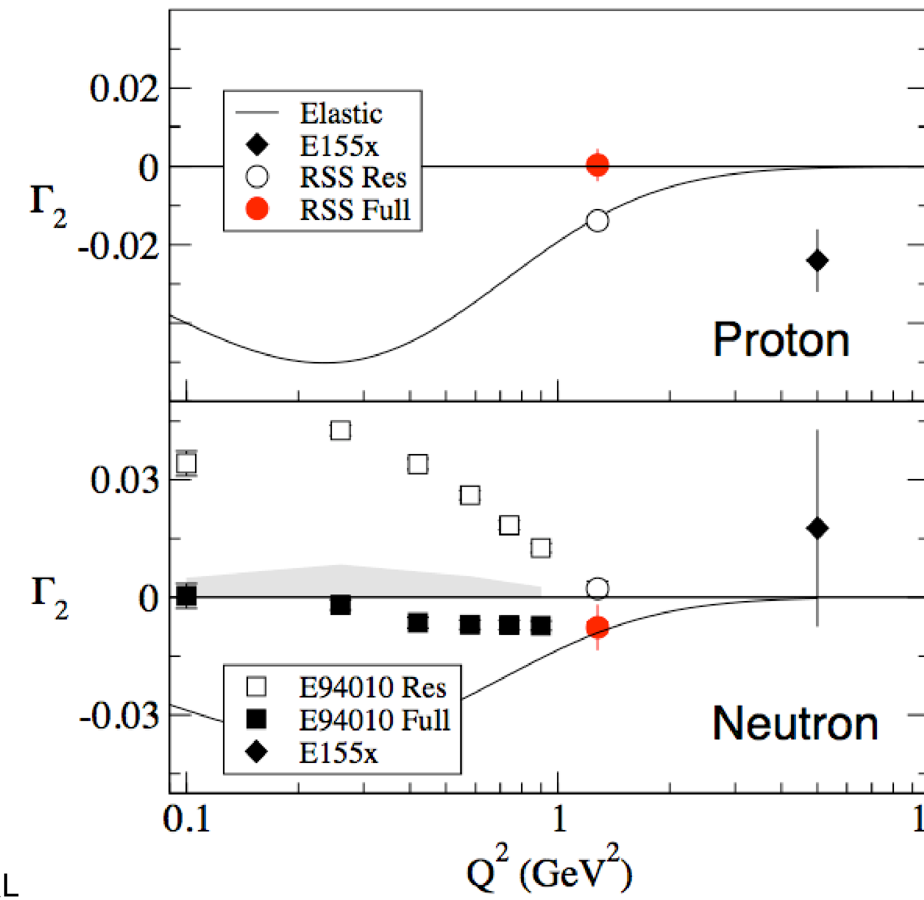
$$g_2 = g_2^{WW} + \overline{g}_2$$

Burkhardt-Cottingham Sum Rule

$$\Gamma_2 = \int_0^1 g_2(x,Q^2)dx = 0$$

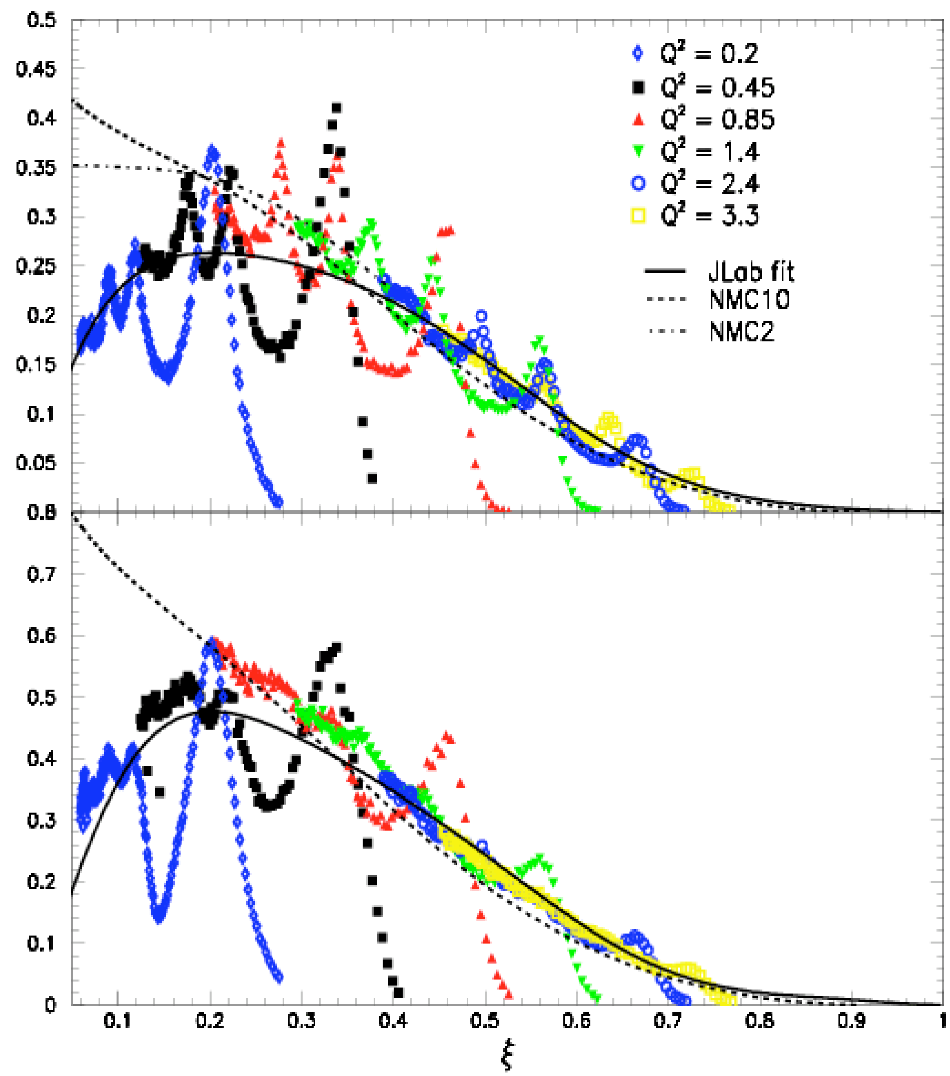
Hall C: Slifer et al. arXiv:0812.0031





Duality

Hall C
PRL85(00)1182
Global duality to 10%
Local duality to 10%
W=1.232, 1.535, 1.680 GeV



Duality - structure functions averaged over resonances behave according to DIS systematics
Global - all resonances
Local - one resonance

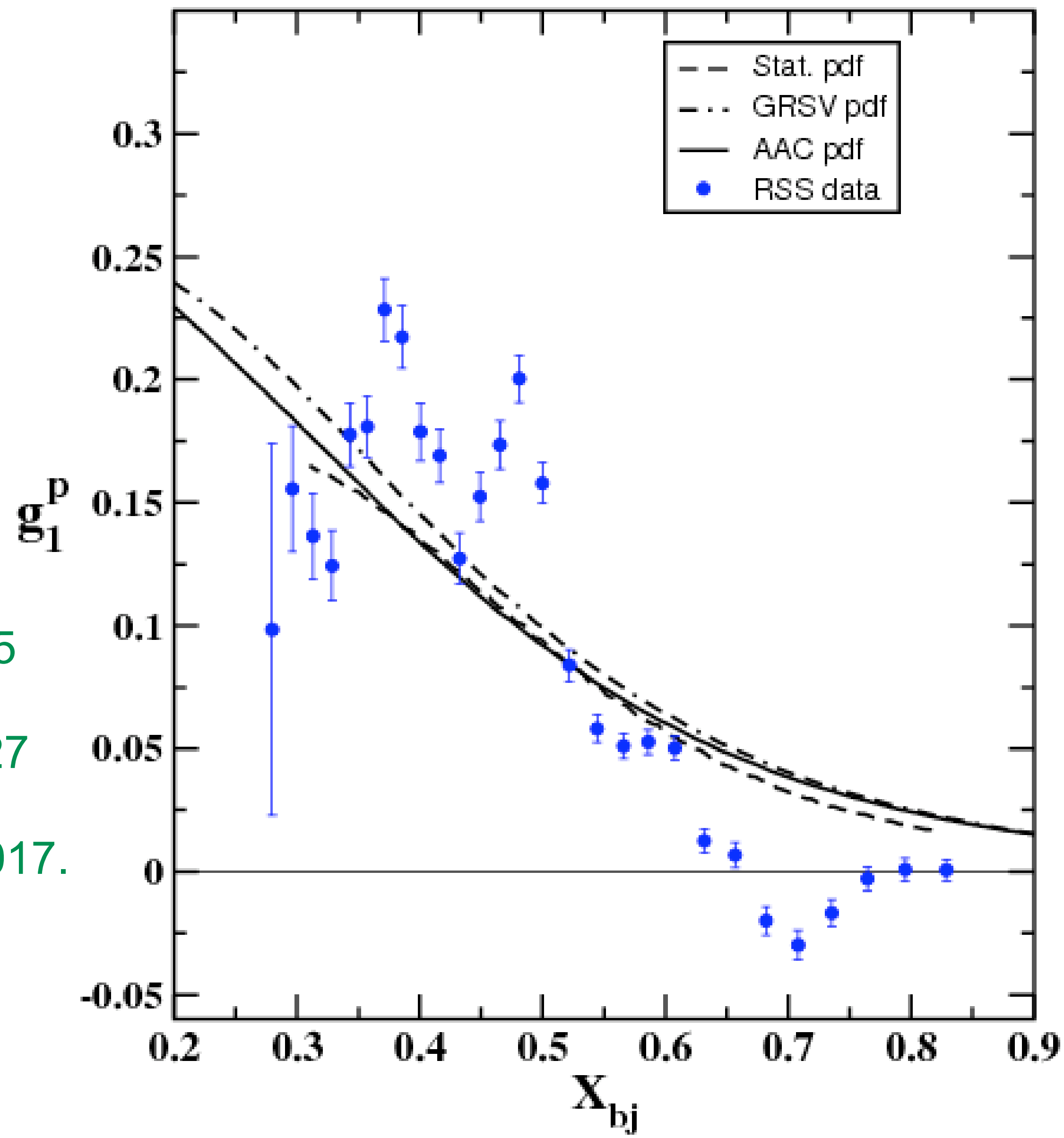
Polarized Duality

Hall C
RSS, Wesselmann, Slifer
Q^2=1.379 GeV^2
Target Mass Corrections
  applied to PDFs
No duality for Δ
PRL98(07)132003

GRSV: Phys. Rev. D 53, (1996) 4775

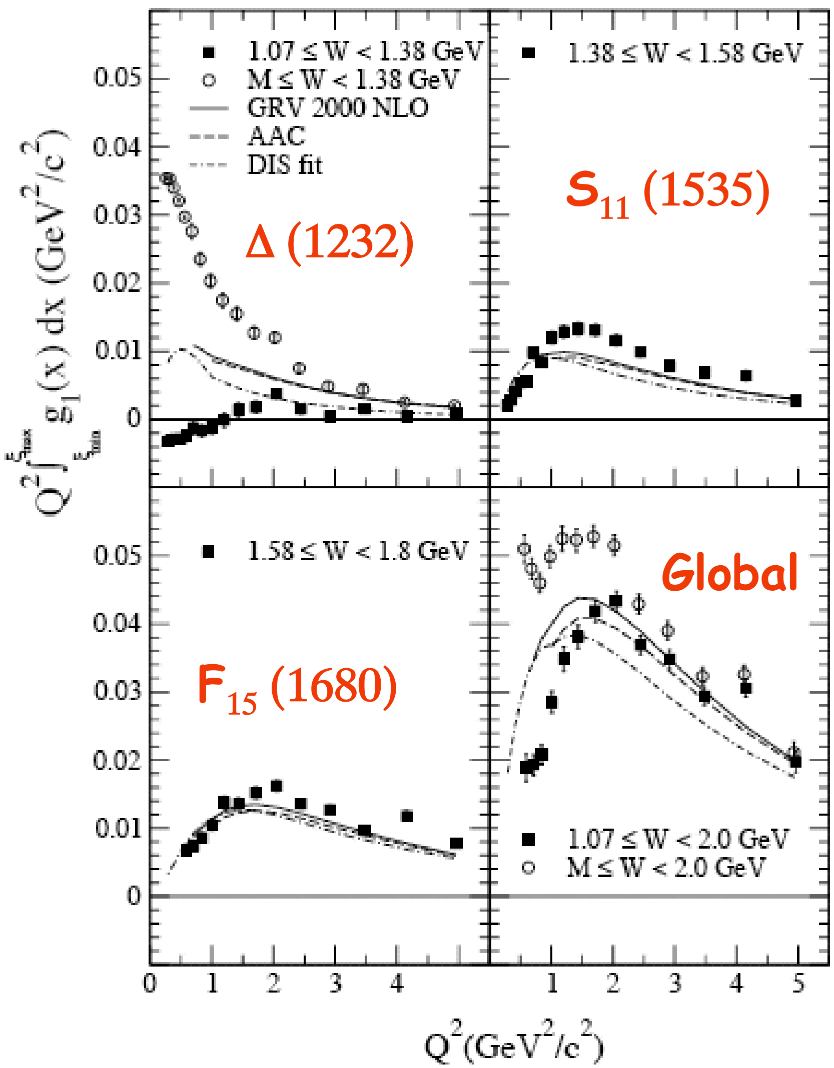
BSB : Eur. Phys. J. C 41, (2005) 327

AAC : Phys. Rev. D 62, (2000) 034017.

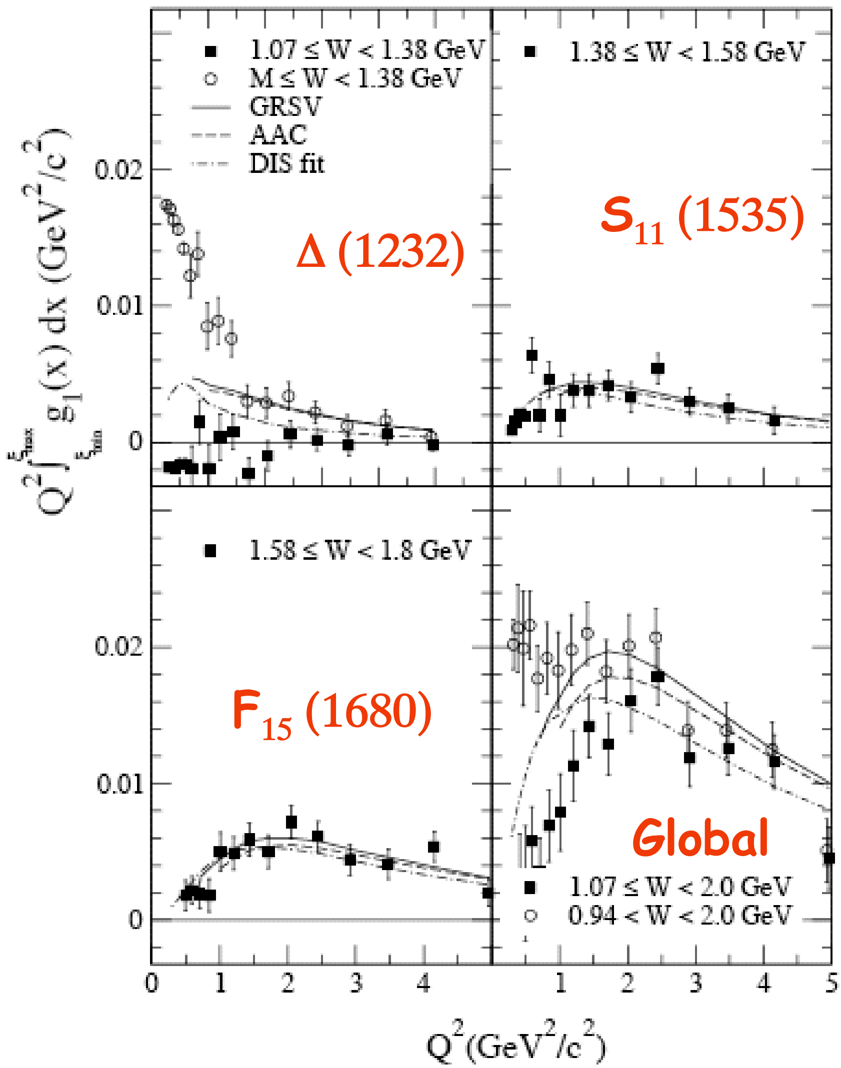


Duality at CLAS (EG1)

Proton



Deuteron



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CLAS Moments $\Gamma_1^{p,d}$

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x,Q^2)dx \qquad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

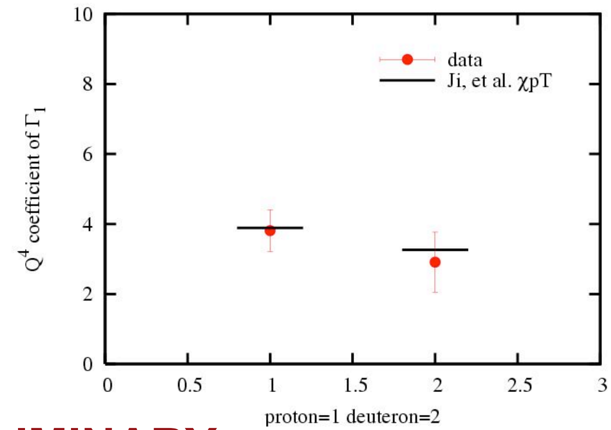
low Q² fit

$$\Gamma_1^d(Q^2) = (1 - 1.5\omega_D)\left\{\Gamma_1^p(Q^2) + \Gamma_1^n(Q^2)\right\}$$

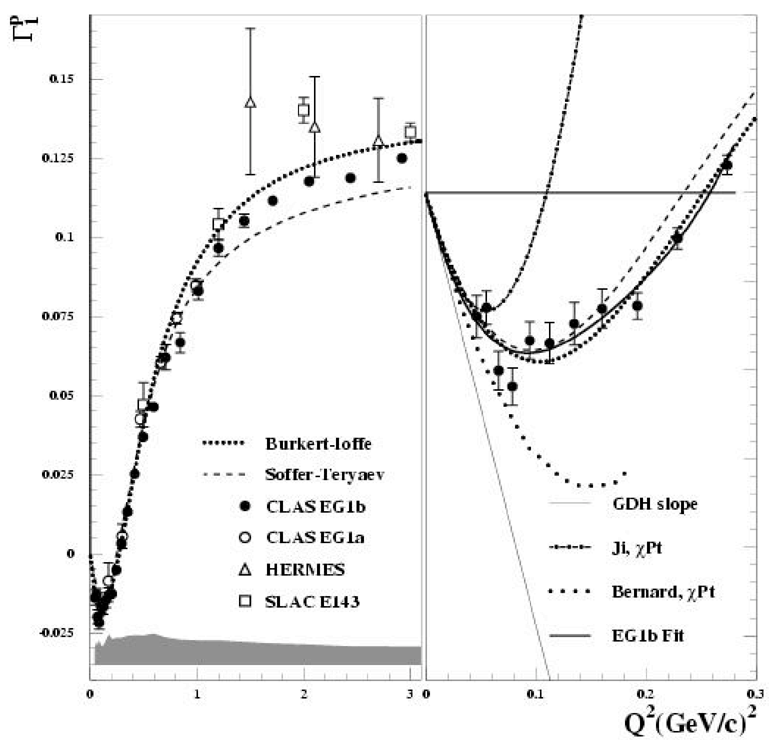
$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^2}{8M^2}Q^2 + 3.89Q^4 + \ldots$$

$$\Gamma_1^n(Q^2) = -\frac{\kappa_n^2}{8M^2}Q^2 + 3.15Q^4 + \ldots.$$

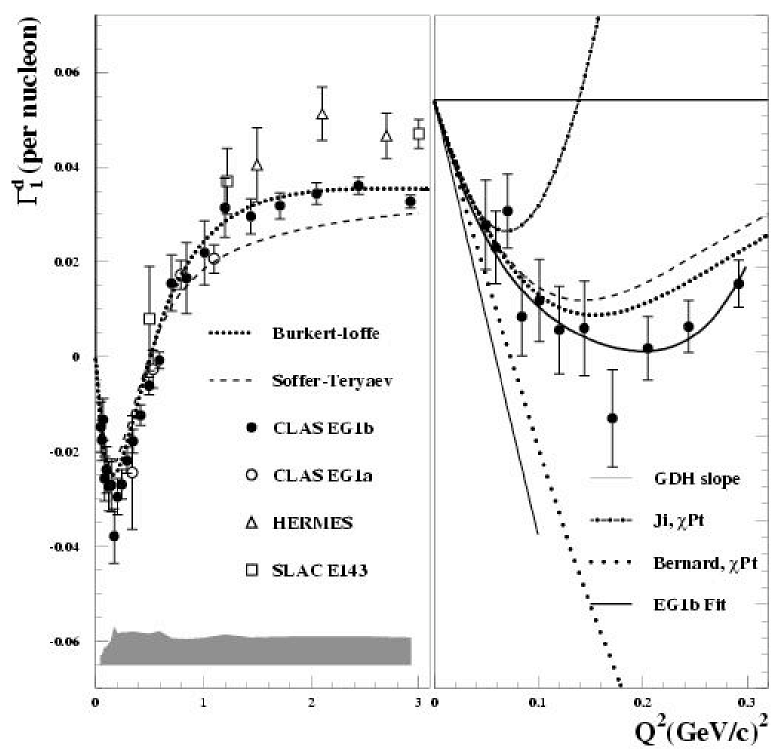
GDH + χpT



PRELIMINARY



PRELIMINARY



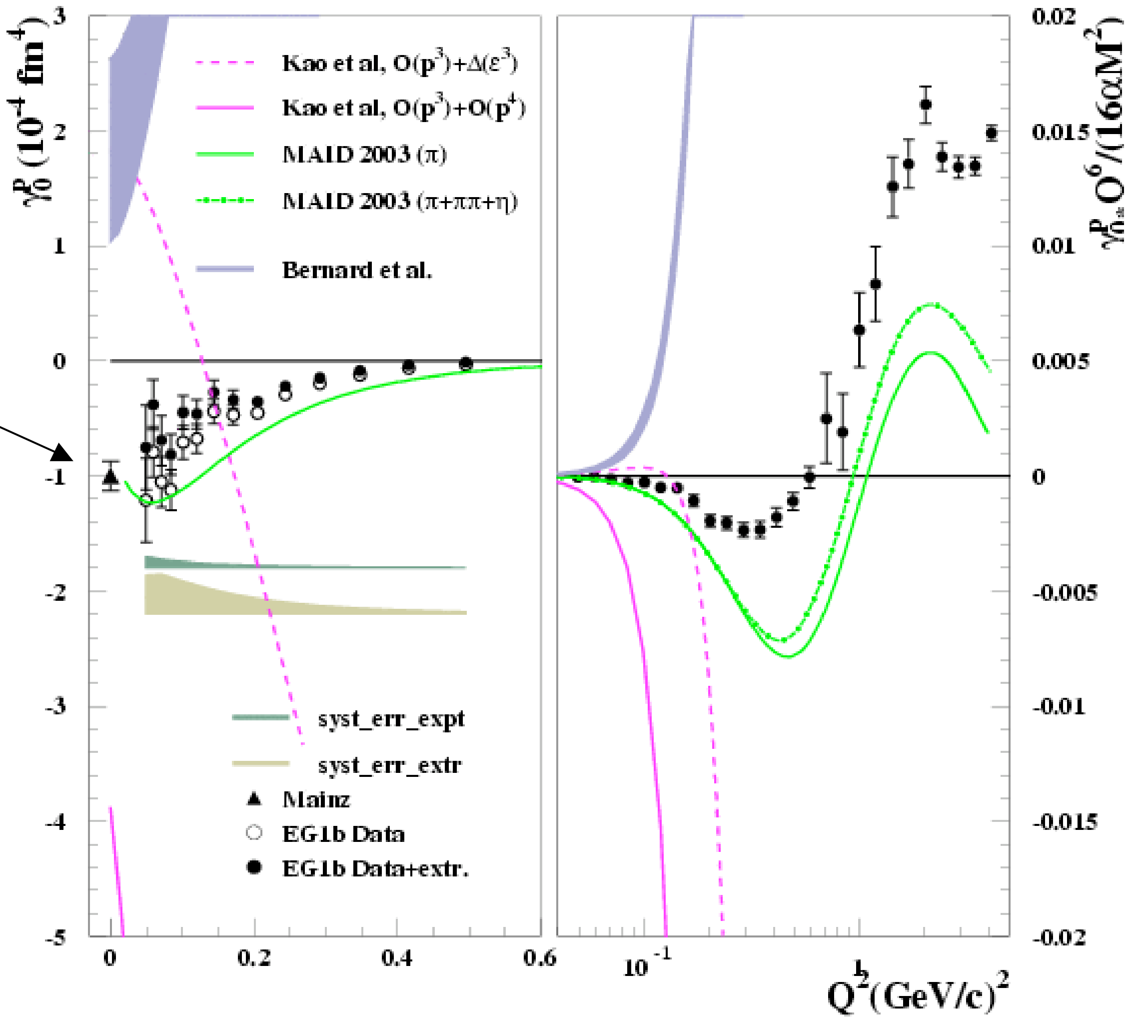
Forward Spin Polarizability

$$\gamma_0(Q^2) = \frac{4e^2M^2}{\pi Q^6}\int_0^{x_0} dx\, x^2 \left\{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)\right\}$$

PRELIMINARY

real photon point

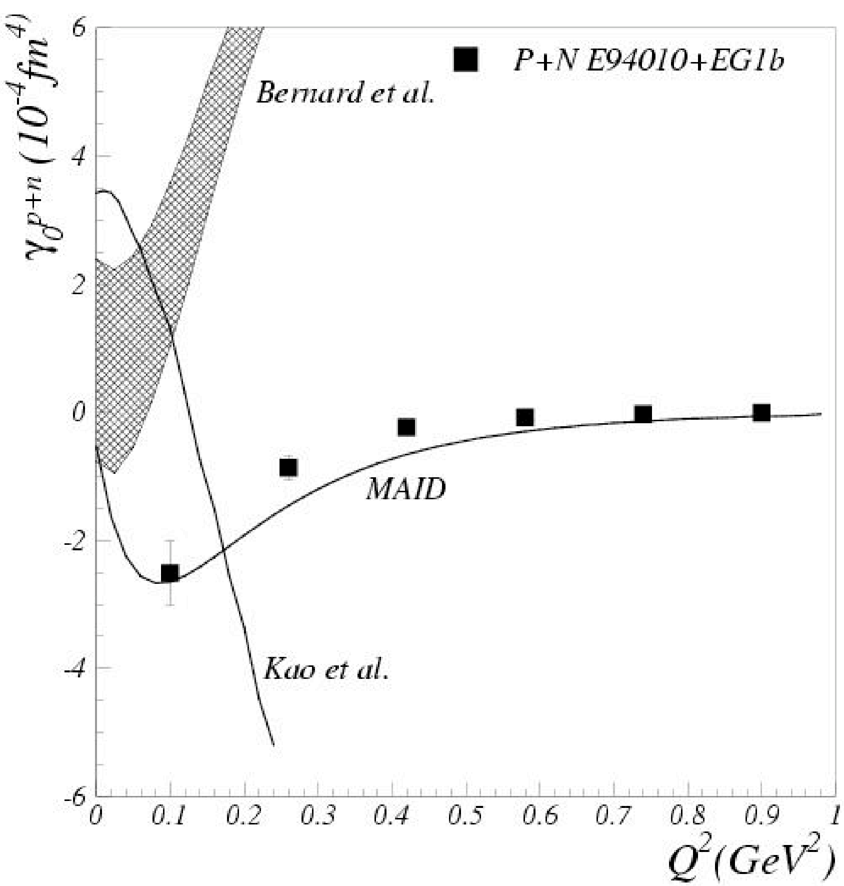
CLAS EG1 Data

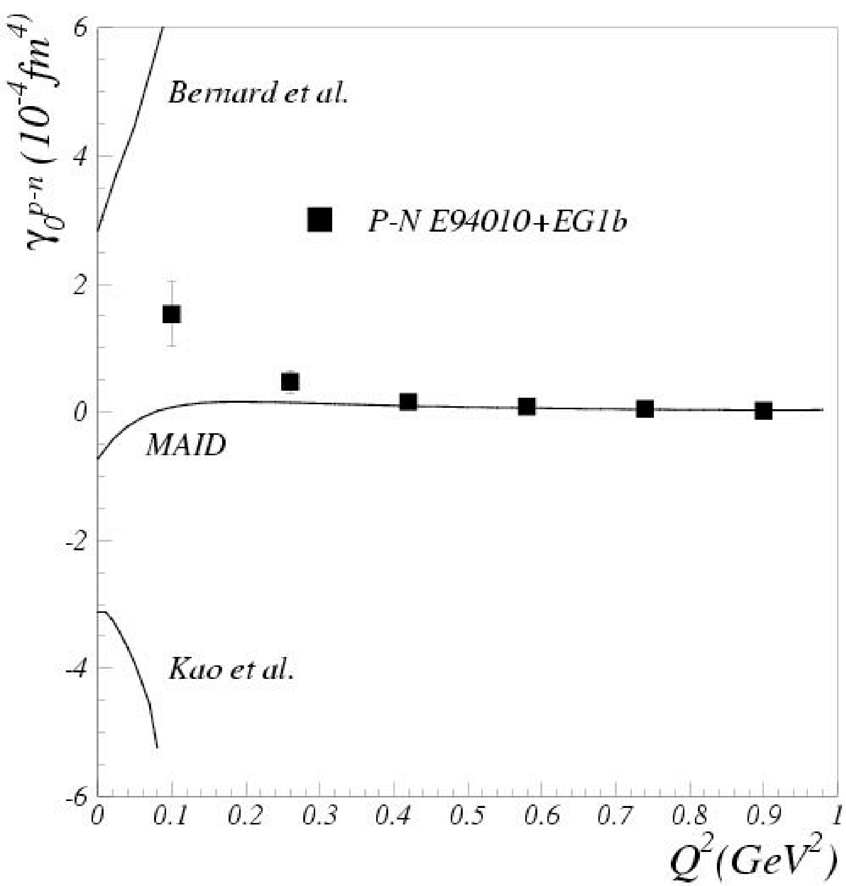


γ_0^{p-n} and γ_0^{p+n}

A Deur
CLAS + Hall A

For isovector (p-n) case
Δ contribution cancels

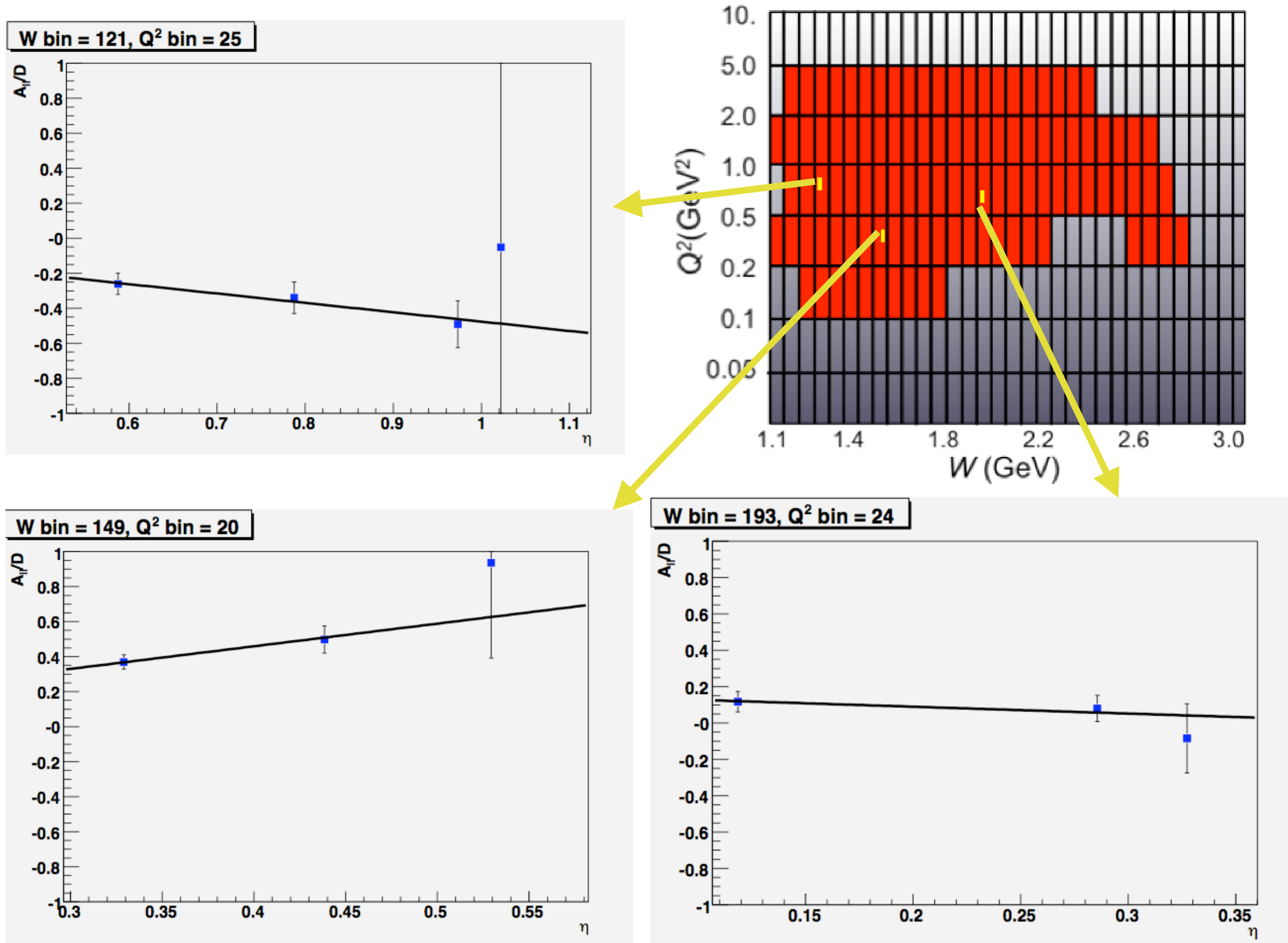




Latest CLAS Data

- Recent results from CLAS EG1b
- Uses data from all 4 beam energies

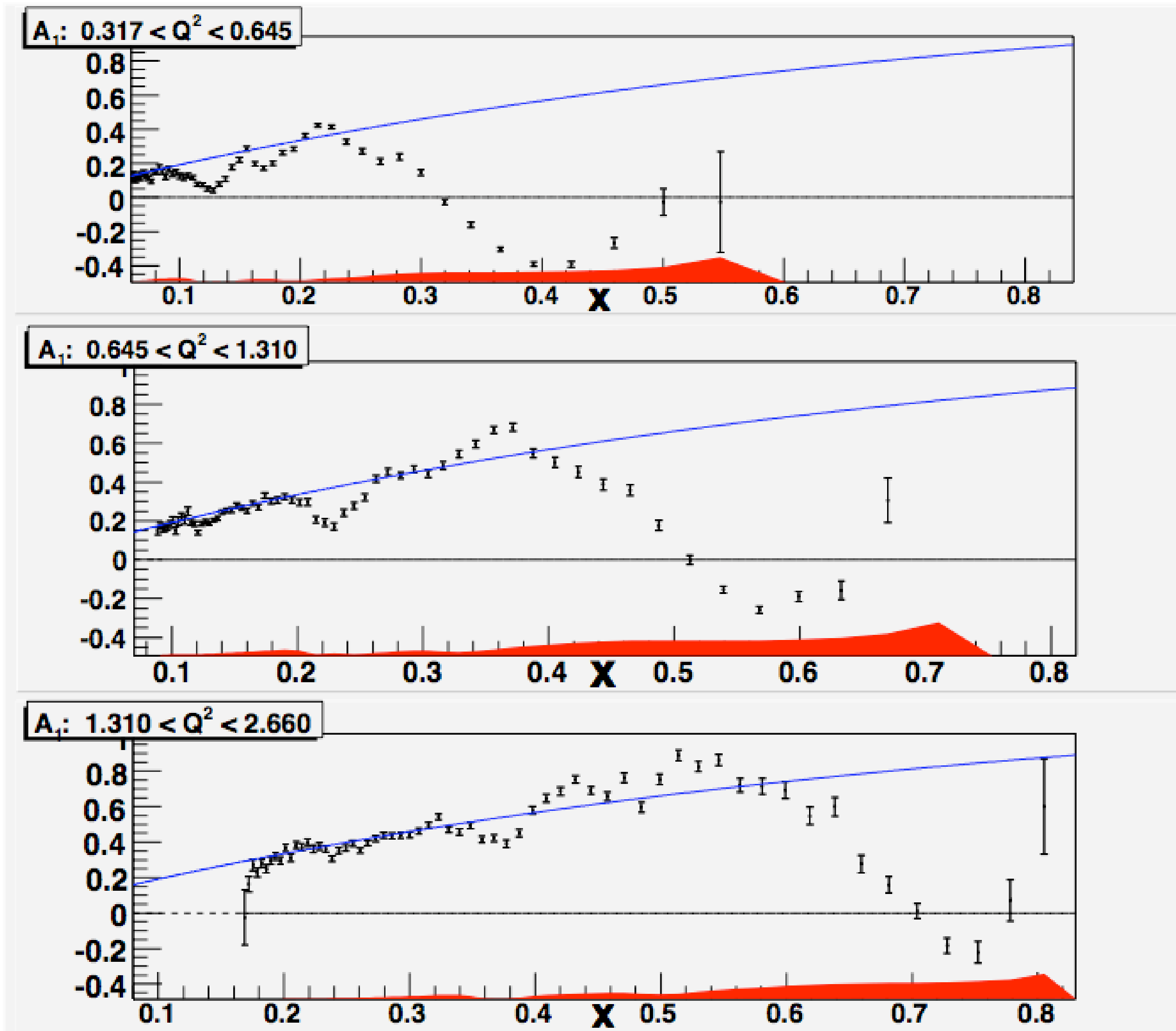
EG1b Fitting $A_1 + \eta A_2$



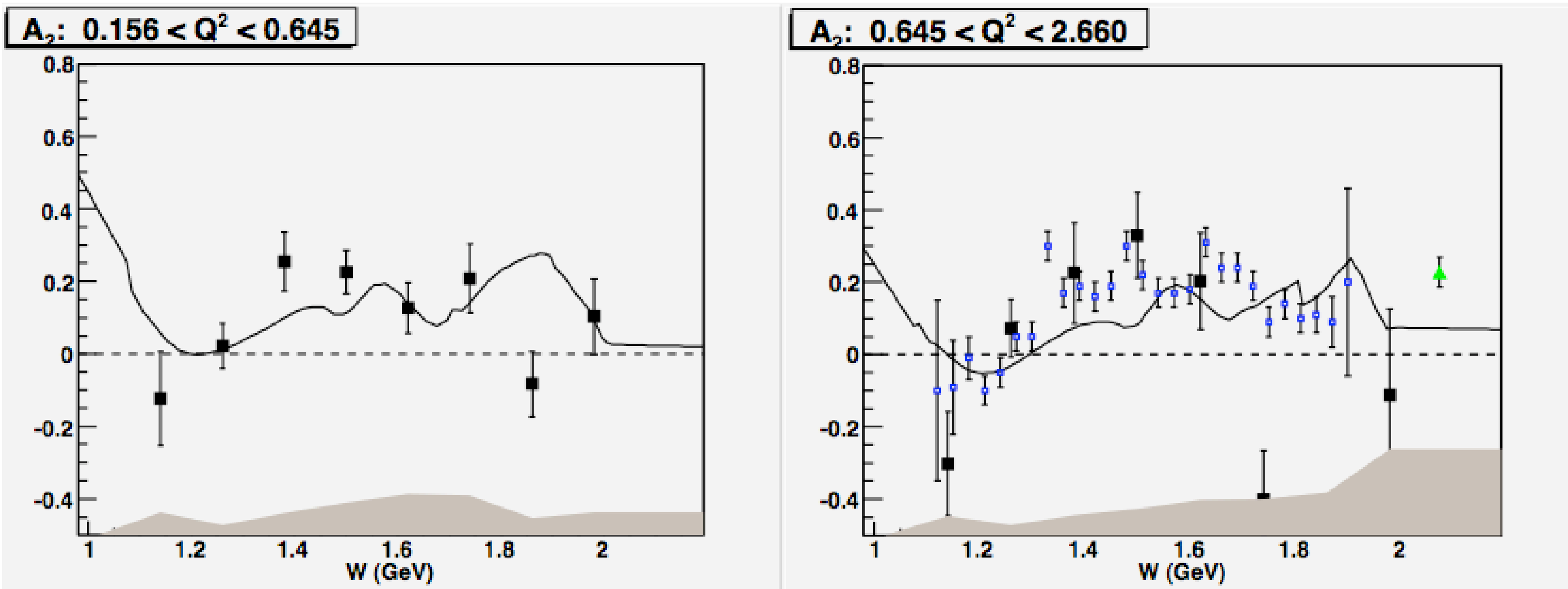
EG1b A_1 vs. x

Blue line:
DIS model of A_1 at Q^2=10 GeV^2

Δ dip is still significant at Q^2 ~ 2 GeV^2



EG1b A_2



Black points: extraction from EG1b
Black line: eg1b model for A_2
Blue points: RSS data

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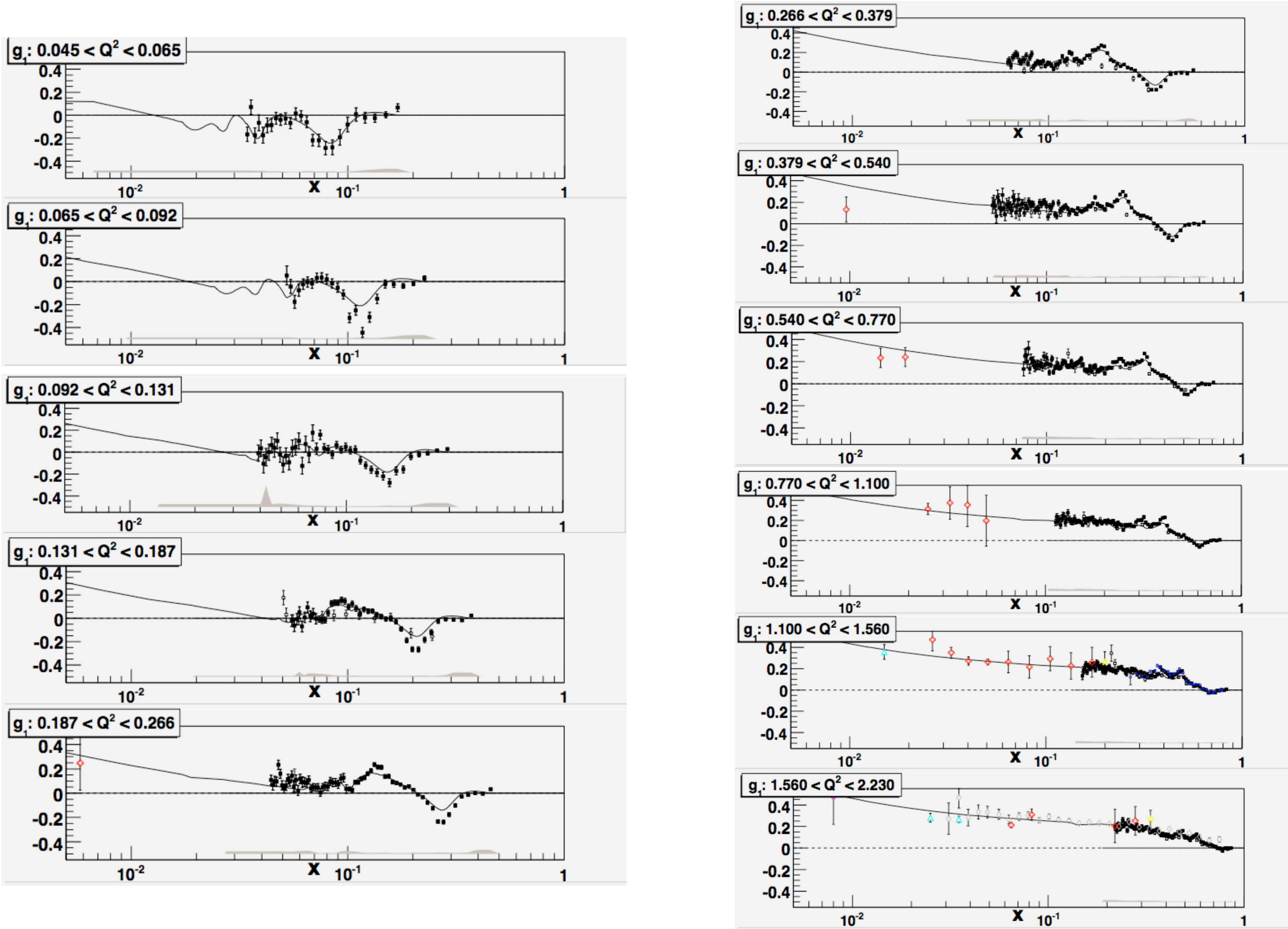
EG1b g_1^p for Various Q^2s



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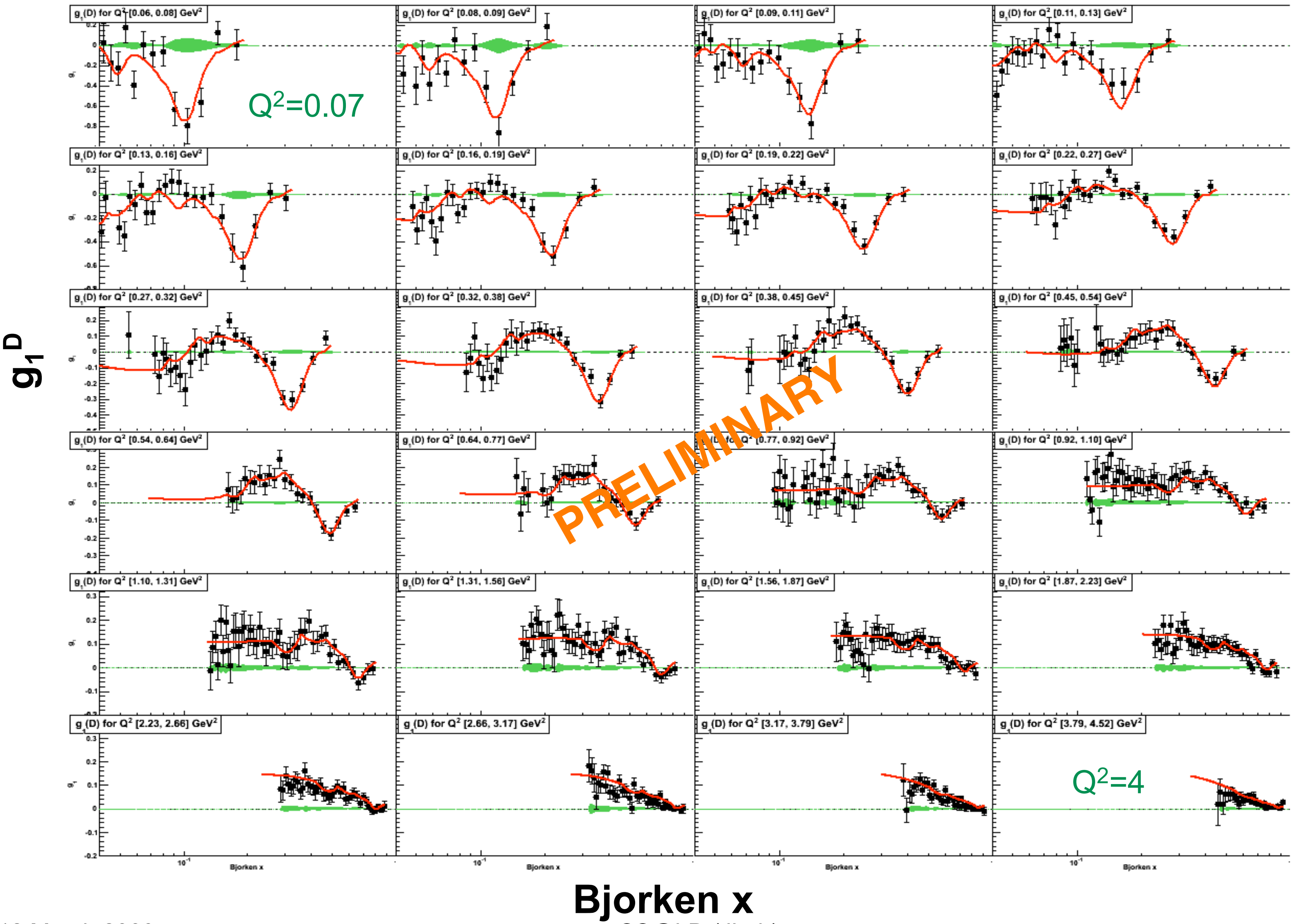
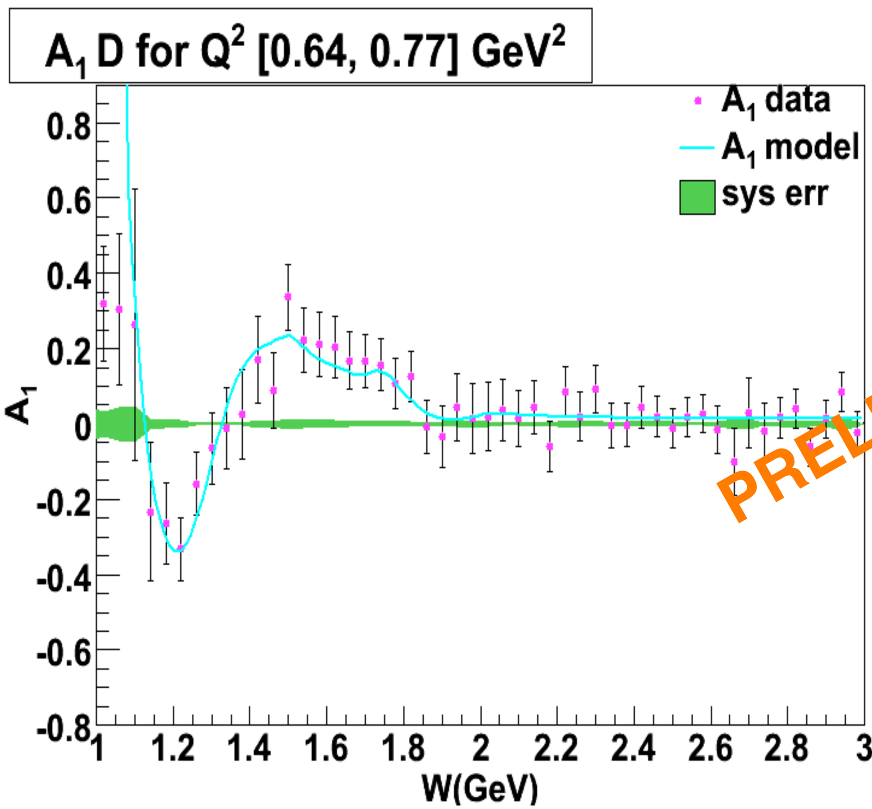
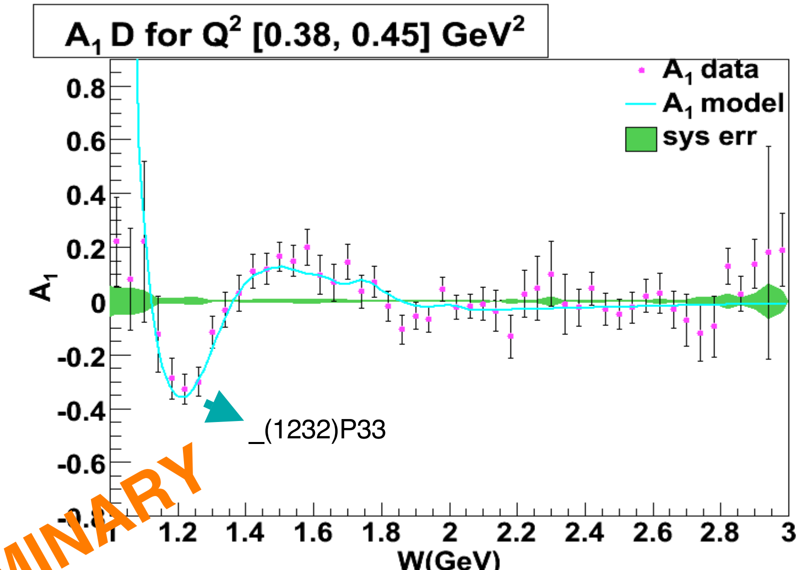
EG1b g_1^d

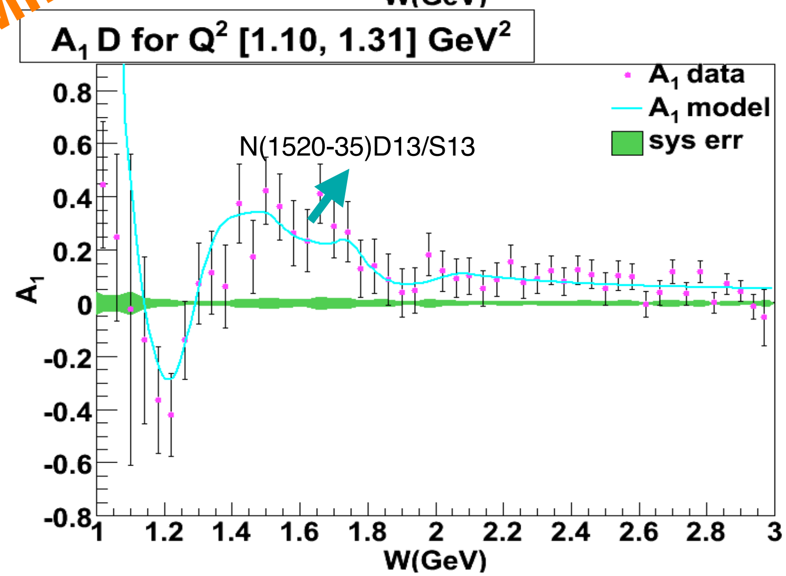


EG1b A_1^d



A3/2 transition is dominant





A1/2 transition is dominant

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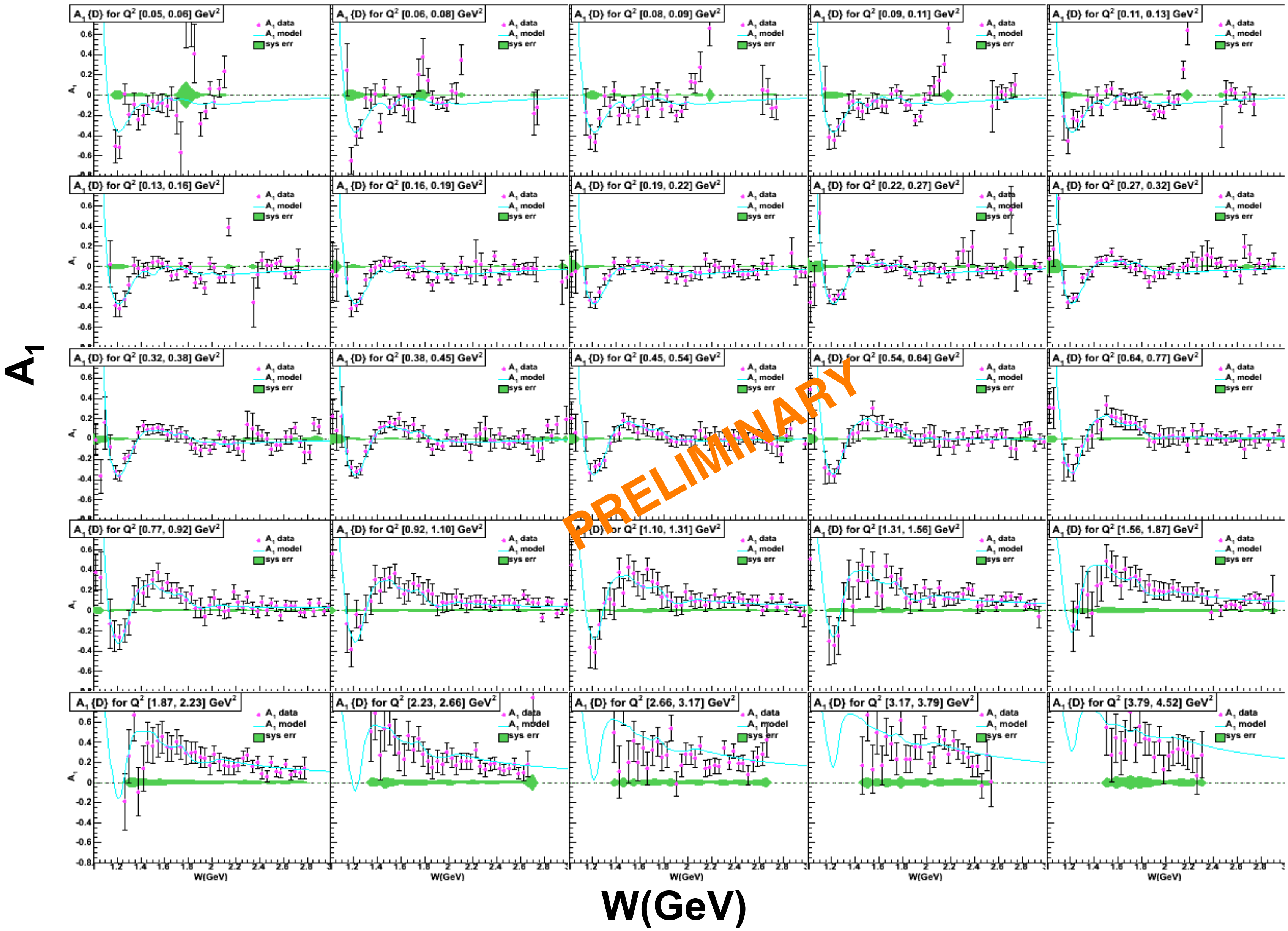
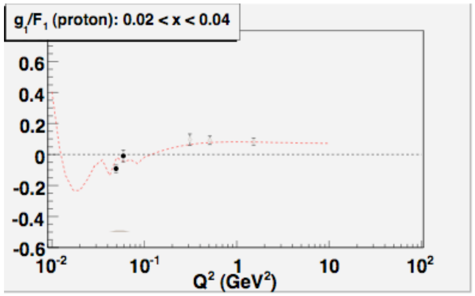
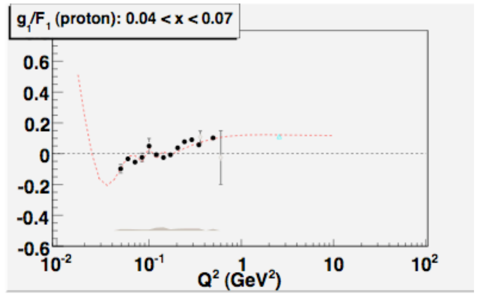
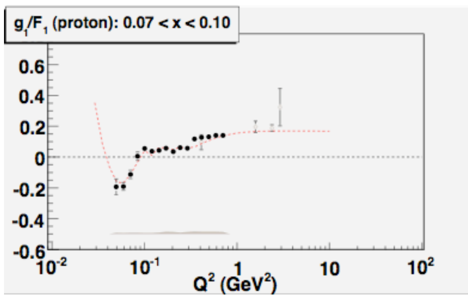
EG1b A_1^d

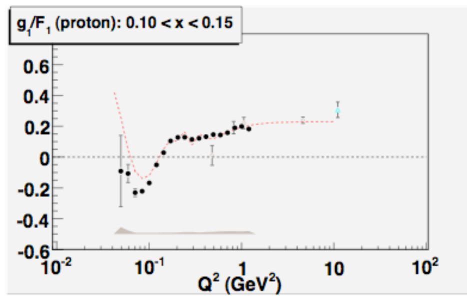


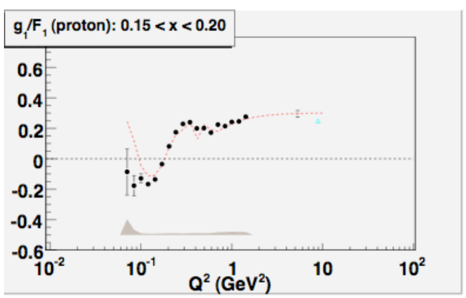
EG1b g_1^p/F_1^p vs. Q^2

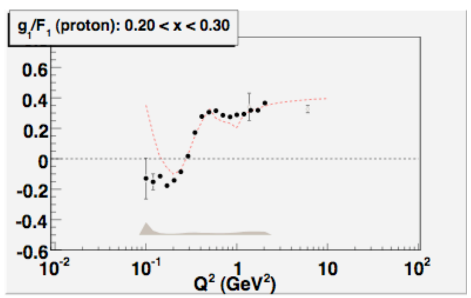


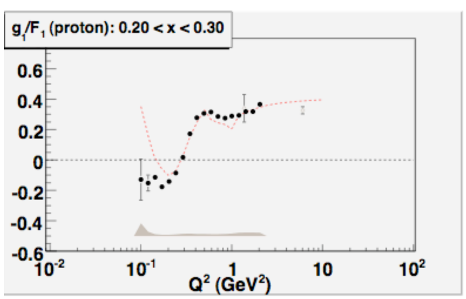


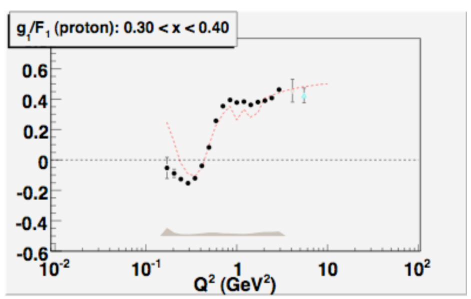


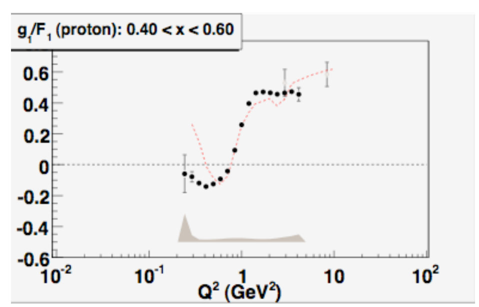








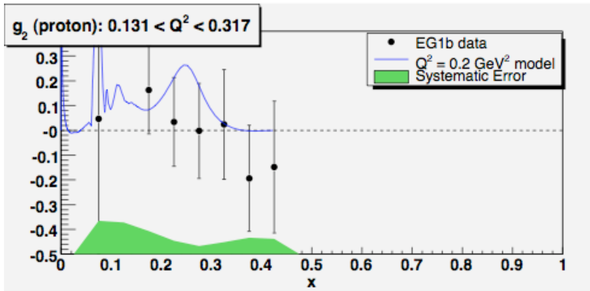


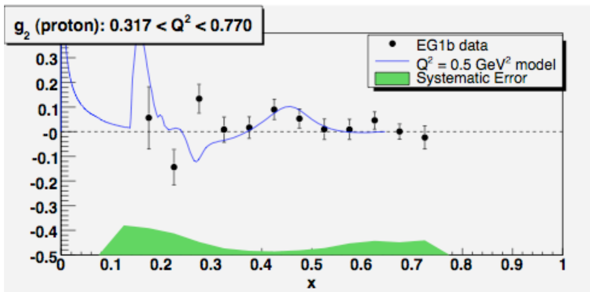


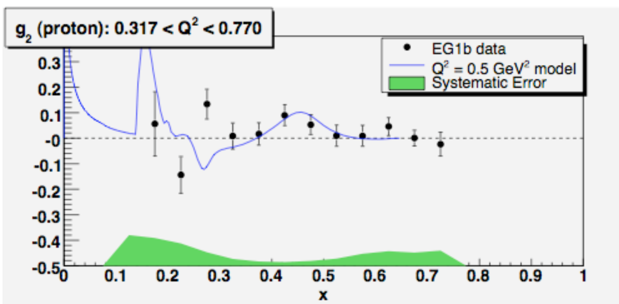
Scaling starts at:

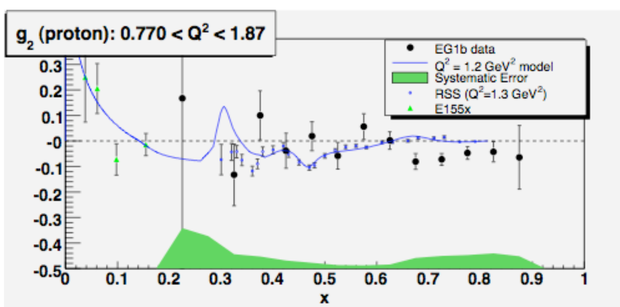
x	Q^2
0.085	0.1
0.125	0.2
0.175	0.3
0.250	0.6
0.350	0.8
0.500	1.0

EG1b g_2



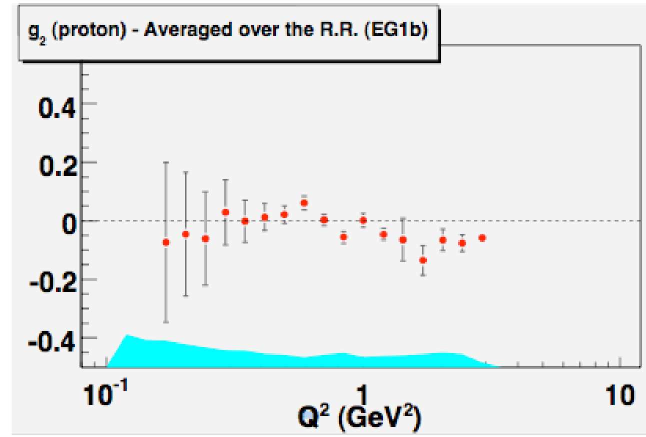


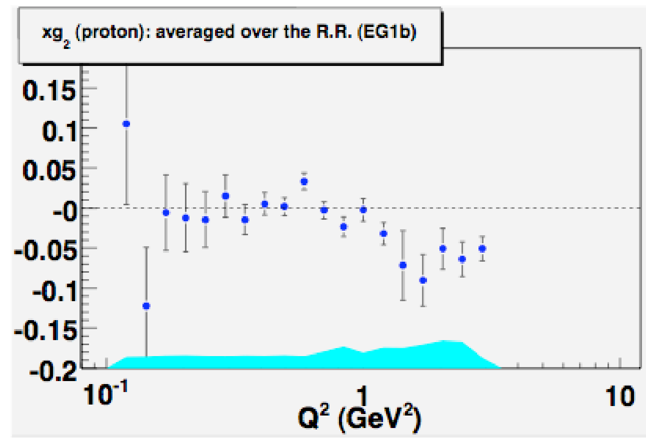


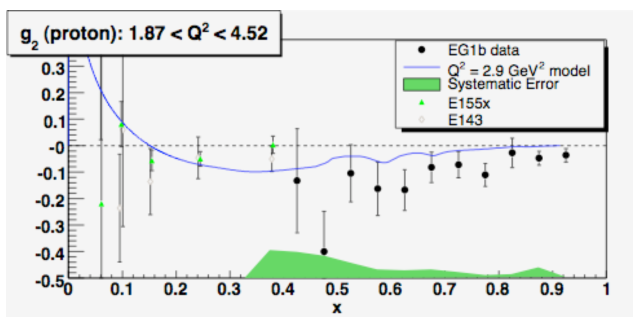


g_2 extracted from A_1 & A_2

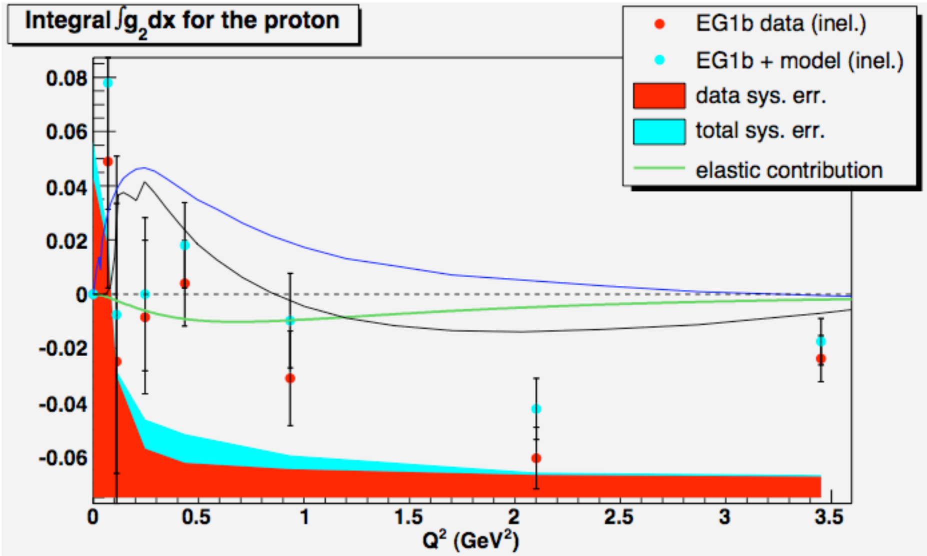
vs. x (left)
vs. Q^2 (right)
for W<2 GeV

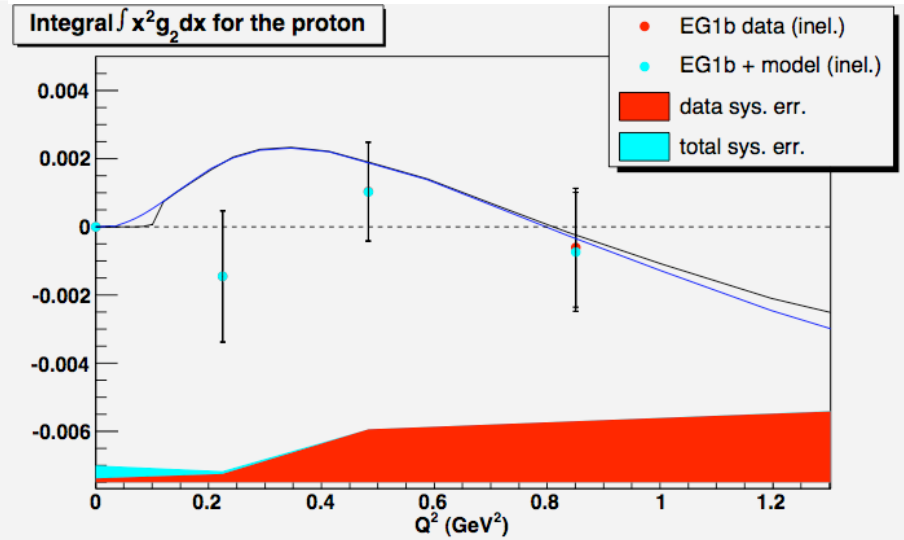


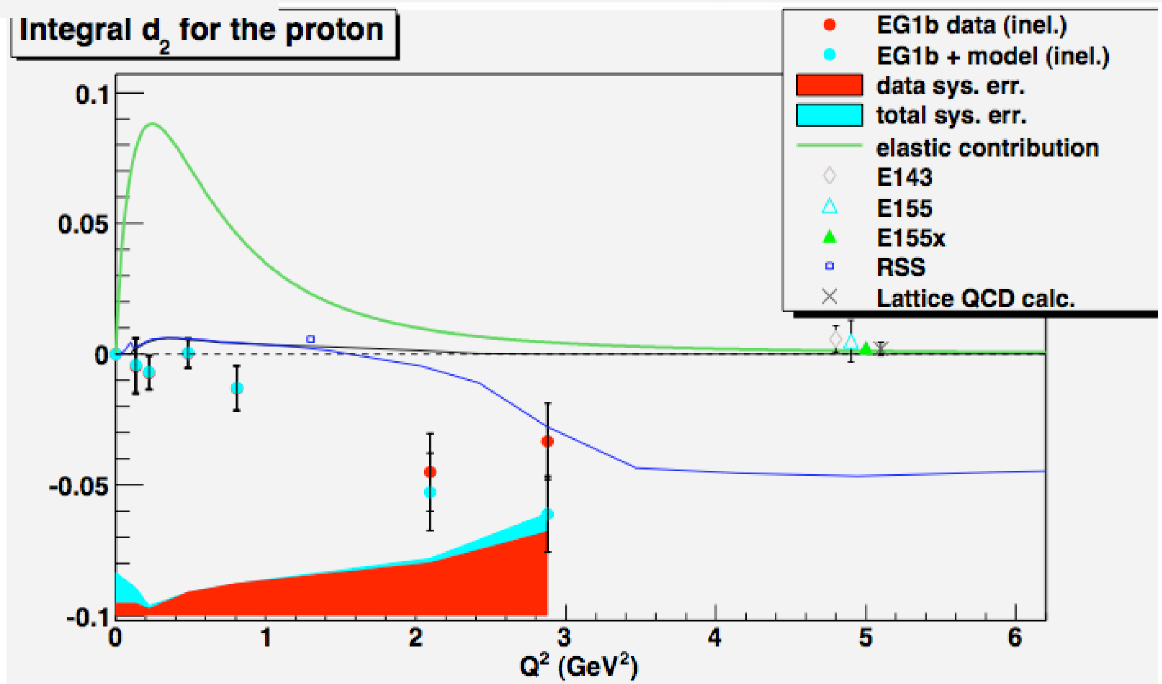




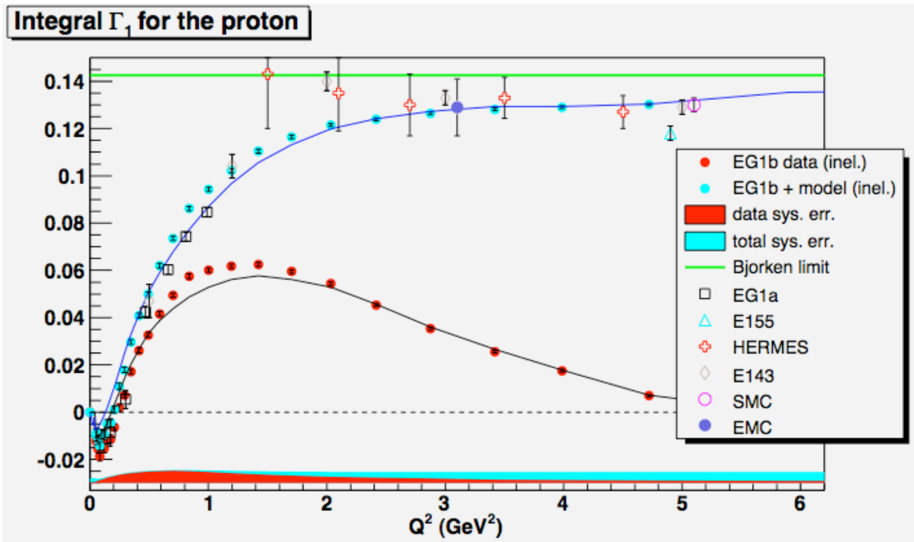
EG1b Γ_2 & d_2

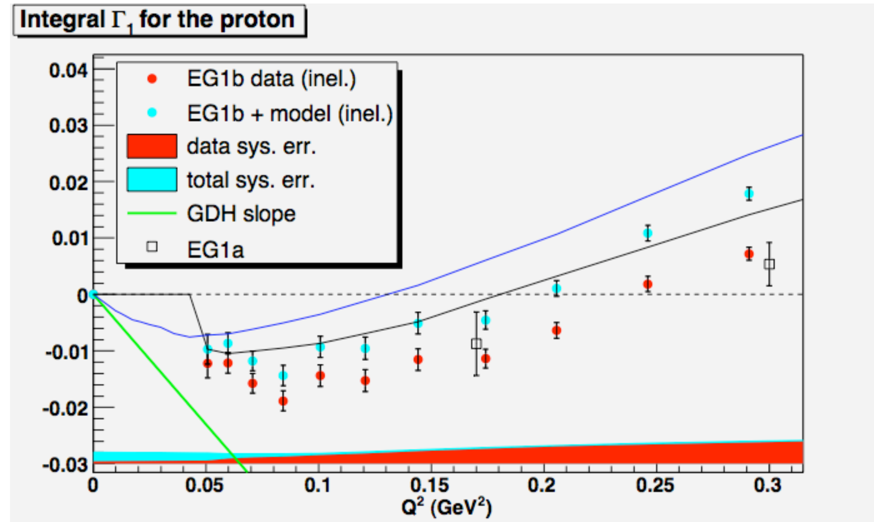


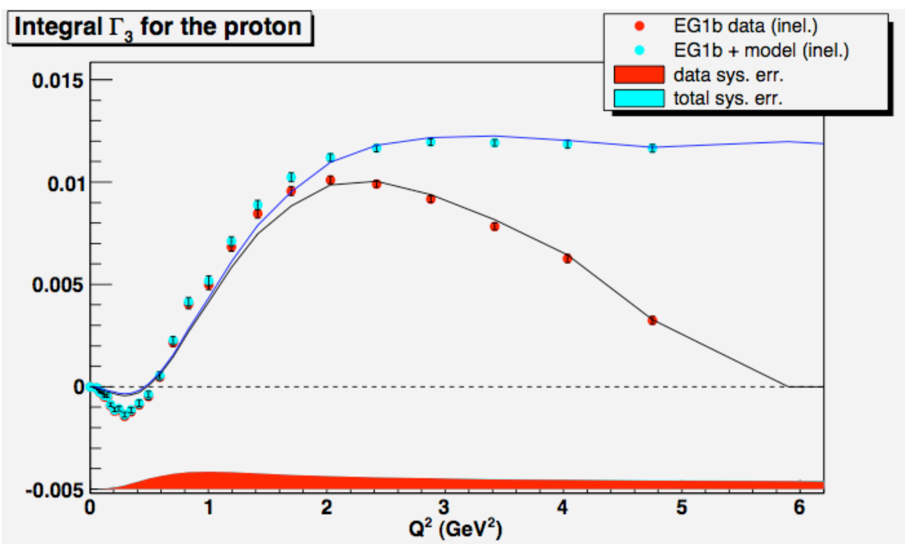


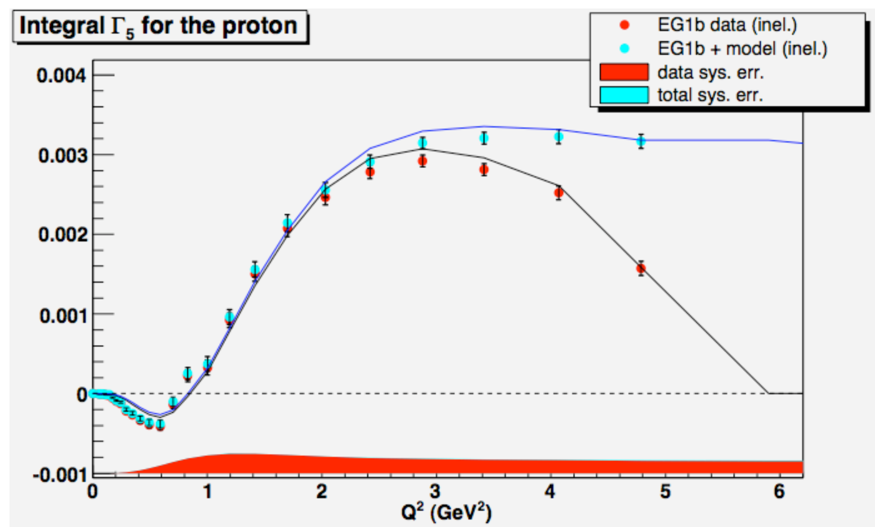


EG1b $\Gamma^p_{1,3,5}$

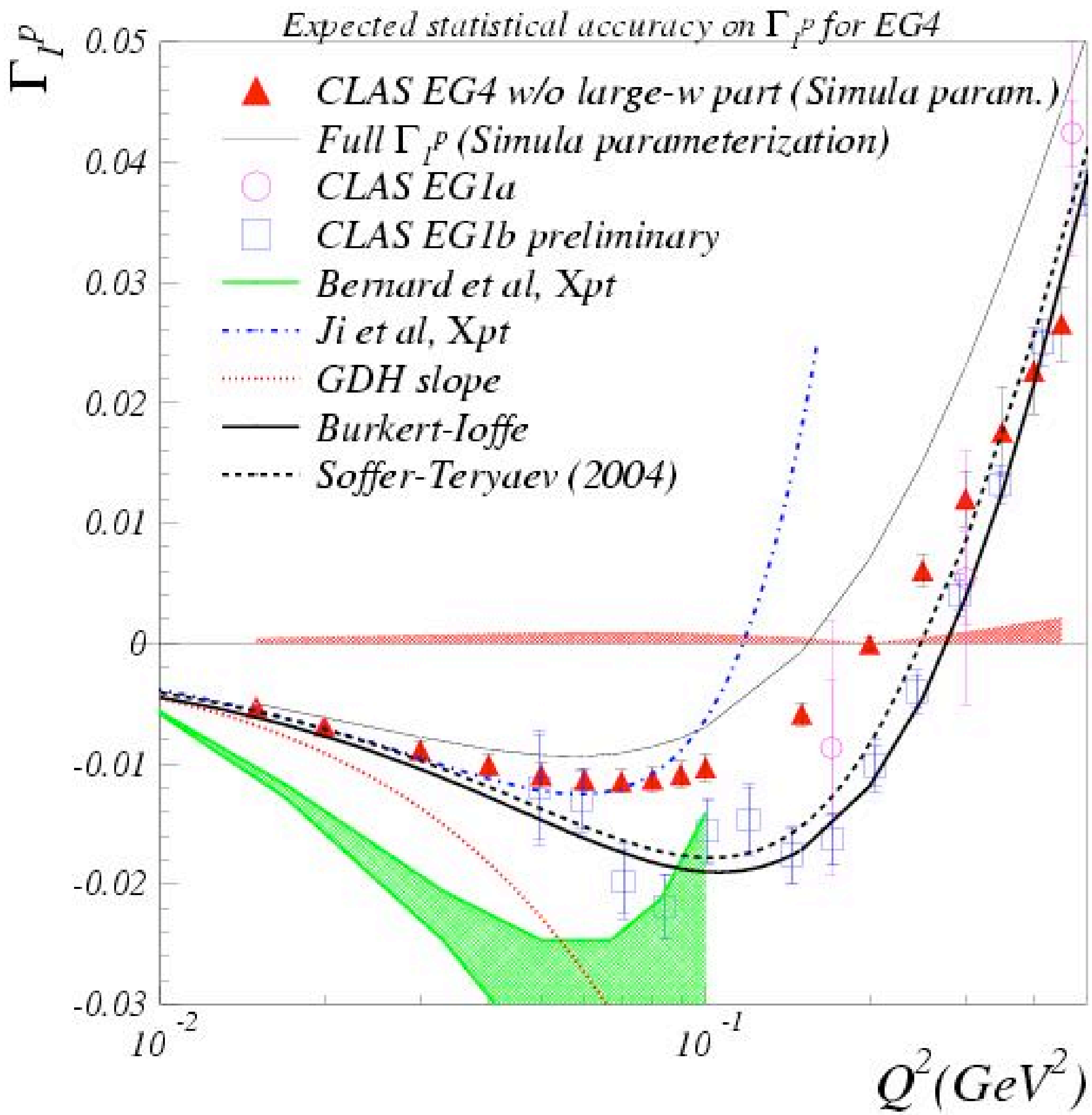








EG4 Γ_1^p Expected

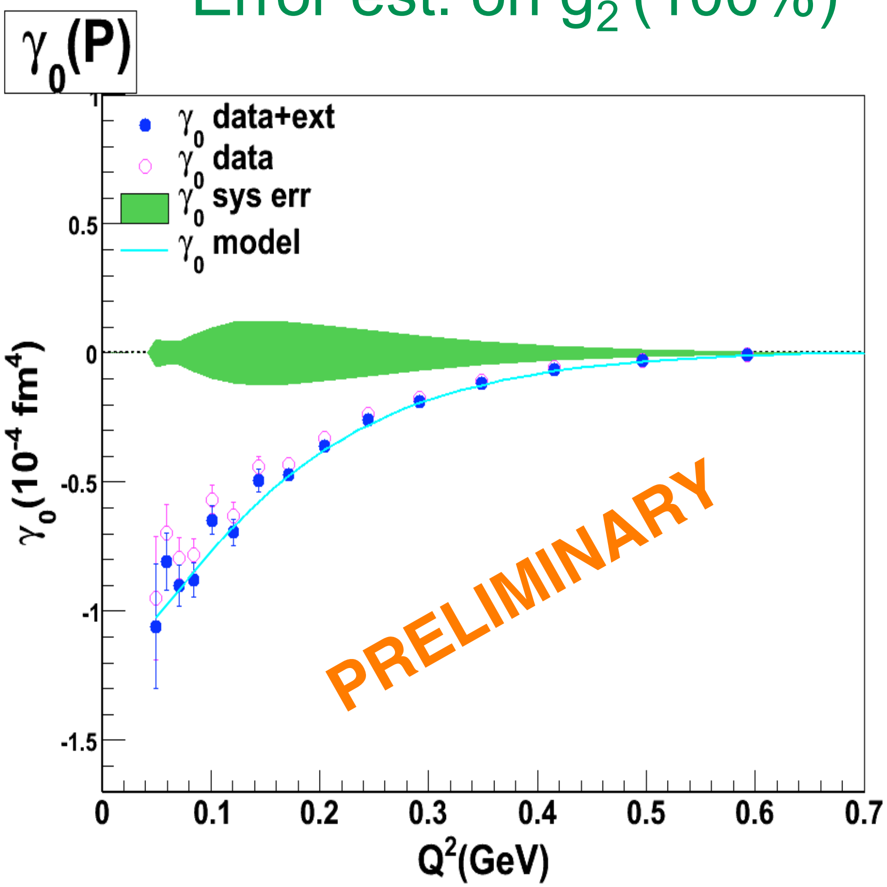


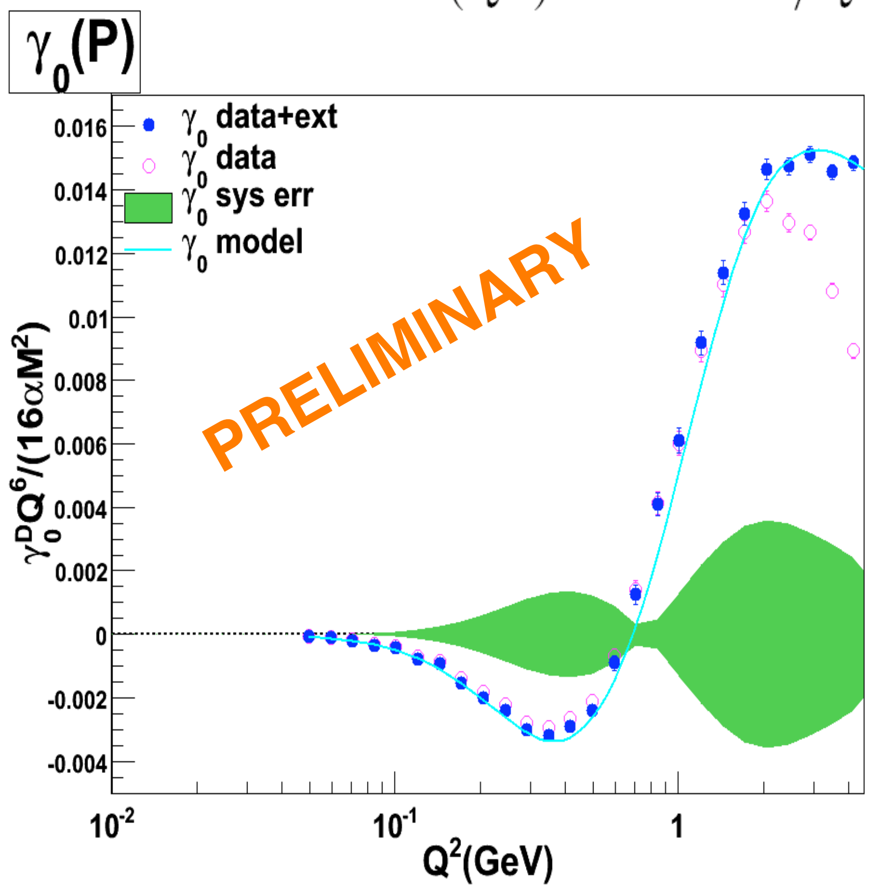
EG1b γ_0^p

$$\gamma_0(Q^2) = C(Q^2)\int_0^{x_0} x^2\left\{g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)\right\}dx$$

Error est. on g_2 (100%)

$$C(Q^2) = 16\alpha M^2/Q^6$$



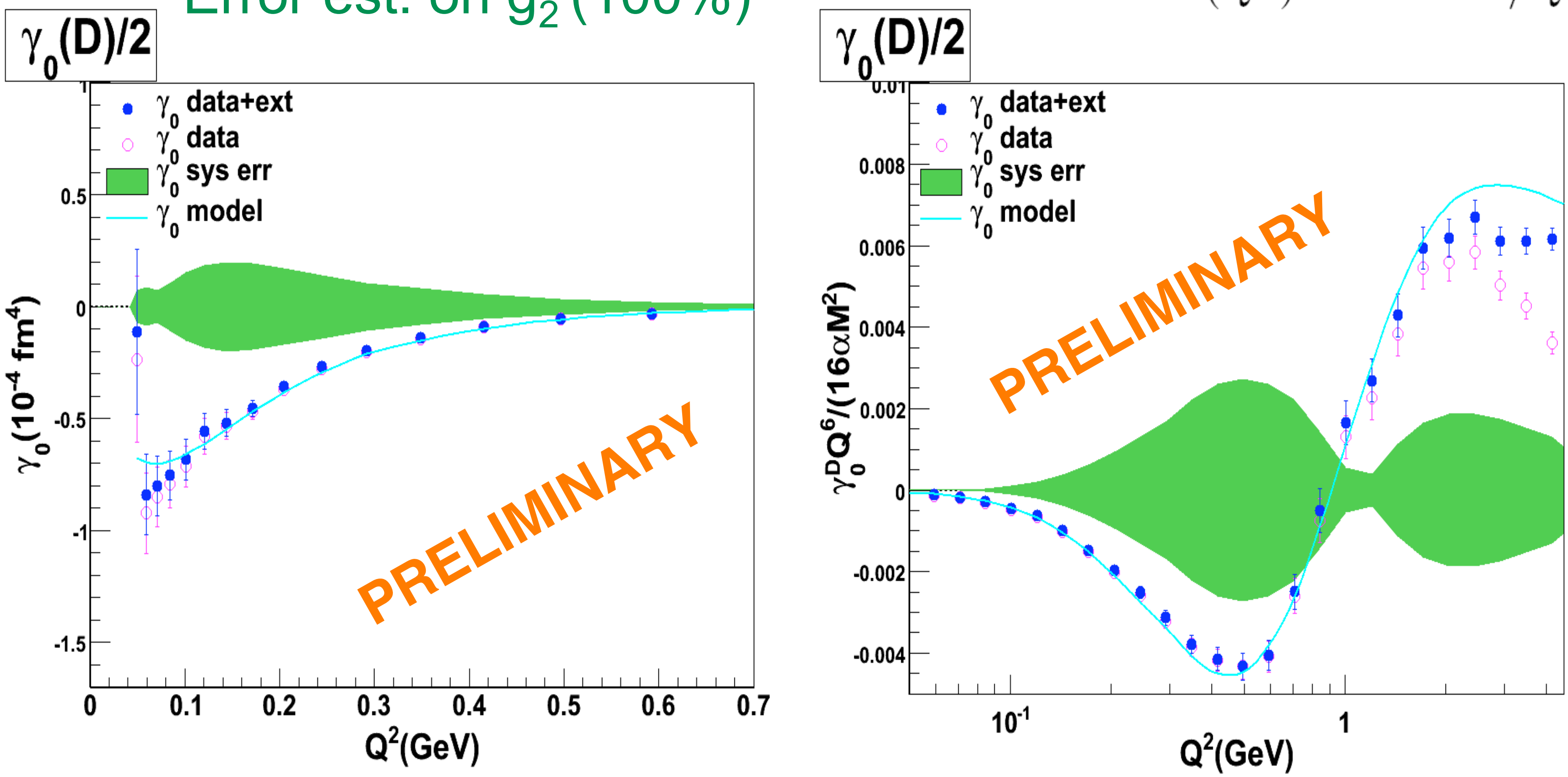


EG1b γ_0^d

$$\gamma_0(Q^2) = C(Q^2)\int_0^{x_0} x^2\left\{g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)\right\} dx$$

$$C(Q^2) = 16\alpha M^2/Q^6$$

Error est. on g_2 (100%)

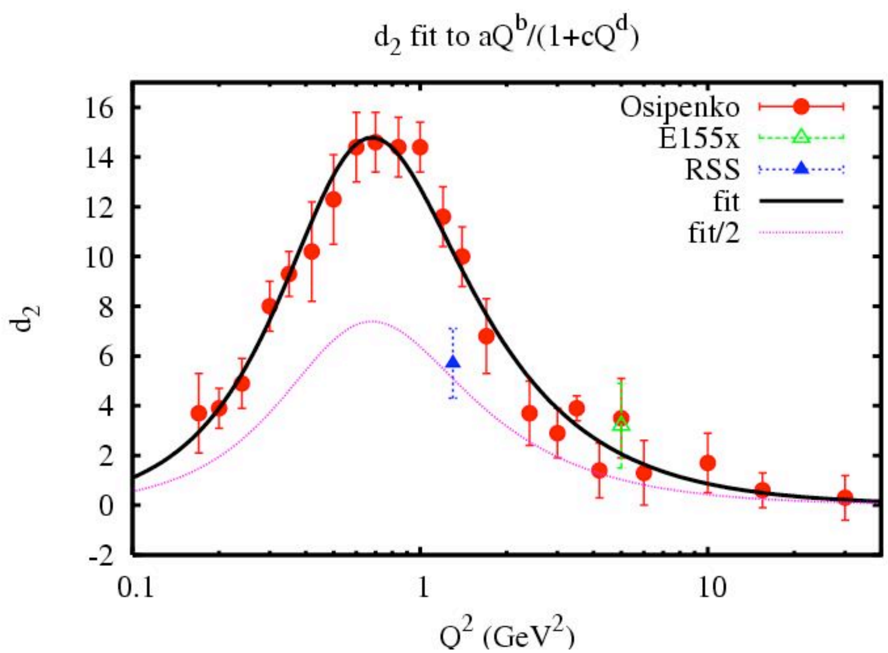


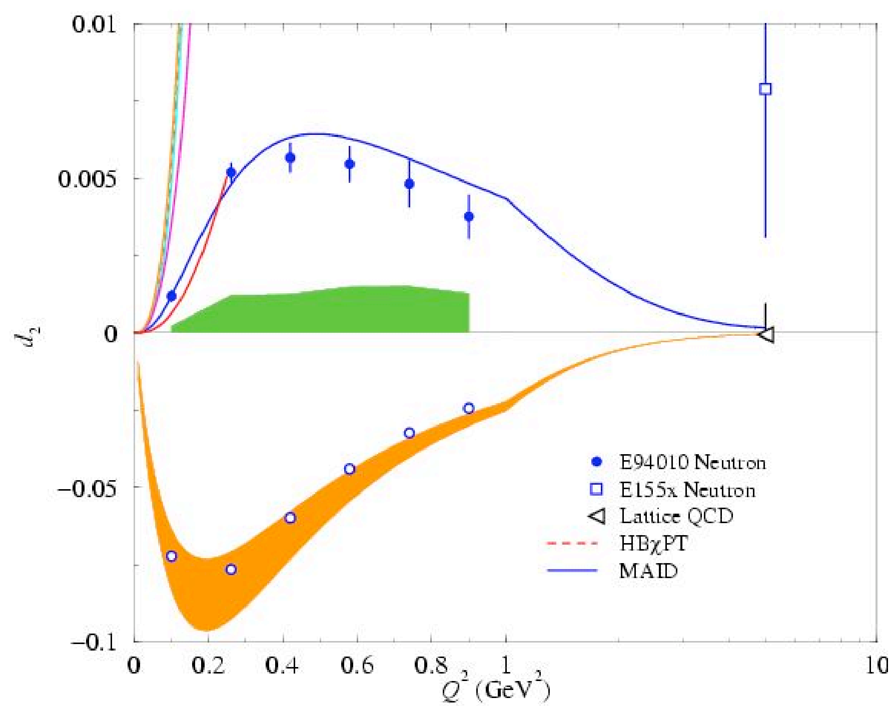
Higher Twist d_2

$$d_2(Q^2) = \int_0^1 dx\, x^2 [2g_1(x,Q^2) + 3g_2(x,Q^2)]$$

CLAS EG1 (proton)
Osipenko, PRD71(05)054007
Model-dependent determination

Hall A (neutron)
E94-010
Amarian, PRL92(04)022301





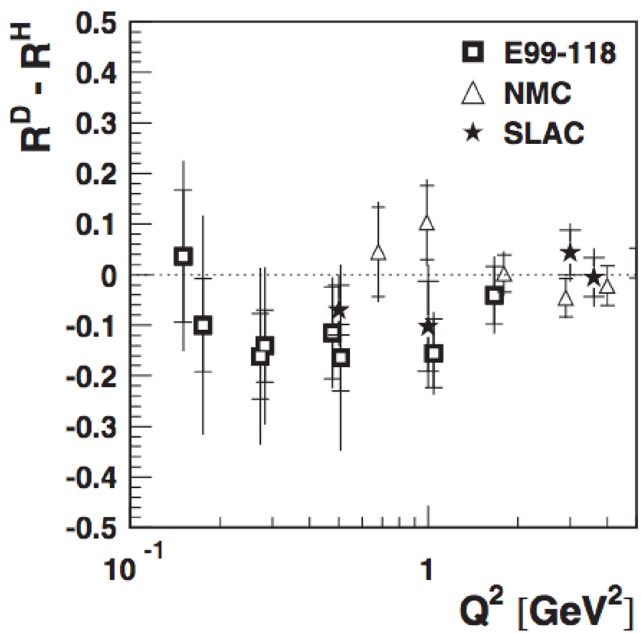
$R = \sigma_L/\sigma_T$ (DIS)

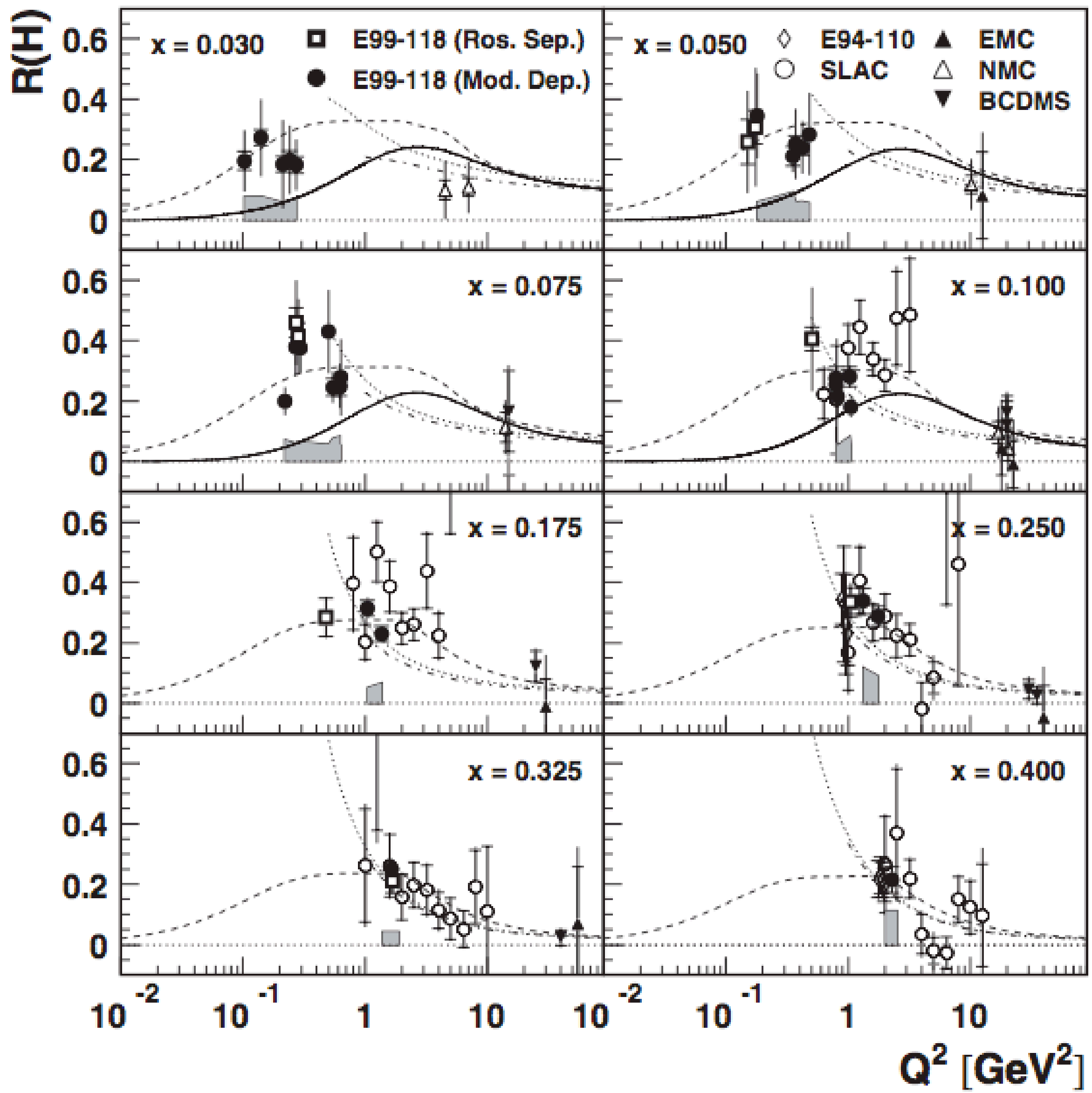
Hall C: E99-118
Tvaskis, PRL98(07)142301

$$\frac{1}{\Gamma}\frac{d^2\sigma}{d\Omega dE'} = \sigma_T + \varepsilon\sigma_L$$

$$R(x, Q^2) \equiv \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1}$$

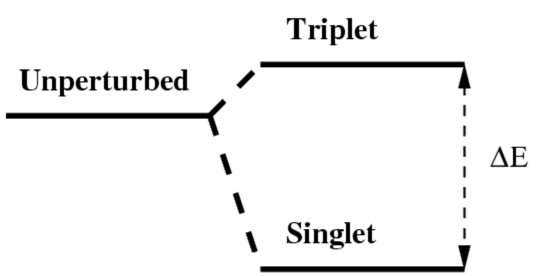
$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}}\frac{2MxF_2}{Q^2\varepsilon}\left(\frac{1+\varepsilon R}{1+R}\right)$$





Hydrogen Hyperfine Splitting

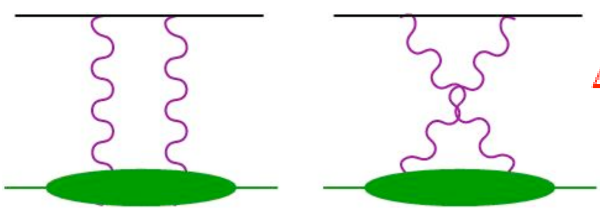
$$E_{\rm HFS}(e^-p) = 1.4204057517667(9)\,{\rm GHz} = (1+\Delta_{QED}+\Delta_R^p+\Delta_S)E_F^p$$



$$\Delta_S = \Delta_Z + \Delta_{\rm pol} \qquad \delta_Z^{\rm rad} = \frac{\alpha}{3\pi}\left[2\ln\frac{\Lambda^2}{m^2} - \frac{4111}{420}\right]$$

Zemach: $\Delta_Z = -2\alpha m_e\langle r\rangle_Z(1+\delta_Z^{\rm rad})$

$$\langle r\rangle_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa} - 1\right]$$



$\Delta_S = -38.62(16)$ ppm $\quad \Delta_Z = -41.0(5)$ ppm

$\Delta_{\rm pol} = 2.38(58)$ ppm

$$\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi(1+\kappa)M}(\Delta_1+\Delta_2) = (0.2264798\ {\rm ppm})(\Delta_1+\Delta_2)$$

$$\tau = \nu^2/Q^2$$

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\} = 6.48(89)$$

$$\Delta_2 = -24m_p^2\int_0^\infty \frac{dQ^2}{Q^4}B_2(Q^2). = -0.57(57)$$

$$B_1 = \int_0^{x_{\rm th}} dx\,\beta(\tau)g_1(x,Q^2)\,,$$

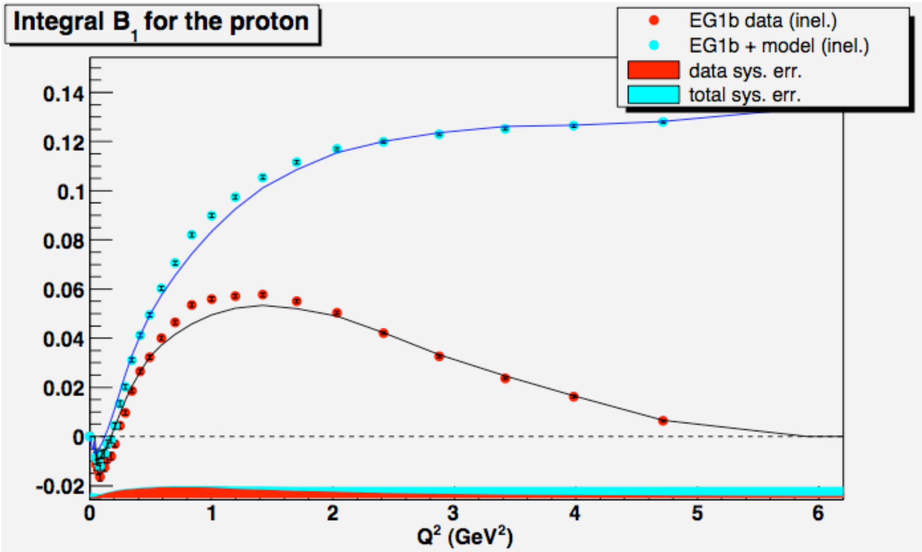
$$B_2 = \int_0^{x_{\rm th}} dx\,\beta_2(\tau)g_2(x,Q^2)\,,$$

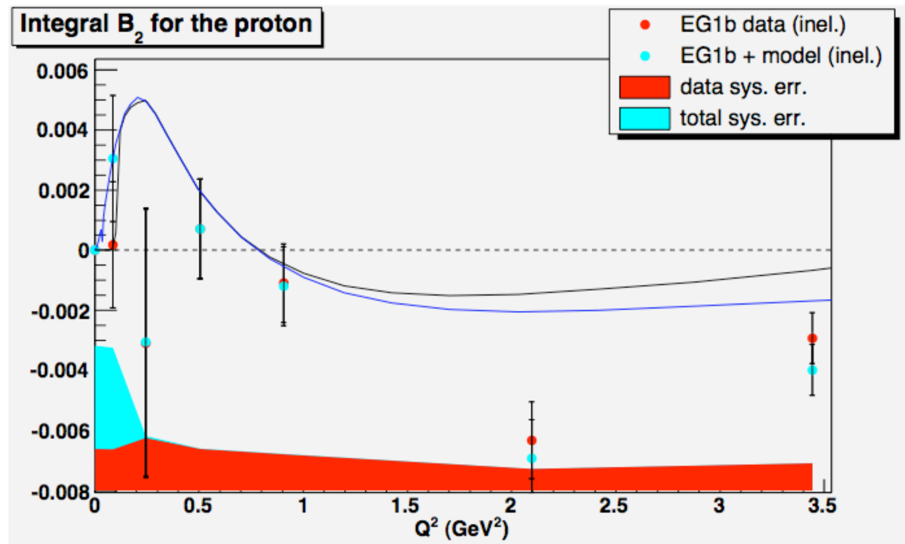
$$\beta(\tau) = \frac{4}{9}\left(-3\tau+2\tau^2+2(2-\tau)\sqrt{\tau(\tau+1)}\right)$$

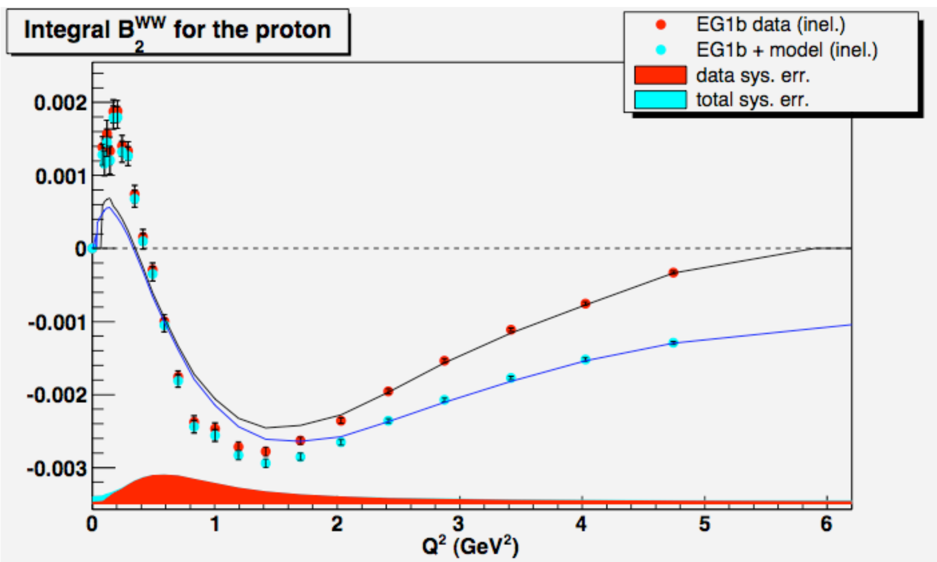
$$\beta_2(\tau) = 1+2\tau-2\sqrt{\tau(\tau+1)}\,,$$

$\Delta_{\rm pol}$ = 1.34(24) ppm from CLAS

EG1b B_1 and B_2







g_2 extracted from A_1 & A_2

vs. x (left)
vs. Q^2 (right)
for W<2 GeV

Moments

- Elastic and resonant scattering are higher twists
- One must be careful when to include elastic
- WW (leading twist) [does not]
- $\Gamma_{1,2}^{(n)} = \langle x^n g_{1,2}(x)\rangle$ [does not]
- $\Gamma_2^{(0)} = 0$ (Burkardt-Cottingham) [does]
- $\Gamma_2^{(0)WW} = 0$ (thus elastic cancels higher twist)
- $\Gamma_2^{(2)WW} = -(2/3)\Gamma_1^{(2)}$
- $d_2 = 3\Gamma_2^{(2)} + 2\Gamma_1^{(2)}$ (higher twist) [does (not)]
- $d_2 = 0$ if $\Gamma_2^{(2)} = \Gamma_2^{(2)WW}$ (no higher twist)
- $B_{1,2}$ [do not]

Commensurate Scaling

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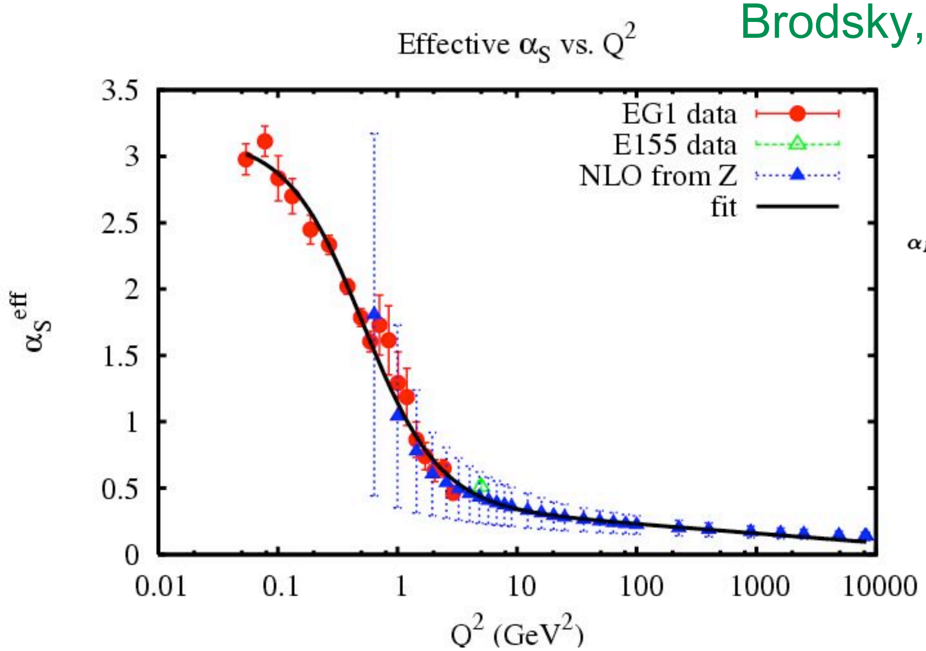
Brodsky, Lu, PRD51(95)3652; Deur, PLB650(07)244



$$\int_0^1 dx\,[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

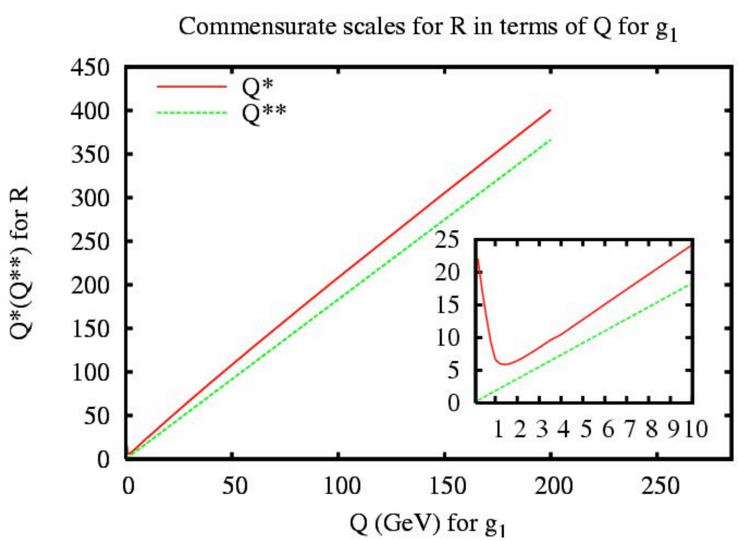
$$R(Q) \equiv 3\sum_f Q_f^2\left[1 + \frac{\alpha_R(Q)}{\pi}\right]$$

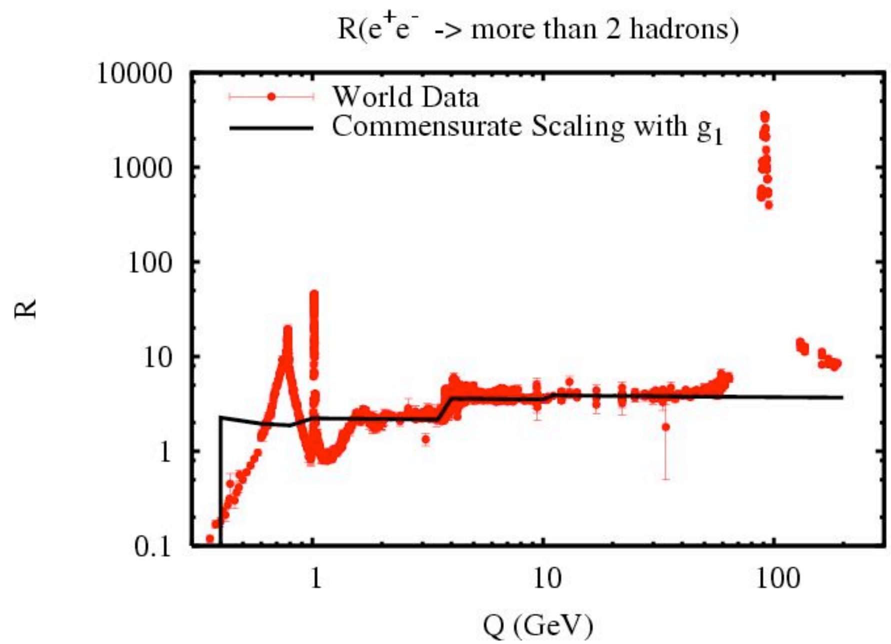
$\alpha_R(Q)$ in terms of $\alpha_{g_1}(Q)$.

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{g_1}(Q^*)}{\pi} + \frac{3}{4}C_F\left(\frac{\alpha_{g_1}(Q^{**})}{\pi}\right)^2 + \left[\frac{9}{16}C_F^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right)\frac{d^{abc}d^{abc}}{C_F N}\frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right]\left(\frac{\alpha_{g_1}(Q^{***})}{\pi}\right)^3,$$

$$\ln(Q^*/Q) = -\frac{7}{4} + 2\zeta_3 + \left(-\frac{11}{96} - \frac{7}{3}\zeta_3 + 2\zeta_3^2 + \frac{\pi^2}{24}\right)\left(\frac{11}{3}C_A - \frac{2}{3}f\right)\frac{\alpha_{g_1}(Q)}{\pi},$$

$$\ln(Q^{**}/Q) = -\frac{233}{216} + \frac{64}{9}\zeta_3 - \frac{20}{3}\zeta_5 + \left(-\frac{13}{54} + \frac{2}{9}\zeta_3\right)\frac{C_A}{C_F}.$$





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Conclusions

- A wealth of data exists for g_1, g_2
- What's still missing:
  - high x: $A_1^{p,d}$ (CLAS12)
  - g_2^p on the proton (transverse target); SANE (Hall C) covers $Q^2>1$ GeV^2; nobody's measuring $Q^2<1$ GeV^2
  - precision and full kinematic coverage for $1< Q^2<10$ (CLAS12)
  - low Q^2 evolution of $g_1^{p,d}$ (EG4)
- What's gained:
  - understanding three regions
    - Q^2 near 0 (χPT)
    - Q^2 from 0.1-10 GeV^2 (TMC, higher twists, resonances, the transition)
    - Q^2 near infinity (pQCD)