Measurements of Quark Momenta in 3 Dimensions

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Mainz, 14 November 2011

Scattering

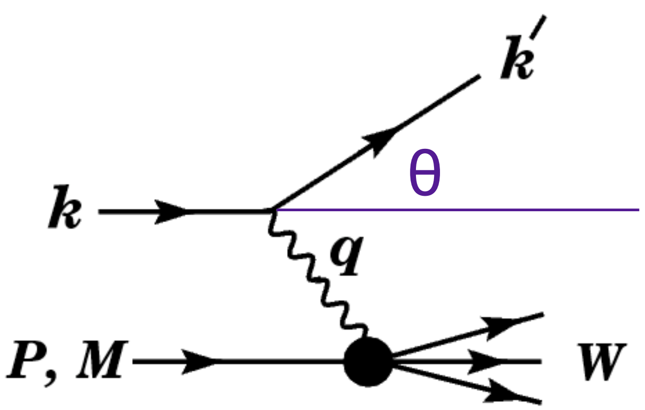
Most of what we know about the world comes from scattering experiments. The qualities of light give very different pictures of the same object.

Rouen Cathedral, Claude Monet, ~1893

eN Scattering

- The only thing we can measure is a cross section.
- But by separating kinematics from nucleon structure, we can identify robust, experimentally determined objects, the structure functions:

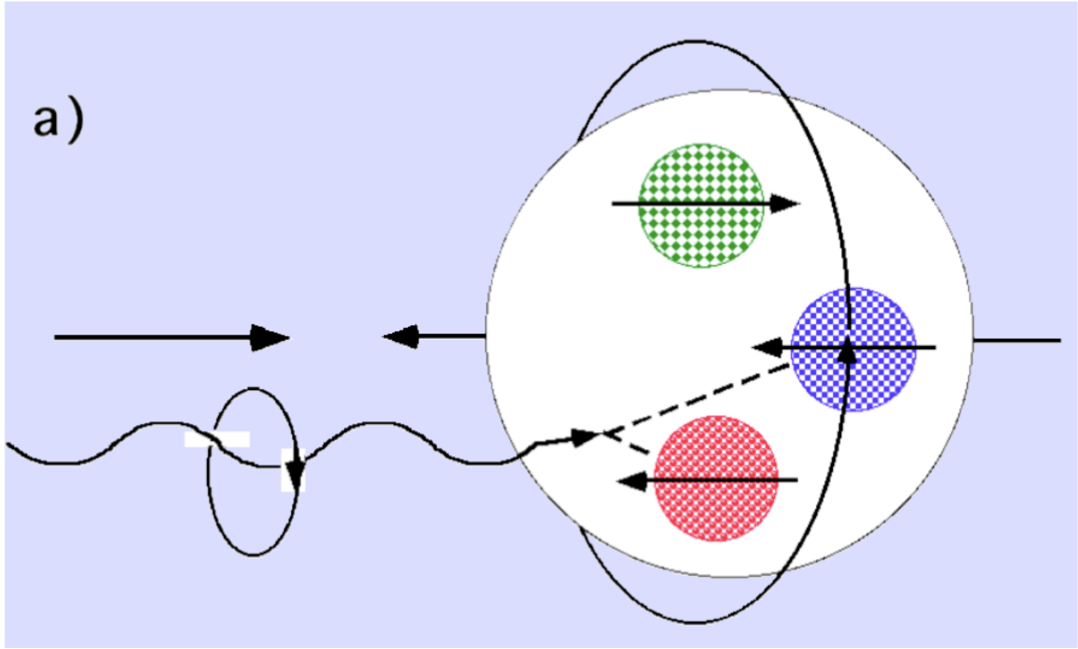
$$\frac{d\sigma}{dx\,dy\,d\psi} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\Big\{F_T + \varepsilon F_L + S_{\|}\lambda_e\sqrt{1-\varepsilon^2}\,2x\left(g_1-\gamma^2 g_2\right) - |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,2x\gamma\left(g_1+g_2\right)\Big\}$$



- Thus, F_T, F_L, g_1, $g_2(x,Q^2)$ can be extracted for all x, Q^2.
- Experiment tells us where these can be interpreted in terms of parton distribution functions (PDFs) in pQCD and where complications show up.
- PDFs are known only through model fitting of structure functions.
- The same holds for transverse momentum dependent distributions (TMDs) and generalized parton distributions (GPDs)

Lorentz invariants:
$q \cdot q = -Q^2$
$p \cdot q/M = \nu$
$(p+q)^2 = W^2$
$(k+p)^2 = s$
$-q \cdot q/(2p \cdot q) = x$
$p \cdot q/p \cdot k = y = (\nu/E)_{lab}$
$E_h/\nu = z$

Structure Functions



- Structure functions have a simple interpretation in the parton model
- Realistically, we measure convolutions of PDFs that are extracted through NLO QCD analyses

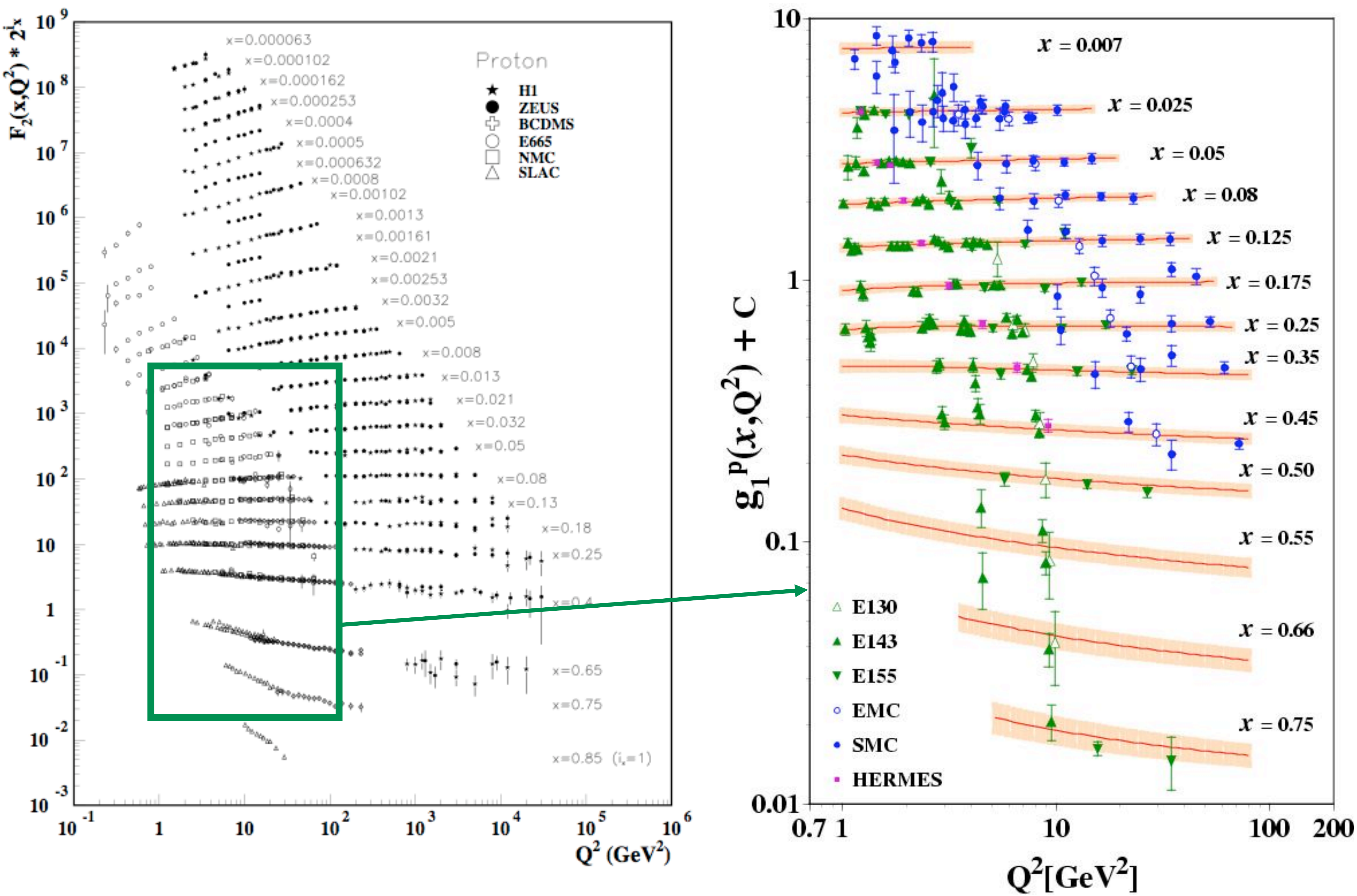
Parton Model:

$$F_1(x,Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^{\uparrow}(x) + q^{\downarrow}(x) + \bar{q}^{\uparrow}(x) + \bar{q}^{\downarrow}(x))$$

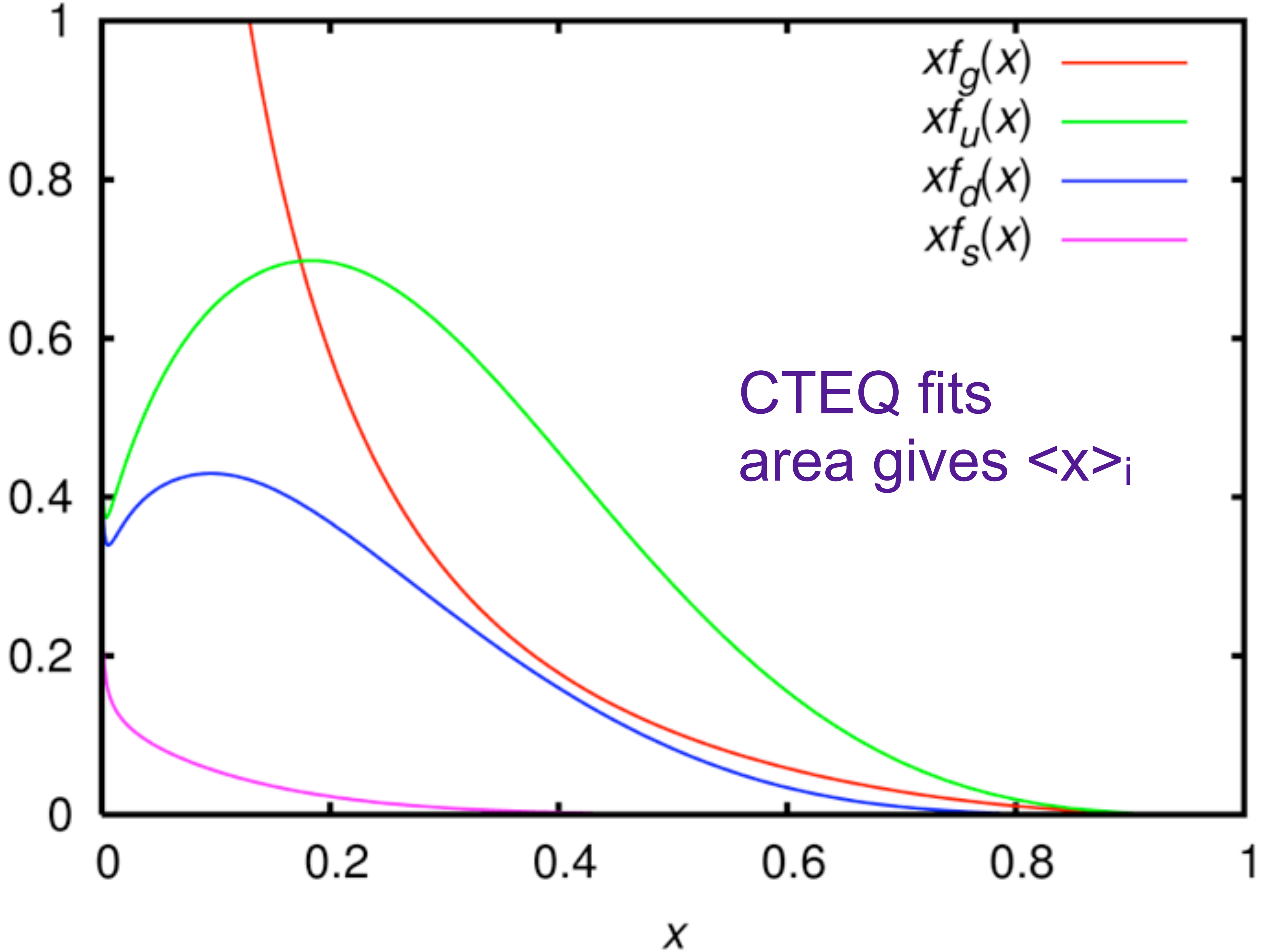
$$F_2(x,Q^2) = 2xF_1(x,Q^2)$$

$$g_1(x,Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^{\uparrow}(x) - q^{\downarrow}(x) + \bar{q}^{\uparrow}(x) - \bar{q}^{\downarrow}(x))$$

$F_2^p(x,Q^2)$ and $g_1^p(x,Q^2)$

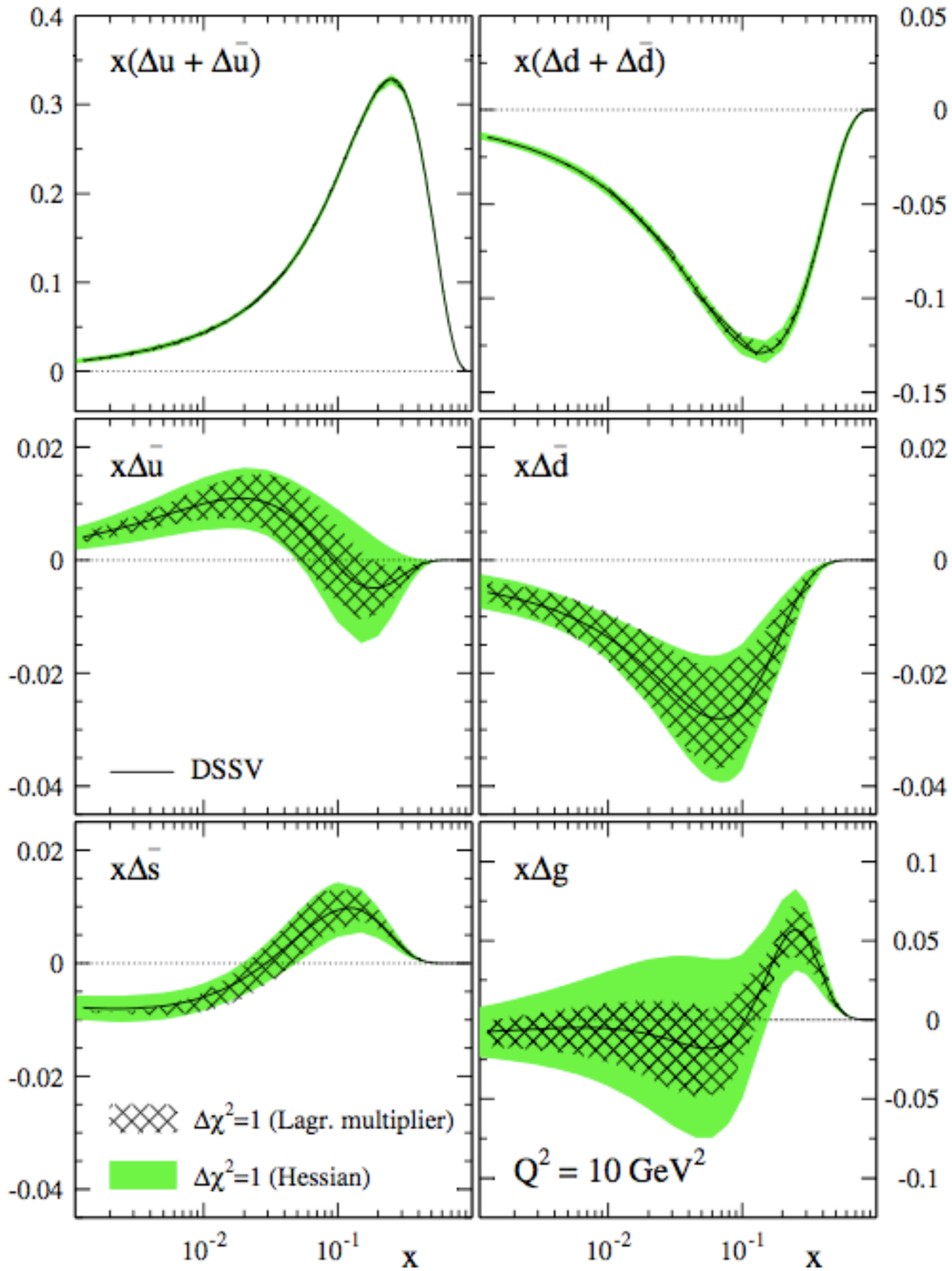


Fits to all the world's data yield the probability distributions for finding a quark or a gluon with momentum fraction x at scale Q^2.



CTEQ fits
area gives $\langle x \rangle_i$

Polarized PDFs



DSSV fits

Q^2 evolution is used to determine Δg

Large uncertainties remain

deFlorian, Sassot, Stratmann, Vogelzang

DSSV PDFs

x range in Eq. (35)	Q^2 [GeV^2]	$\Delta u + \Delta\bar{u}$	$\Delta d + \Delta\bar{d}$	$\Delta\bar{u}$	$\Delta\bar{d}$	$\Delta\bar{s}$	Δg	$\Delta\Sigma$
0.001–1.0	1	0.809	−0.417	0.034	−0.089	−0.006	−0.118	0.381
	4	0.798	−0.417	0.030	−0.090	−0.006	−0.035	0.369
	10	0.793	−0.416	0.028	−0.089	−0.006	0.013	0.366
	100	0.785	−0.412	0.026	−0.088	−0.005	0.117	0.363
0.0–1.0	1	0.817	−0.453	0.037	−0.112	−0.055	−0.118	0.255
	4	0.814	−0.456	0.036	−0.114	−0.056	−0.096	0.245
	10	0.813	−0.458	0.036	−0.115	−0.057	−0.084	0.242
	100	0.812	−0.459	0.036	−0.116	−0.058	−0.058	0.238

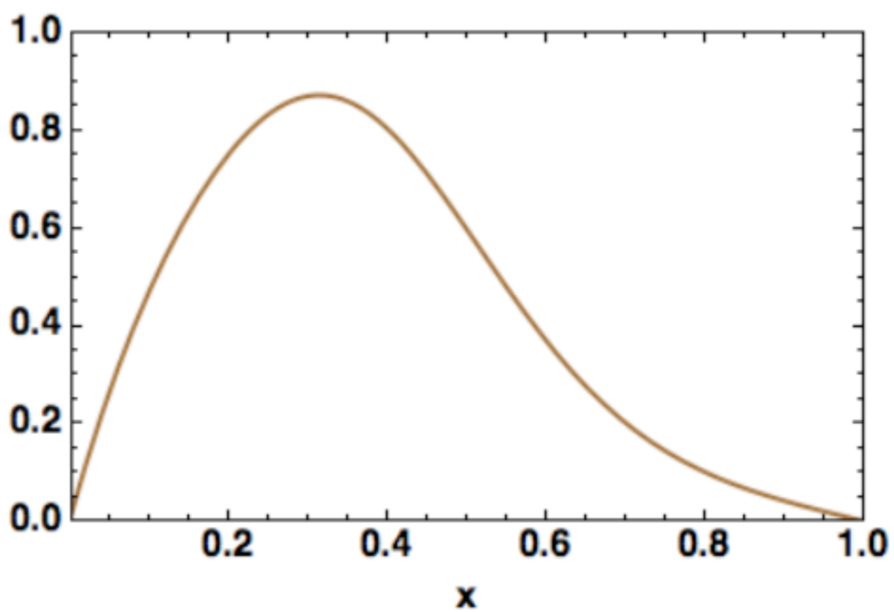
$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

- Significant contributions from $x<0.001$
- ΔG vanishes with increasing Q^2
- At Q^2=4 GeV^2, L_z = 0.474 (large)
- Errors on ΔG are still very large

TMDs

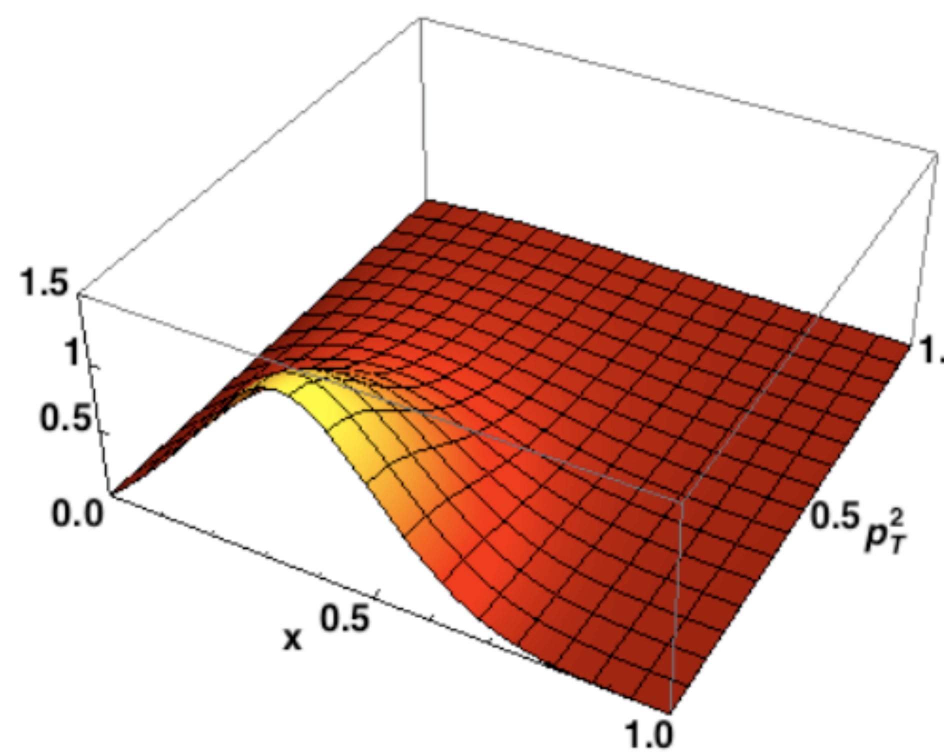
- Any confined quark must have transverse momentum
- Therefore, colinear PDFs cannot give the whole story
- Transverse momentum is related to L_z
- There has been much recent work trying to understand transverse momentum distributions (TMDs)

$x f_1^u(x)$



Standard collinear PDF

$x f_1^u(x, p_T^2)$



TMD

Cahn Effect

Cahn, Phys. Lett. **B78**(1978)269

- Parton model for electron-quark scattering
- Transverse momentum-dependence at higher twist

$$\mathcal{M}_{LL} = \bar{u}_L(p')\gamma_\mu u_L(p)\,\bar{u}_L(k')\,\gamma^\mu u_L(k)$$

$$|\mathcal{M}_{LL}|^2 = |\mathcal{M}_{RR}|^2 = 4s^2$$

$$|\mathcal{M}_{LR}|^2 = |\mathcal{M}_{RL}|^2 = 4u^2$$

$$s = (p+k)^2 = 2\,p\cdot k$$

$$u = (p-k')^2$$

$$p = (xP, p_\perp \cos\varphi, p_\perp \sin\varphi, xP) = xP + p_\perp$$

k: incident electron
k': scattered electron
p: incident quark
p': scattered quark

y = (E-E')/E {fractional energy transfer}
$p_\perp$ = quark transverse momentum
φ = azimuthal angle of struck quark

$$\sigma \propto s^2 + u^2 \propto (1 - (P_\perp/Q)\sqrt{1-y}\cos\varphi)^2$$

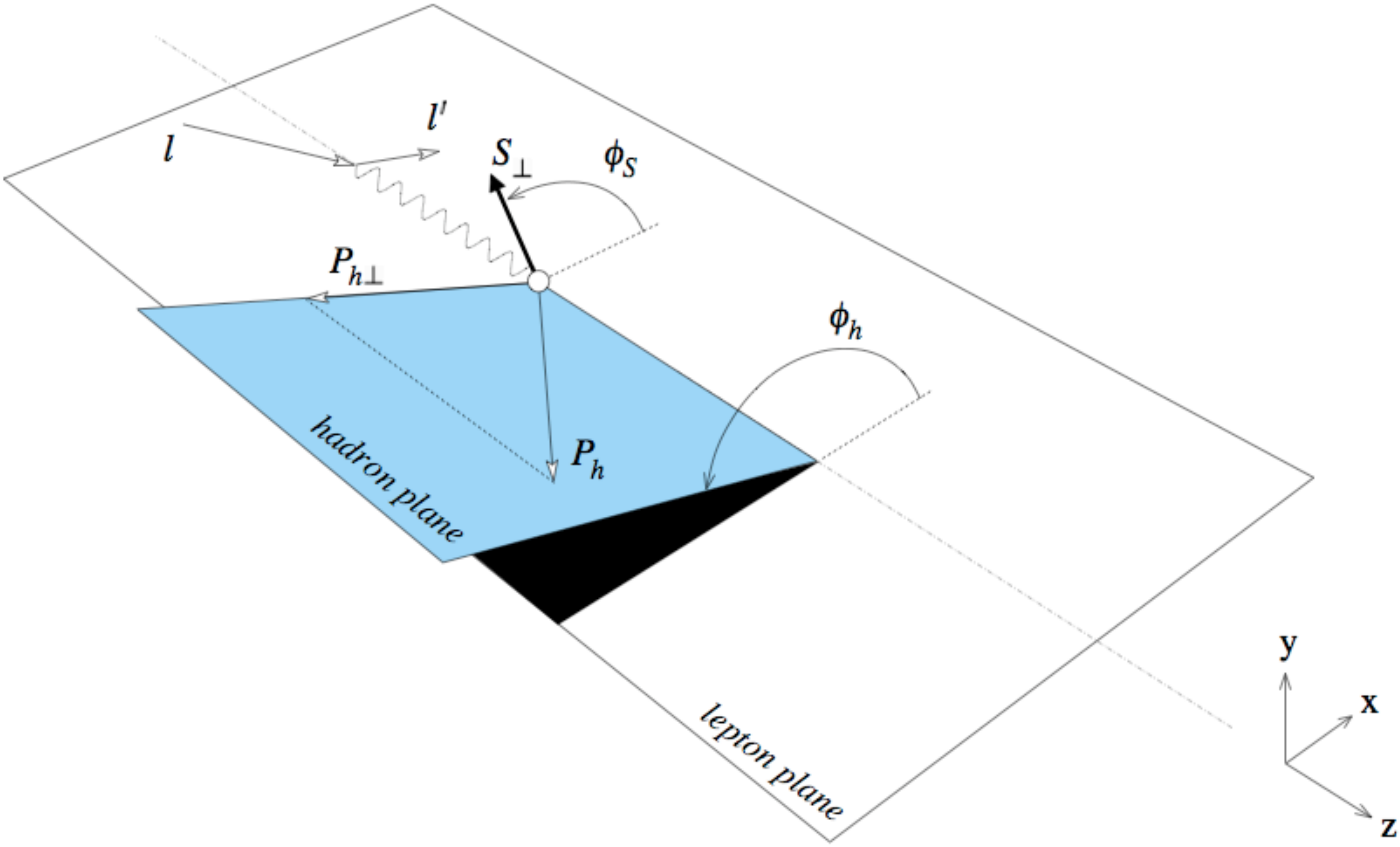
$$+ (1-y)^2\,(1 - (P_\perp/Q\sqrt{1-y})\cos\varphi)^2$$

$$\langle\cos\varphi\rangle_{eP} = -\left(\frac{2p_\perp}{Q}\right)\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

$$\langle\cos 2\varphi\rangle_{eP} = \left(\frac{2p_\perp^2}{Q^2}\right)\frac{(1-y)}{1+(1-y)^2}$$

Azimuthal moment suppressed by 1/Q

Semi-Inclusive DIS



SIDIS Cross Section

Bacchetta, et al., JHEP 2(2007)093

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\},$$

Leading Twist

Sub-Leading Twist (extra factor of 1/Q)

0 (i.e. $R=\sigma_L/\sigma_T=0$)

A_{UL} = {UL terms} / {UU terms}

A_{LL} = {LL terms} / {UU terms}

etc.

TMD Structure Functions

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$$

Unpolarized fragmentation function; integrates to $D_1(z,Q^2)$

Unpolarized structure function; integrates to $F_1(x,Q^2)$

$$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right]$$

Polarized structure function; integrates to $g_1(x,Q^2)$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

The Collins fragmentation function

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right]$$

The Sivers structure function

And there are more...

Primary TMDs

Red: T-odd

Black: survive p_T integration

Yellow box: chiral-odd

quark pol. / nucleon pol.

nucleon pol. \ quark pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Helicity

Boer-Mulders

Pretzelocity

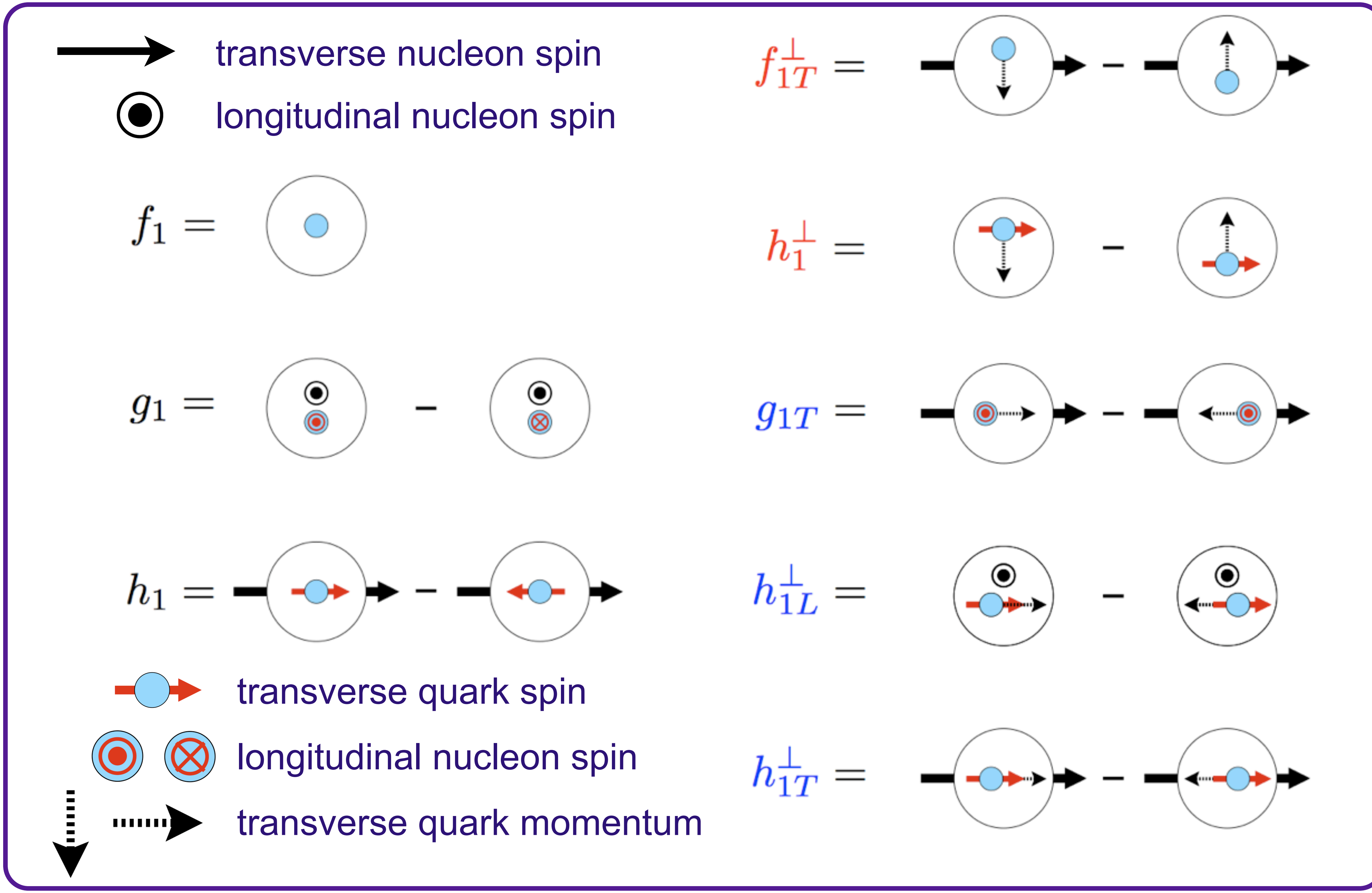
Sivers

Worm Gear

Transversity

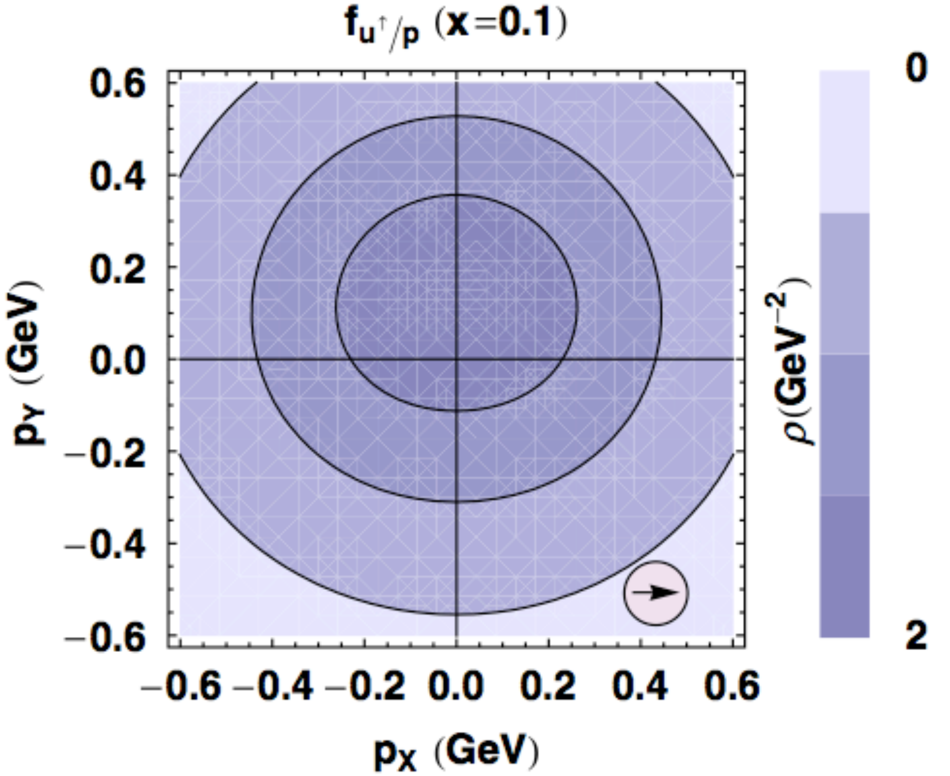
Intuitive TMDs

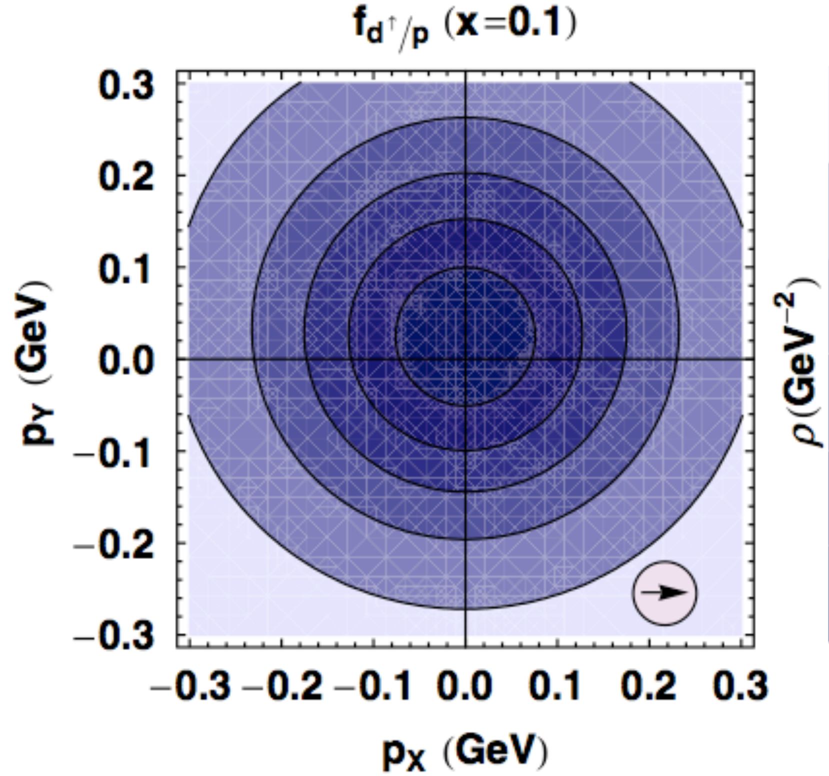
from Bacchetta



Diquark Spectator Model

Bacchetta, PR**D78**(08)074010

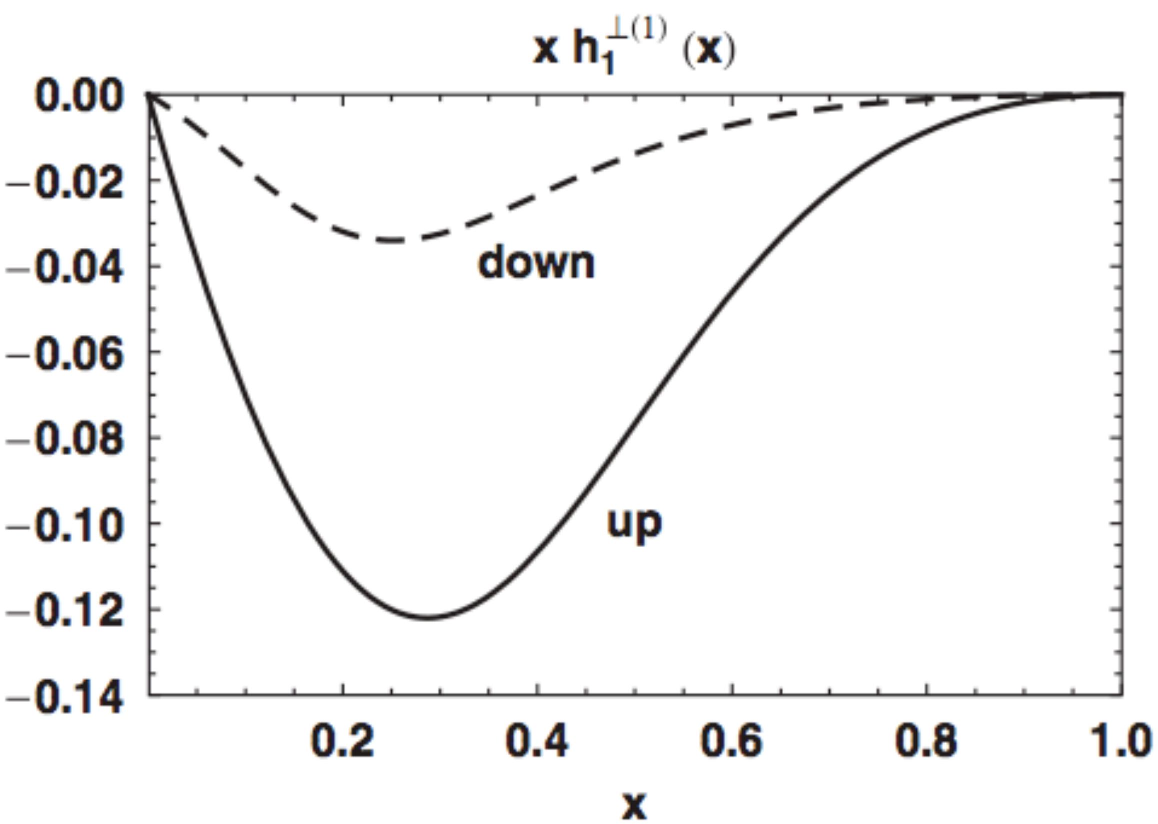


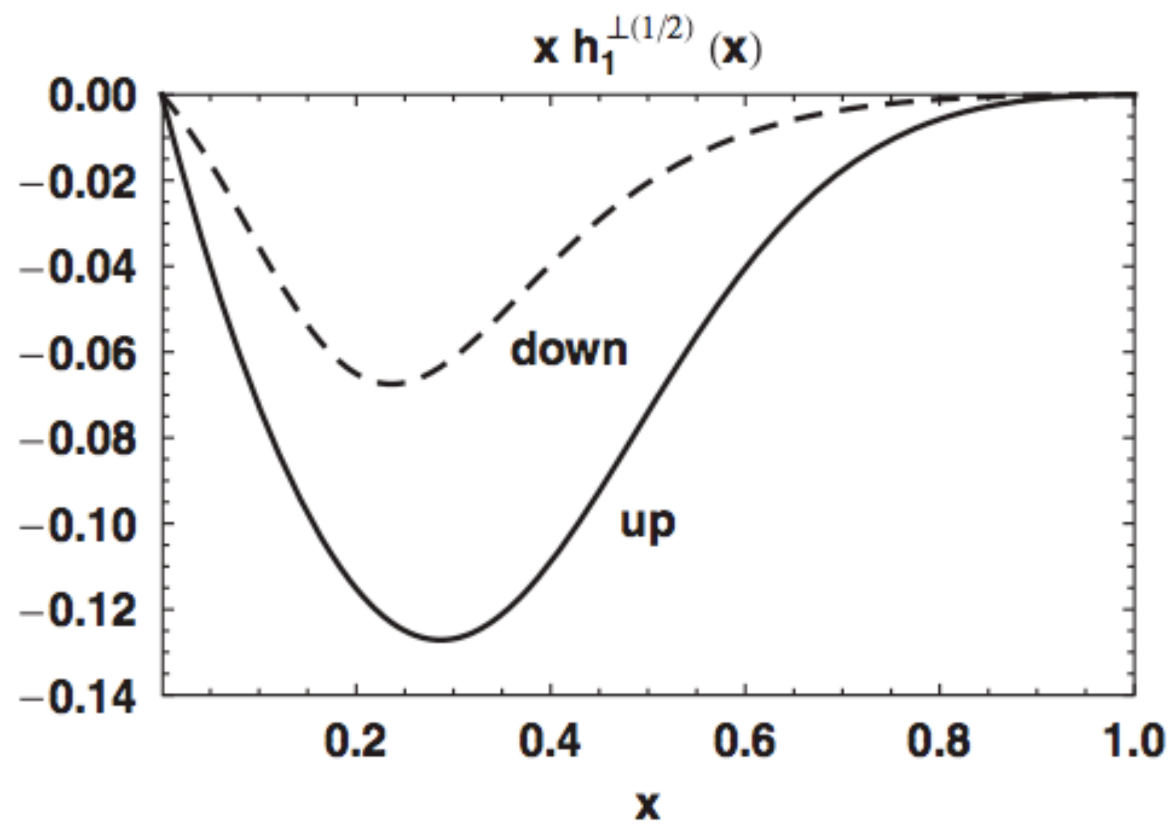


$h_1^\perp$

Boer-Mulders

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$





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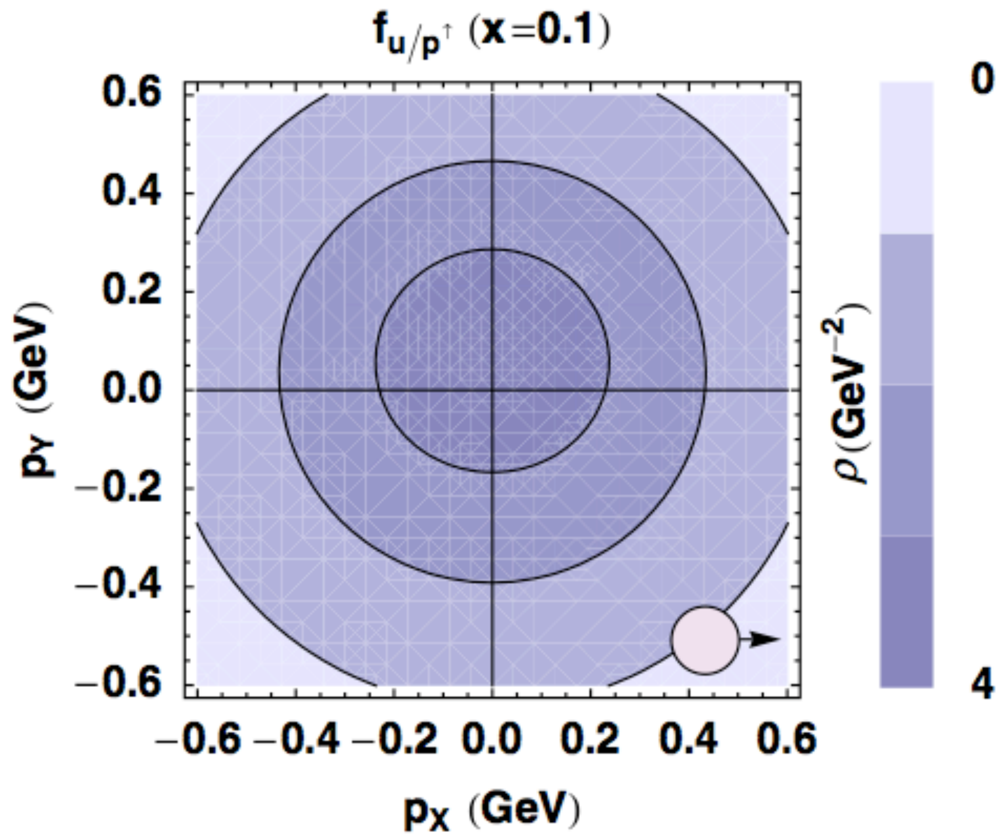
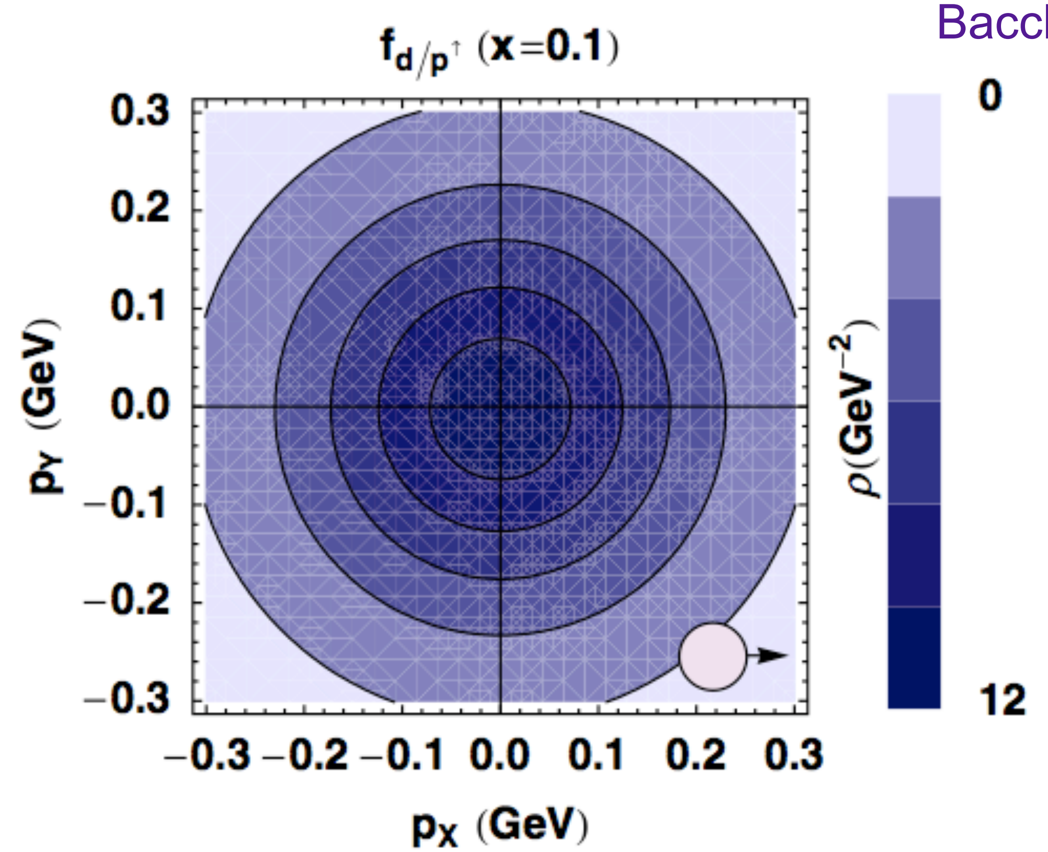
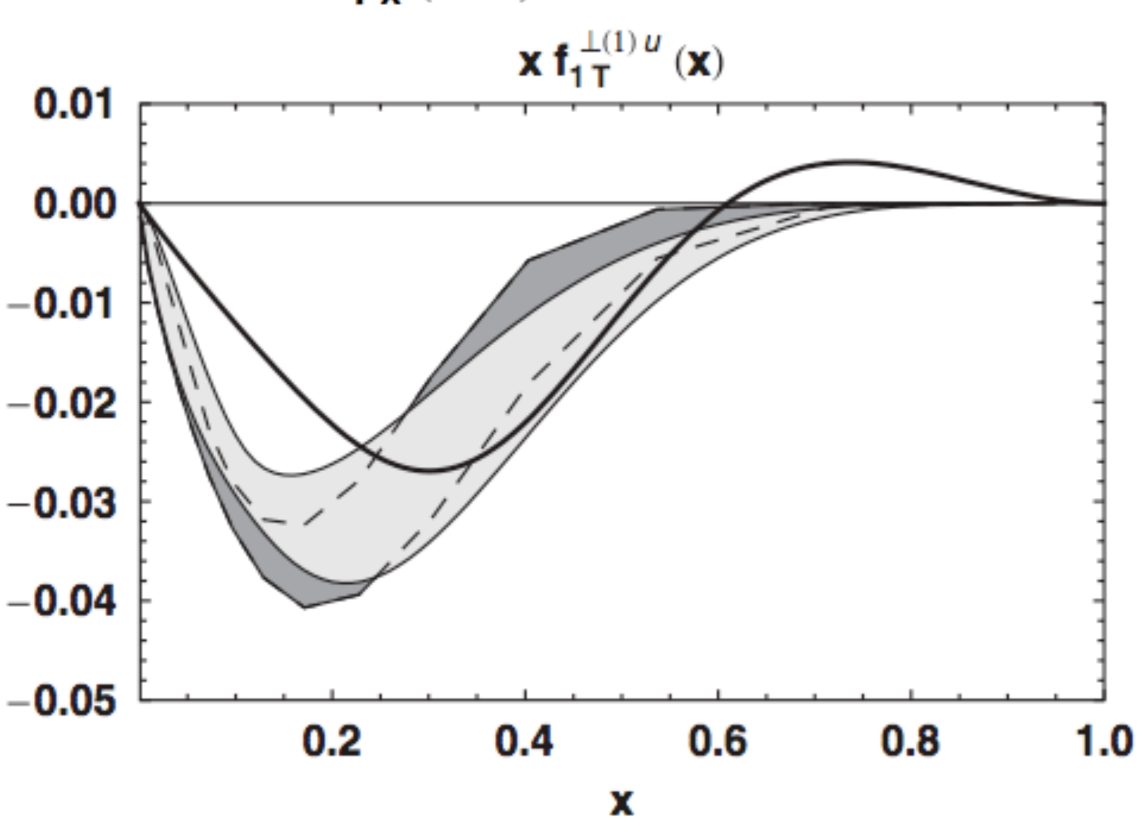


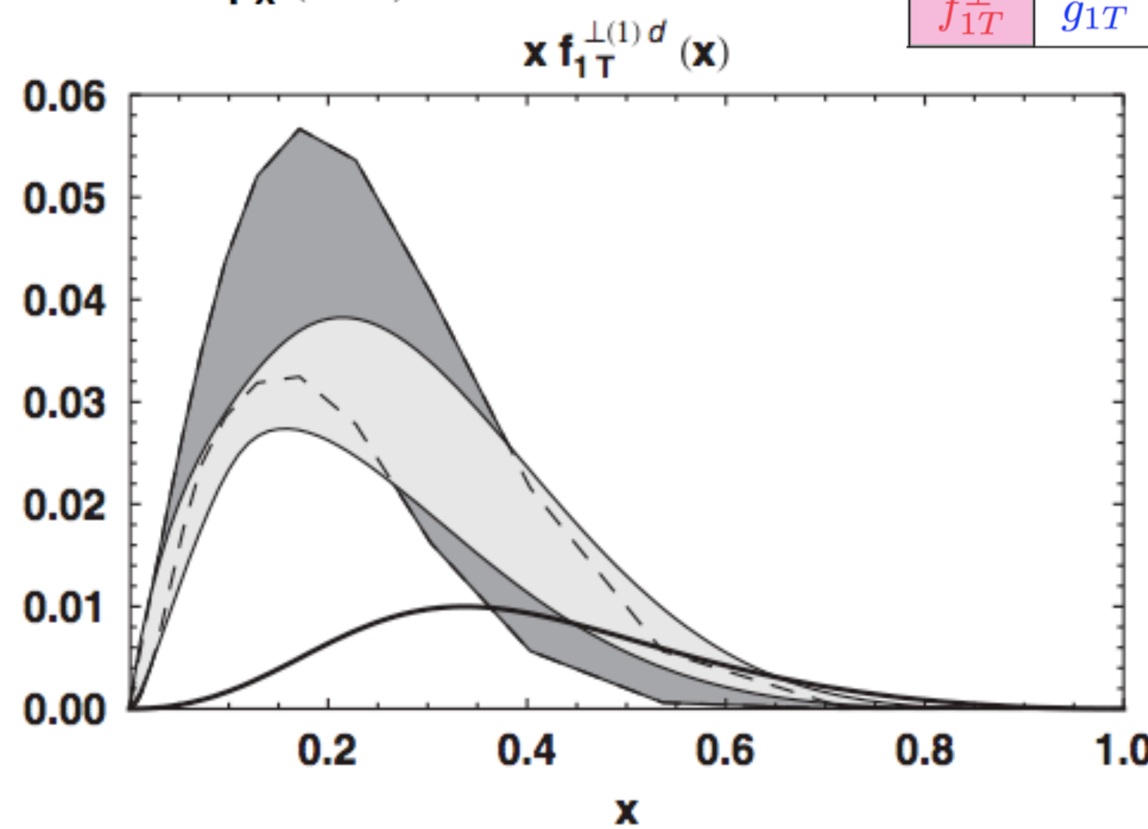


$$f_{1T}^{\perp}$$

Sivers

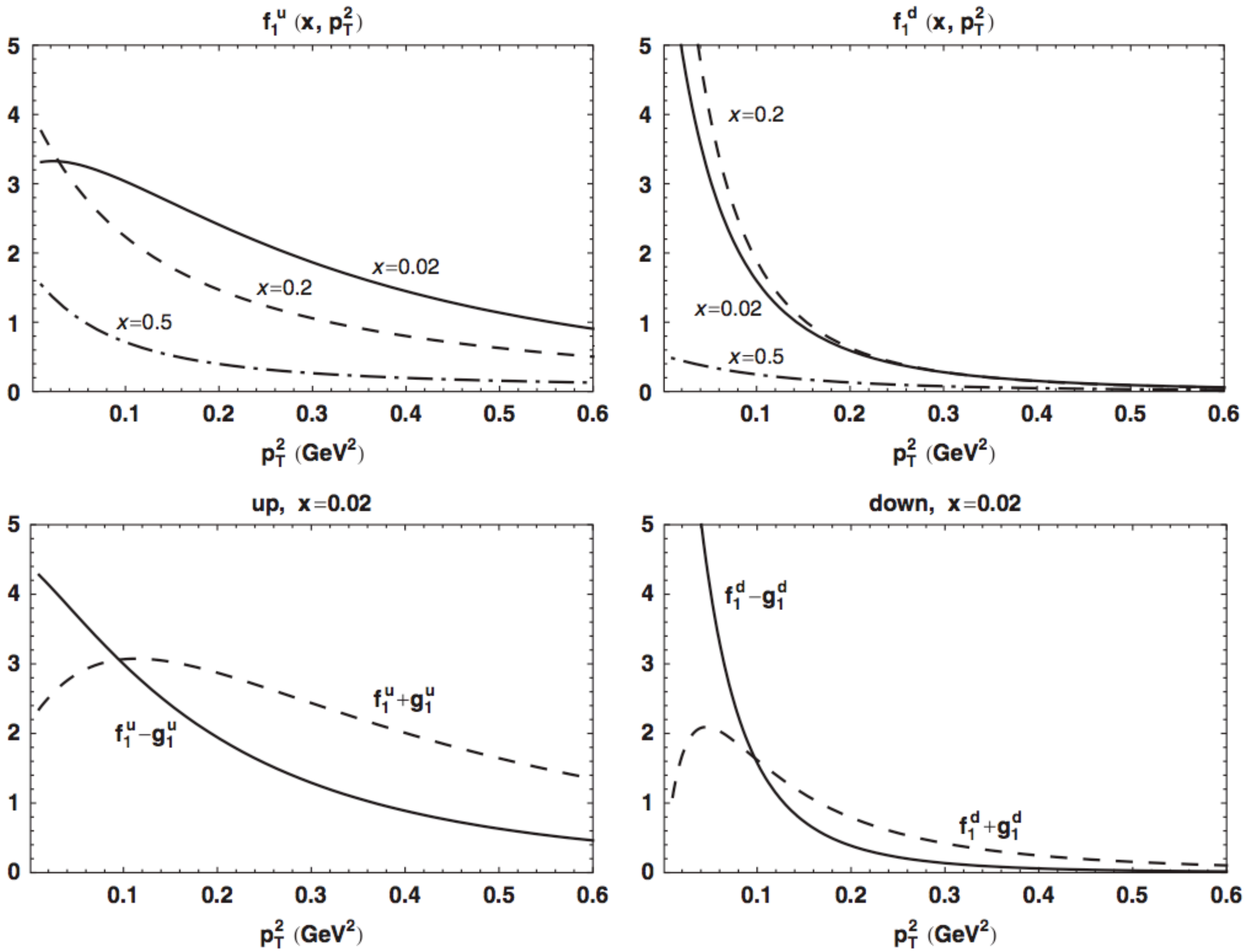
f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$





Diquark Spectator Model

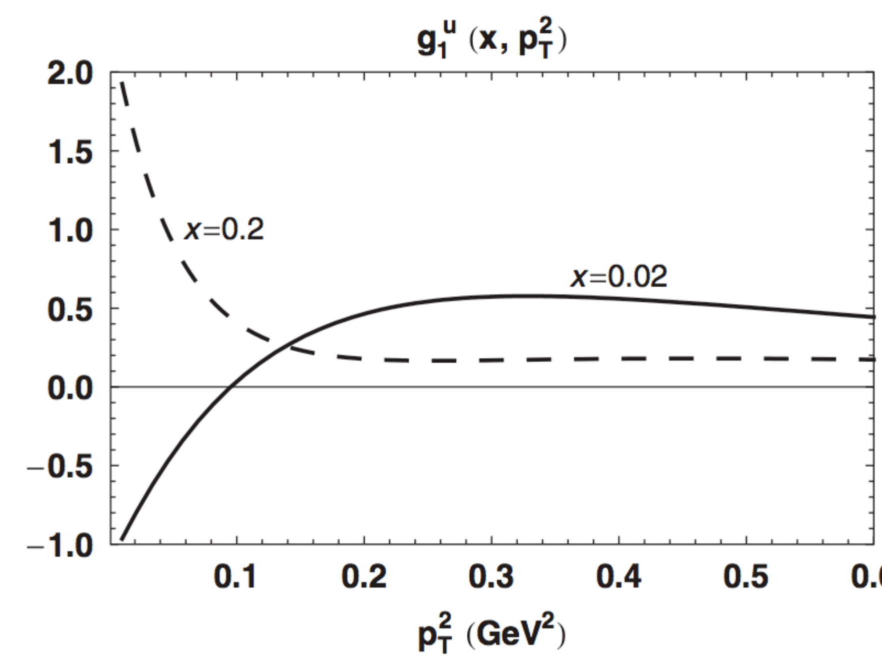
Bacchetta, PR**D78**(08)074010



f_1 g_{1L}

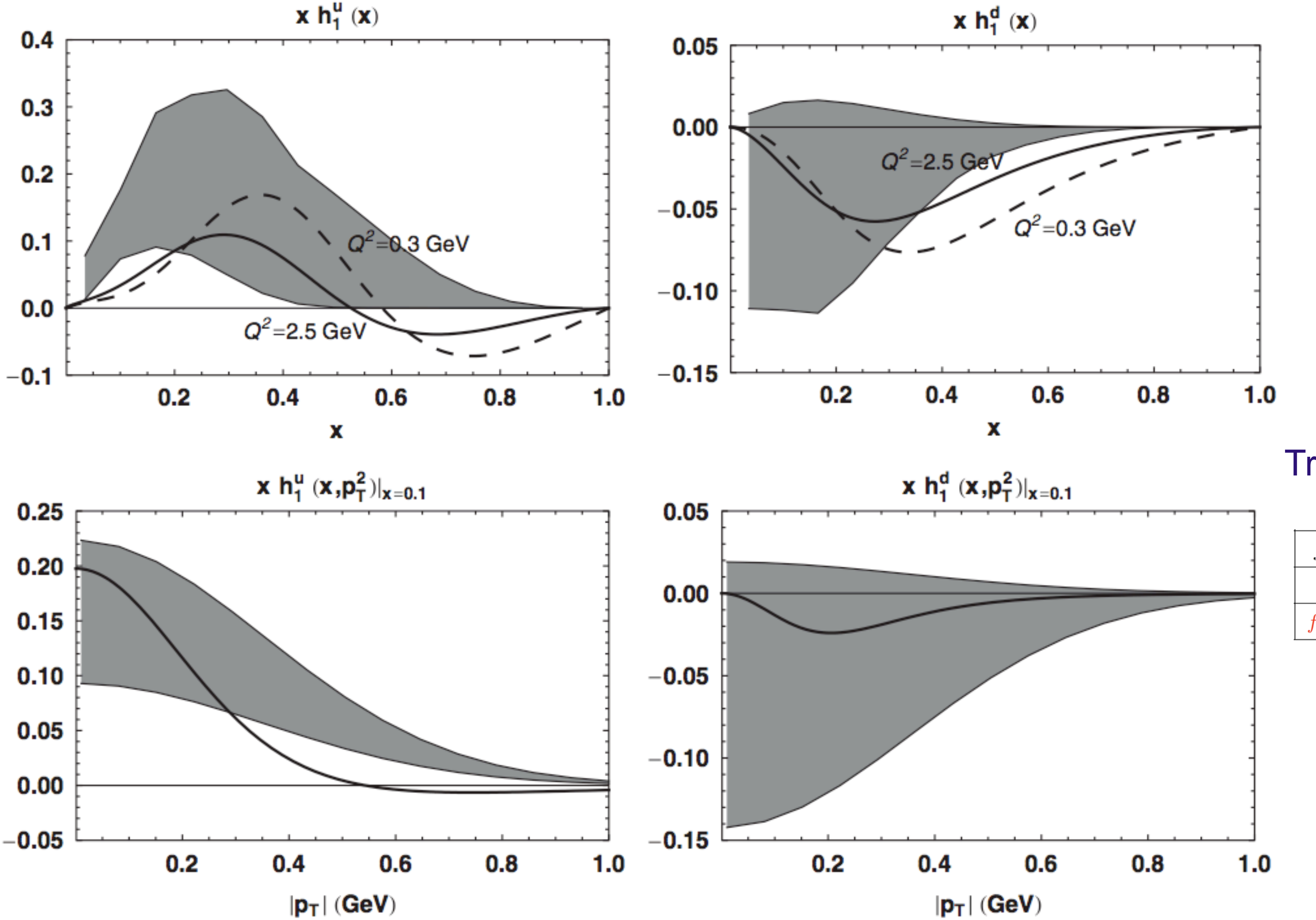
f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$



Diquark Spectator Model

Bacchetta, PR**D78**(08)074010

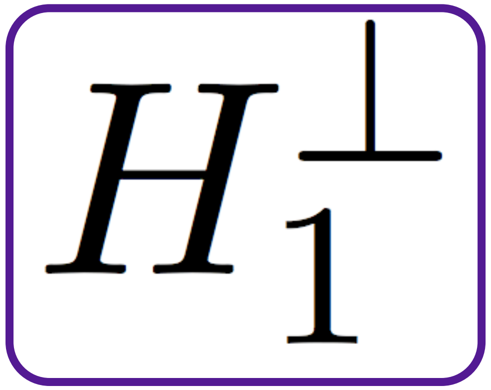


h_1

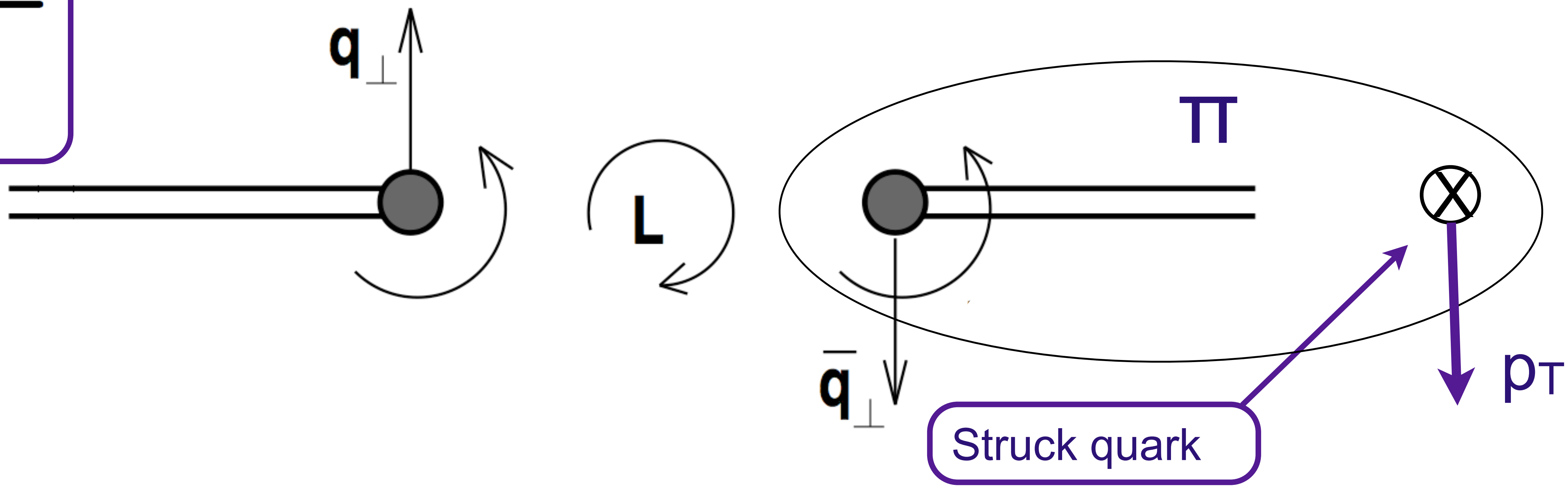
Transversity

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Collins Fragmentation



Artru, Acta Phys. Polon. **B39**(1998)2115



- Imagine that the $q\bar{q}$ pair is created in a $^3P_0$ spin state with vacuum quantum numbers $J^{PC}=0^{++}$
- Quark spins are opposite the orbital ang. mom. L=1
- Pion (with no spin) acquires transverse momentum
- This simple model breaks down if the fragmentation string does not conserve J (i.e. if there are torques)

Function Zoo

Leading Twist TMDs

N/q	U	L	T
U	$\mathbf{f_1}$		$h_1^\perp$
L		$\mathbf{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\mathbf{h_1}$ $h_{1T}^\perp$

Sub-Leading Twist TMDs

N/q	U	L	T
U	$f^\perp$	$g^\perp$	$h, \mathbf{e}$
L	$f_L^\perp$	$g_L^\perp$	$\mathbf{h_L}, e_L$
T	$f_T, f_T^\perp$	$\mathbf{g_T}, g_T^\perp$	$h_T, e_T, h_T^\perp, e_T^\perp$

Leading Twist FFs

q/h	U	L	T
U	$\mathbf{D_1}$		$D_{1T}^\perp$
L		$\mathbf{G_{1L}}$	$G_{1T}^\perp$
T	$H_1^\perp$	$H_{1L}^\perp$	$\mathbf{H_1}$ $H_{1T}^\perp$

Sub-Leading Twist FFs

q/h	U	L	T
U	$D^\perp$	$D_L^\perp$	$D_T, D_T^\perp$
L	$G^\perp$	$\mathbf{G_L^\perp}$	$\mathbf{G_T}, G_T^\perp$
T	$H, \mathbf{E}$	$\mathbf{H_L}, E_L$	$H_T, E_T, H_T^\perp, E_T^\perp$

Complementarity of pp, ep, ee

Drell-Yan

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, f_1^q(x, p_T^2) \otimes f_1^{\bar{q}}(\bar{x}, \bar{p}_T^2)$$

$q_T = p_T + \bar{p}_T$

SIDIS

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)$$

$q_T = z p_T + k_T$

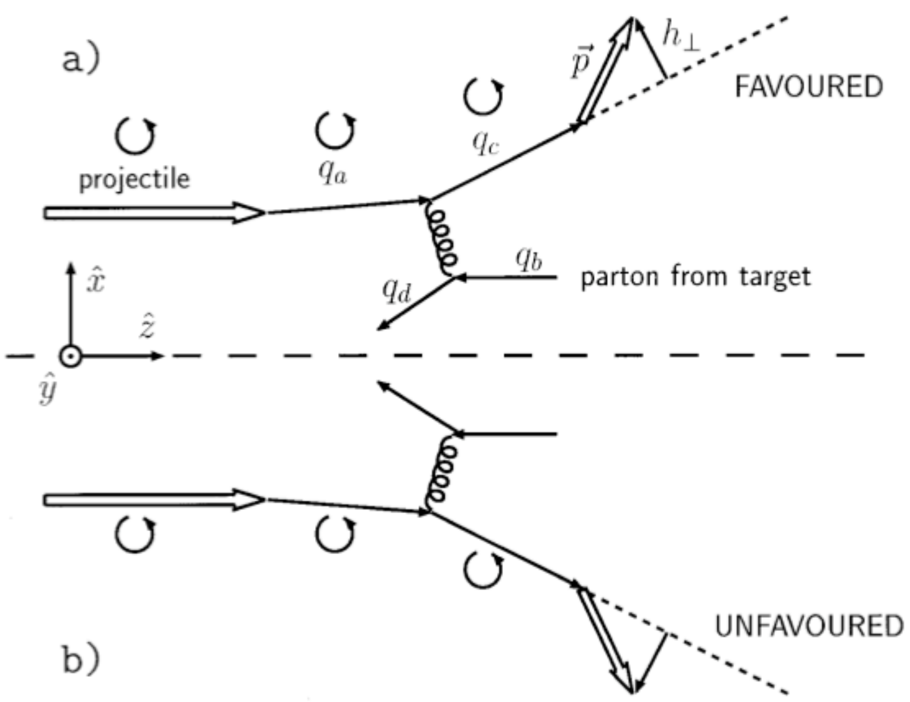
e^+e^- Annihilation

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, D_1^q(z, k_T^2) \otimes D_1^{\bar{q}}(\bar{z}, \bar{k}_T^2)$$

$q_T = k_T + \bar{k}_T$

Polarized pp Asymmetries

Artru, ZP**C73**(1997)527



$$H_1^\perp$$

Fig. 1. Inclusive pion production. Two events (**a**) and (**b**), symmetric with respect to the $\hat{y}\hat{z}$ plane, are represented. Without polarization, they would have the same probability. In the polarized case, the Collins effect favours the case (**a**). The arrows labelled q_i represent the momenta of the quarks in the subprocess. The spins are denoted by the arch-like arrows. The Collins effect acts at the last stage, where the quark q_c fragments into the pion carrying momentum **p**. $h_\perp$ is the pion's transverse momentum with respect to the quark q_c

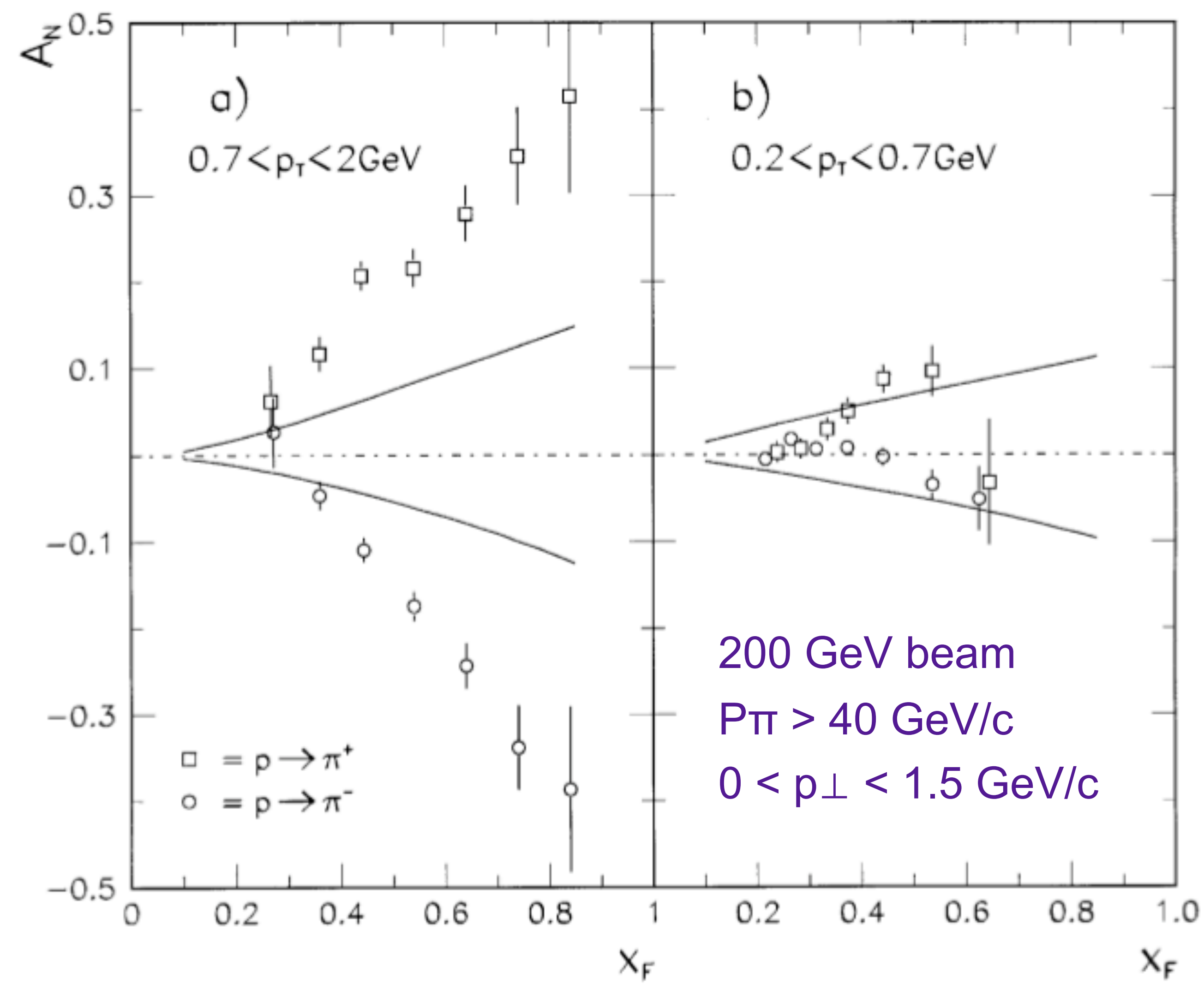


Fig. 5. Single spin asymmetry measured by E704 collaboration for charged pions at $0.2 < p_\perp < 2.0$ GeV [6]. The curves are our model results calculated with quark transverse polarizations $\Delta_\perp u/u = -\Delta_\perp d/d = x^2$ and $\beta = 1$

Bravar, PRL**77**(1996)2626 FNAL E704

$$\bar{p}\uparrow + p \to \pi^-(\pi^+) + X$$

$$A_N = \frac{1}{P_B\langle\cos\phi\rangle}\frac{N\uparrow - N\downarrow}{N\uparrow + N\downarrow}$$

ϕ is angle between beam polarization axis and the normal to the production plane

Collins effect gives the right trend to explain the large asymmetries seen

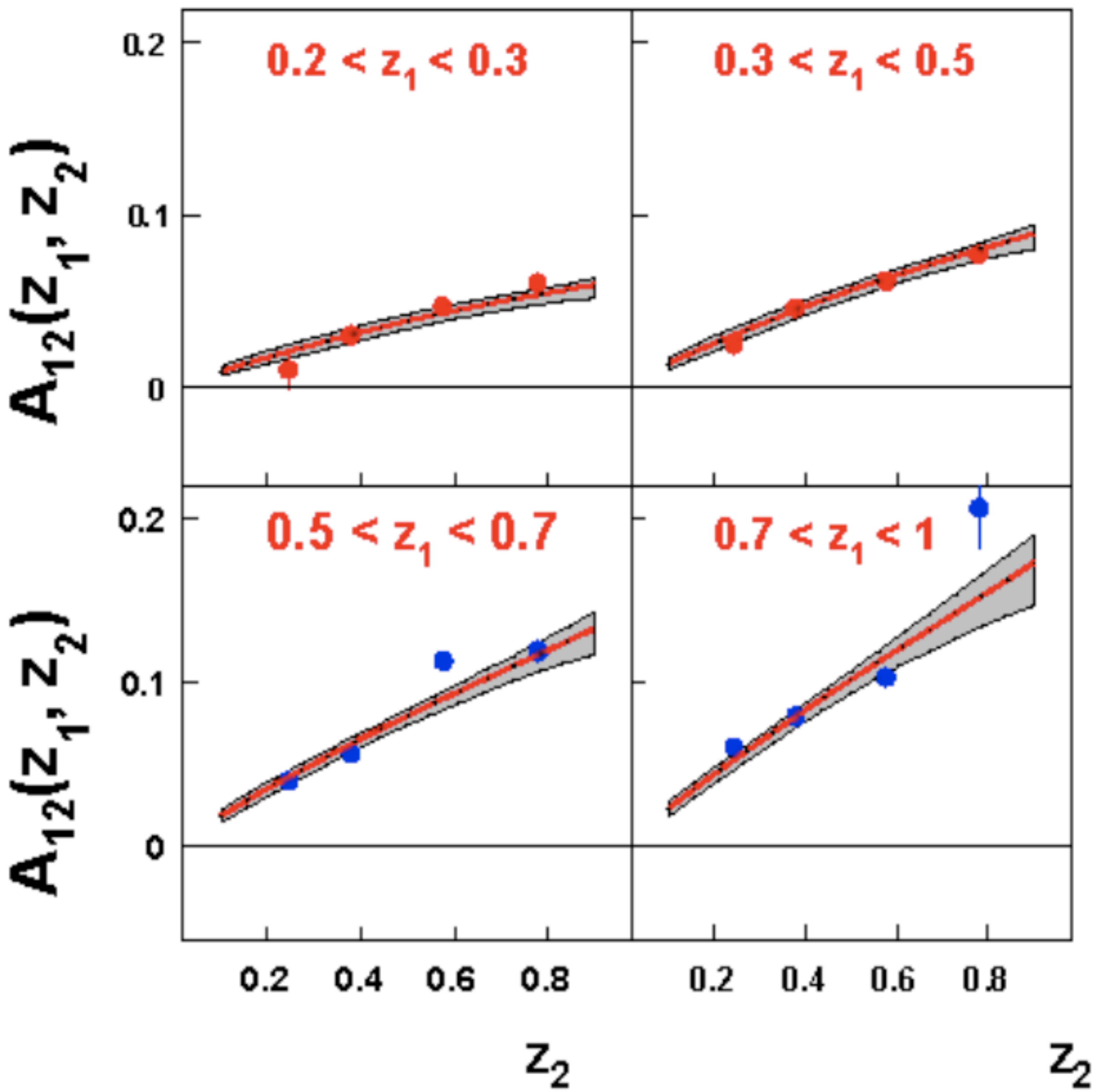
Collins Fragmentation

Seidl, PR**D78**(08)032011 (Belle)

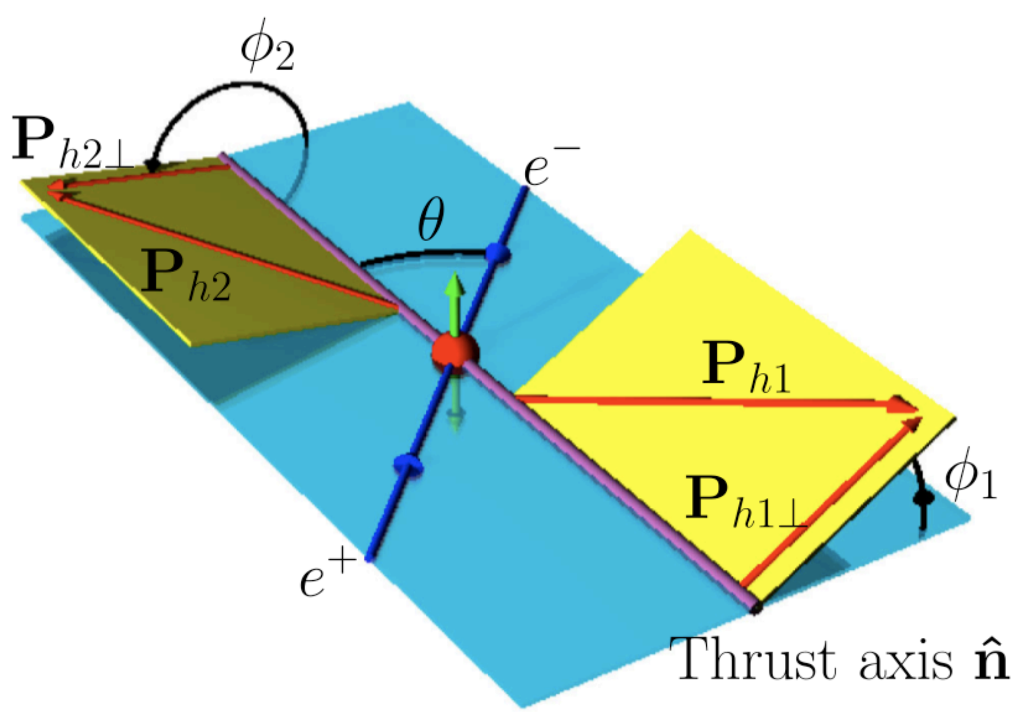
$$N(\phi_1 + \phi_2) \sim a_{12} cos(\phi_1 + \phi_2),\ a_{12} \sim H_1^{\perp}(z_1) H_1^{\perp}(z_2)$$

$$A_{12} = a_{12}^{\pi^+,\pi^-} / a_{12}^{(\pi^+,\pi^+),(\pi^-,\pi^-),(\pi^+,\pi^-)}$$

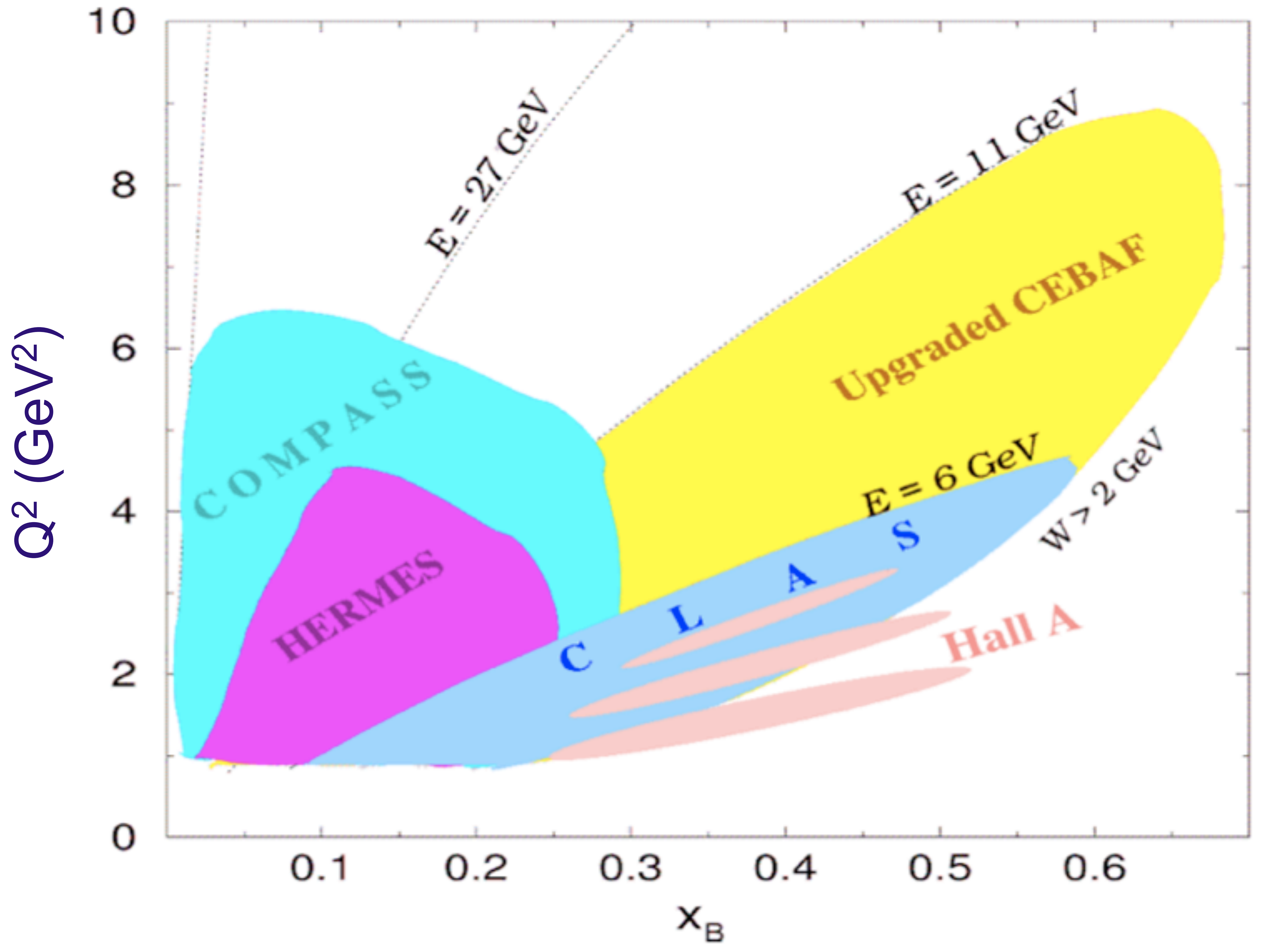
Anselmino, AIPCP**1149**(09)465 [Fits]



- $e^+e^- \rightarrow \pi\pi$
- A_{12}: Ratio cancels QCD radiative and acceptance effects
- CM energy ~10.5 GeV; L=550 fb^{-1}



Kinematic Coverage



Transversity+Collins

$$\mathcal{N}_q^T(x) = N_q^T\, x^\alpha (1-x)^\beta \, \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

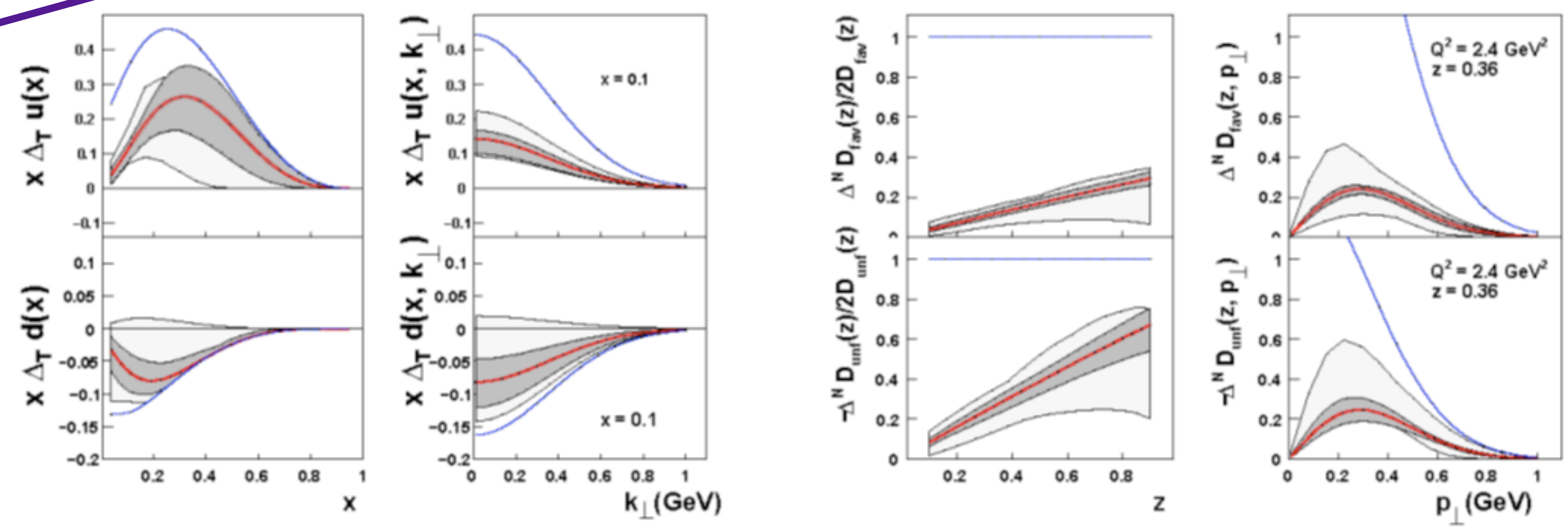
$$\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \, \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta} \; ; \; h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M_h}\, e^{-p_\perp^2/M_h^2}$$

$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\,[f_{q/p}(x) + \Delta q(x)]\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi \langle k_\perp^2\rangle_T} ,$$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, D_{h/q}(z)\, h(p_\perp)\, \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi \langle p_\perp^2\rangle} ,$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot \boldsymbol{k}_T}{M_h} h_1 H_1^\perp \right]$$

Notation is not yet standard!

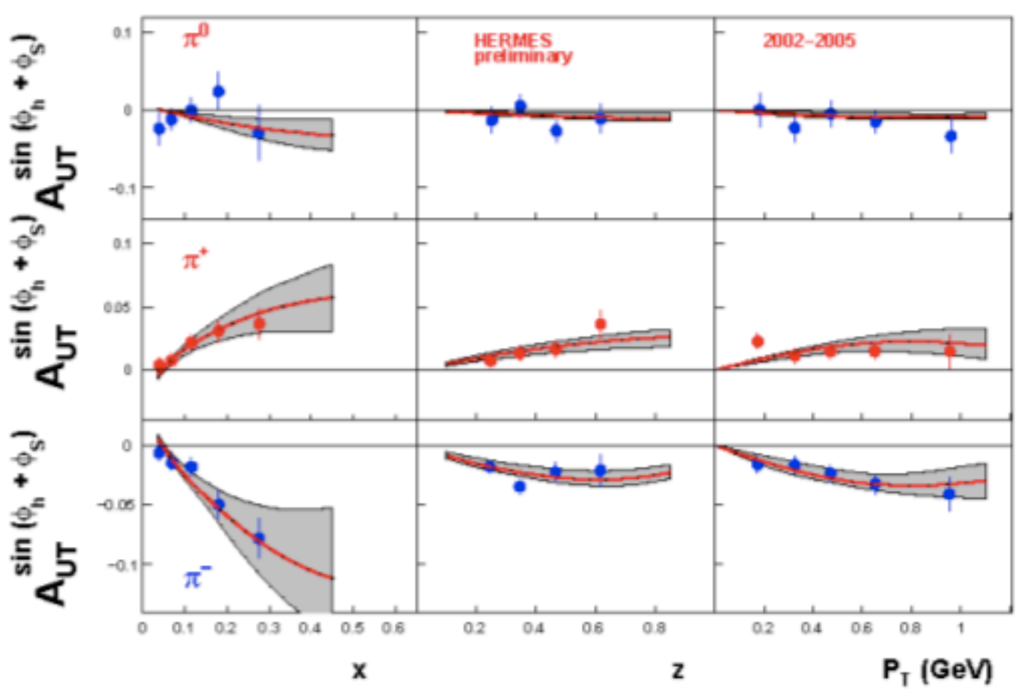


Anselmino, AIPCP**1149**(09)465 [Fits]

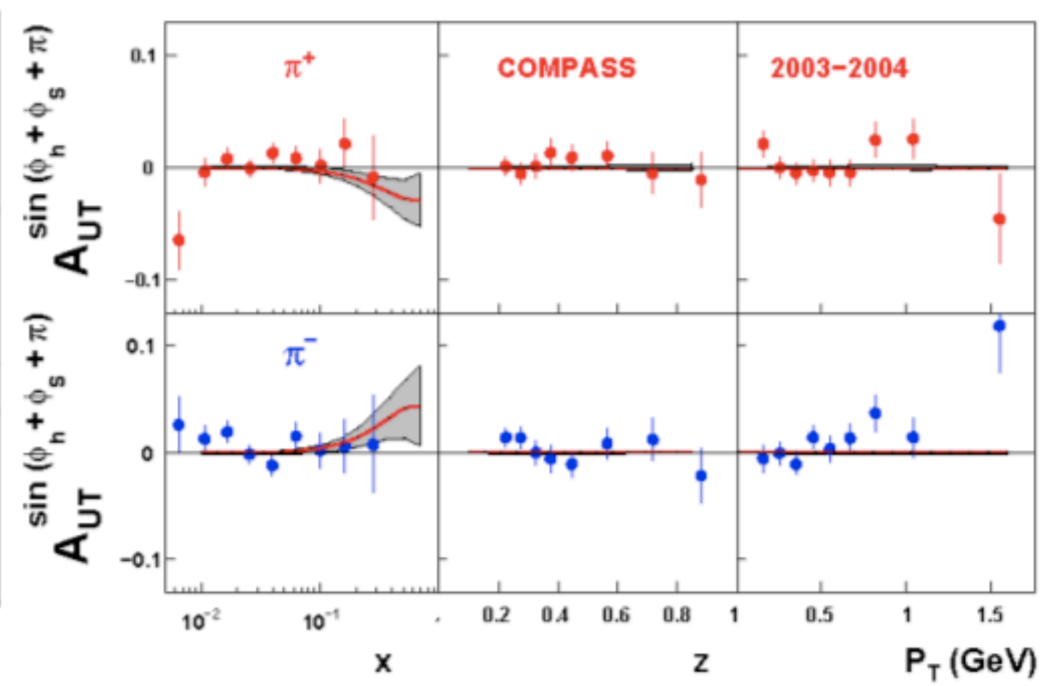
Diefenthaler, arXiv:0706.2242 [HERMES

Alekseev, arXiv:0802.2160 [COMPASS]

proton



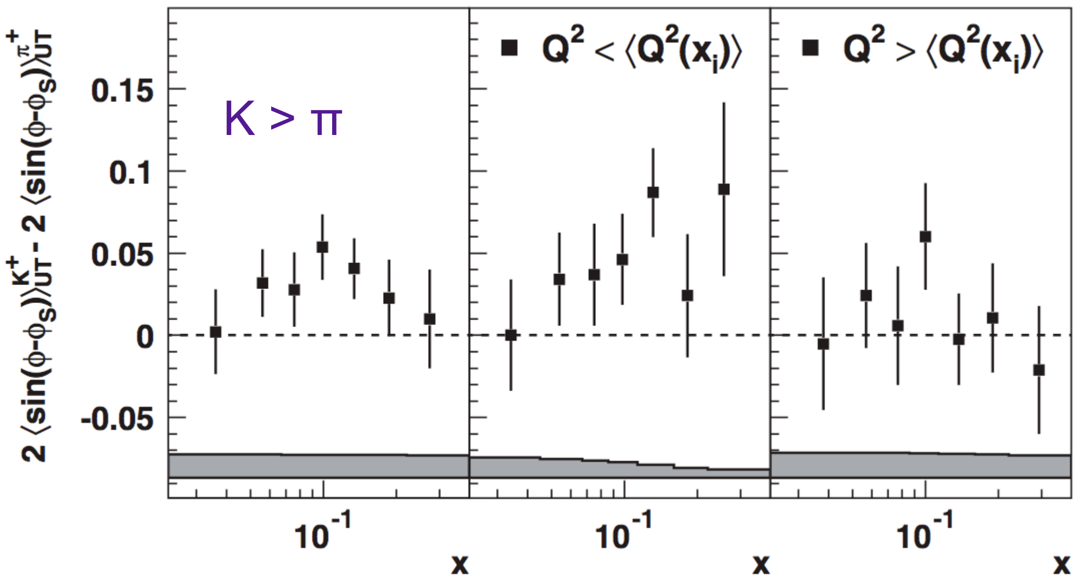
deuteron

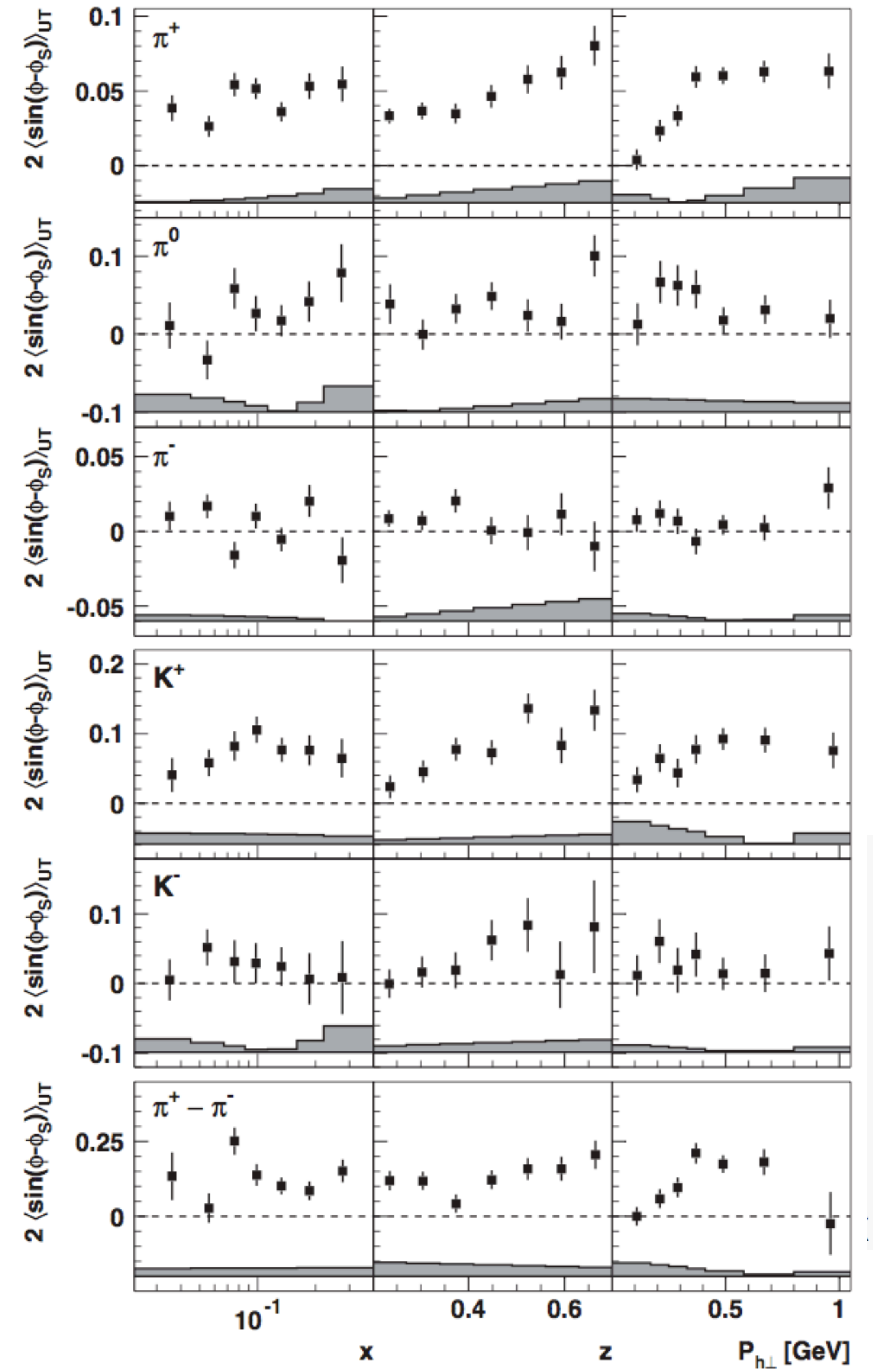


f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	$h_1,\, h_{1T}^\perp$

Sivers

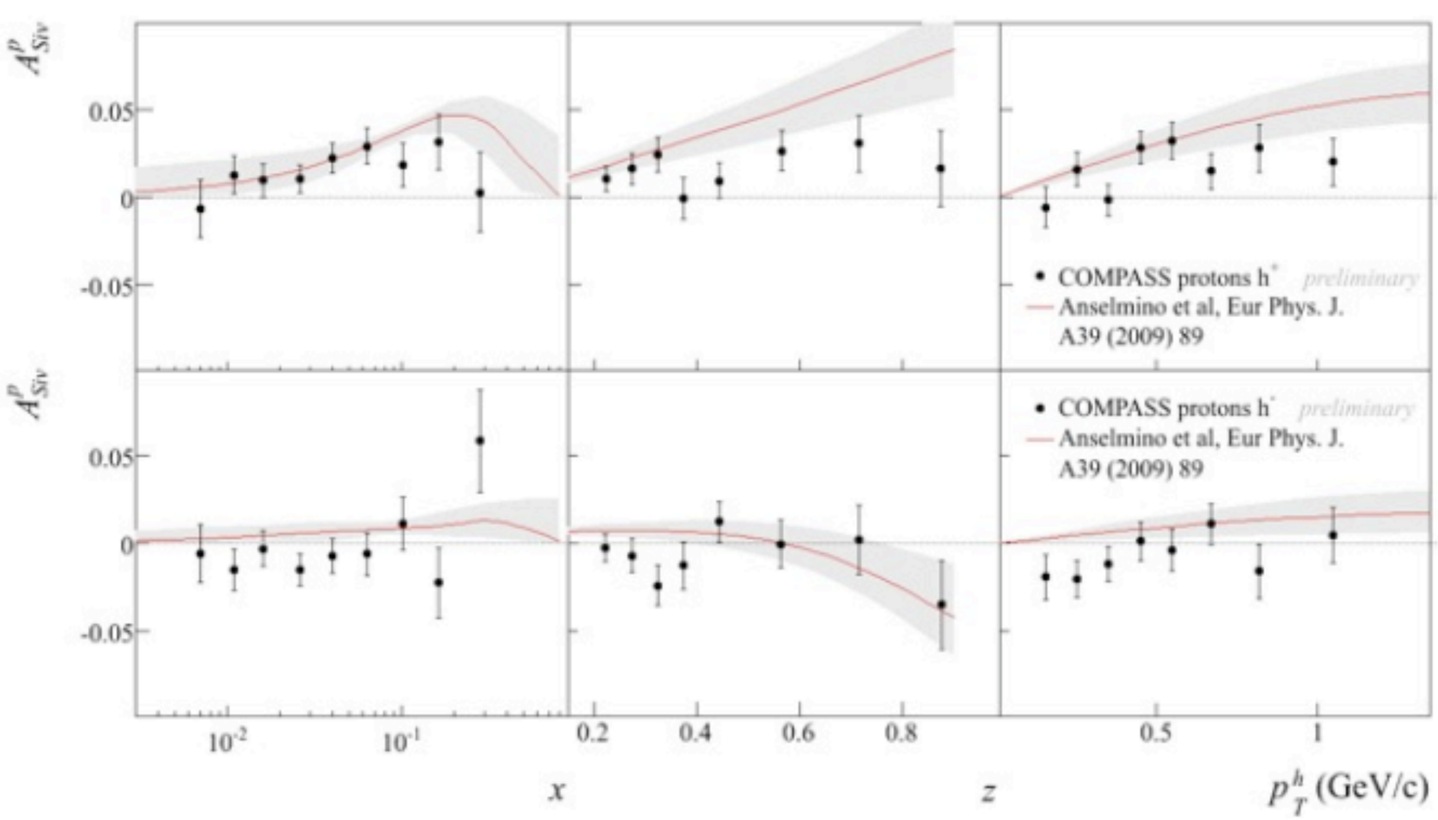
Airapetian, PRL103(09)152002





$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right]$$

f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

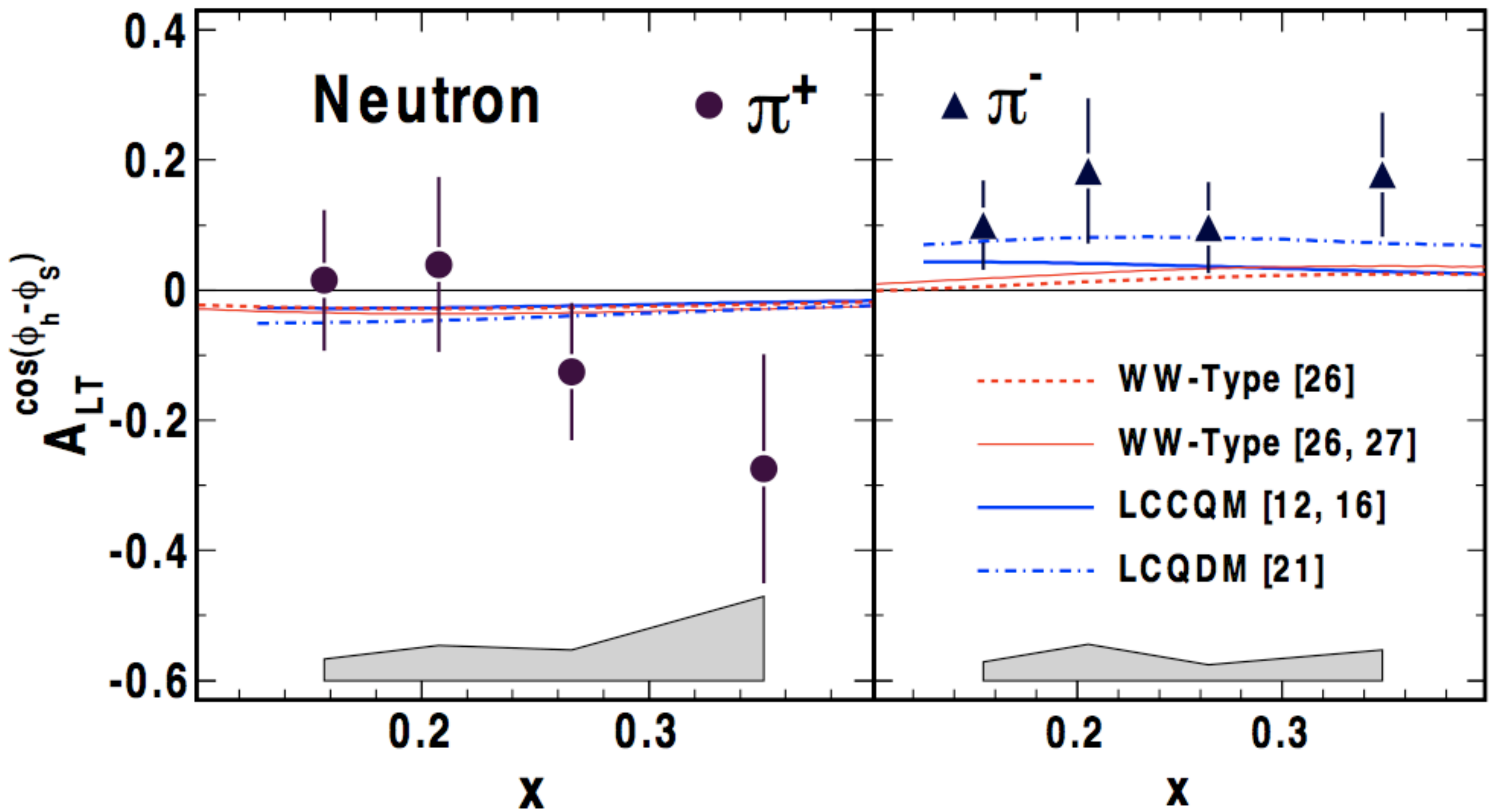


Worm-Gear TMD

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right]$$

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Jin Huang, Hall A, Jefferson Lab, arXiv1108.0489

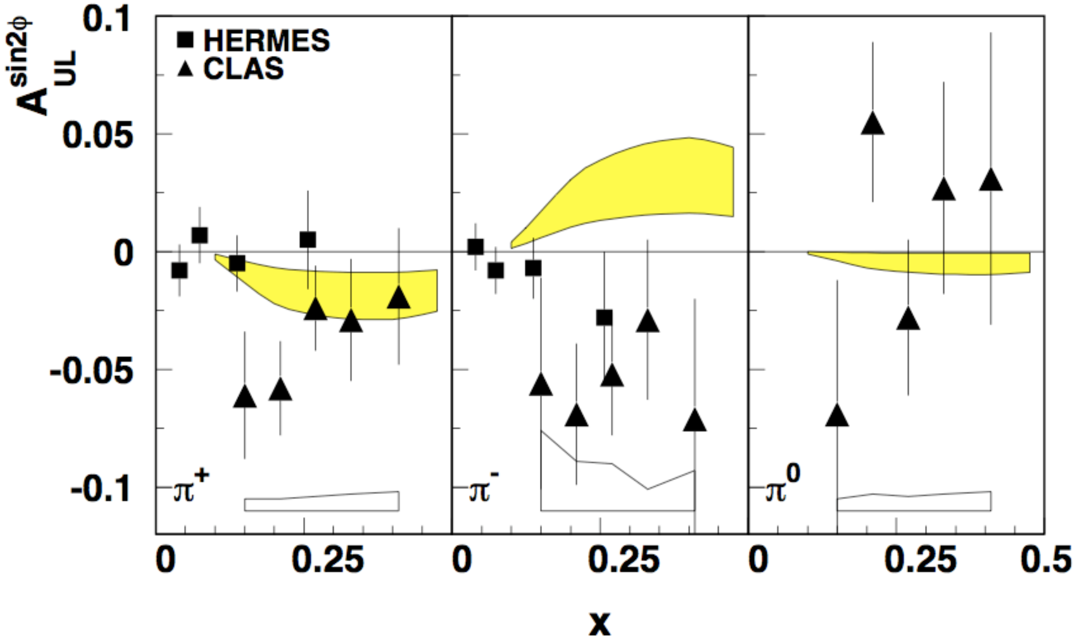


Worm-Gear TMD

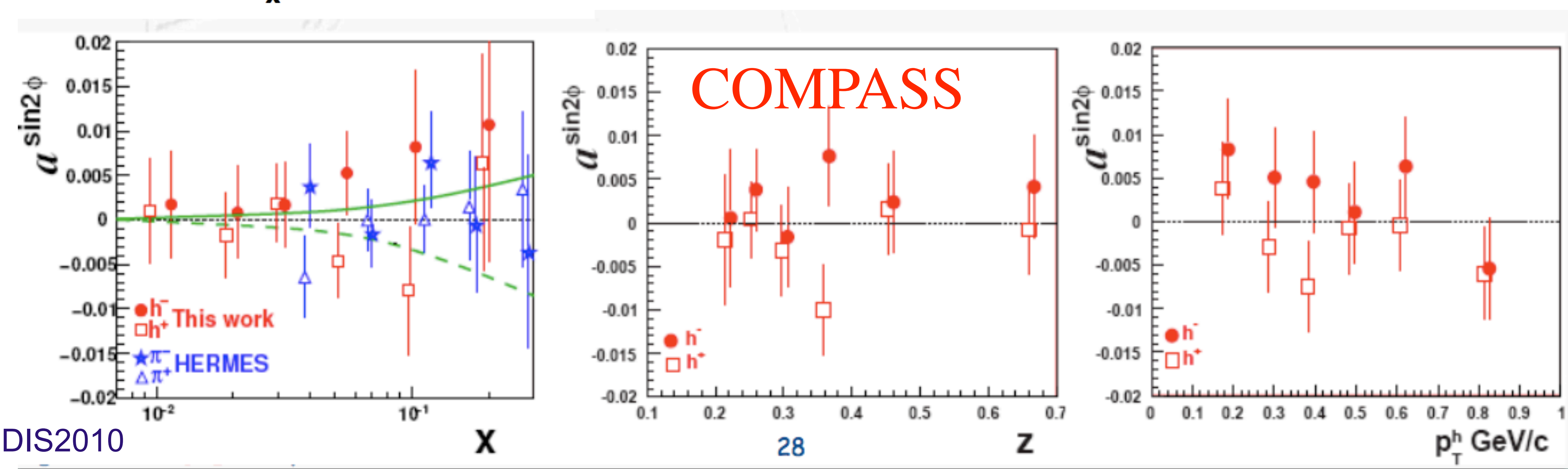
$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$

Worm Gear

Collins



f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$

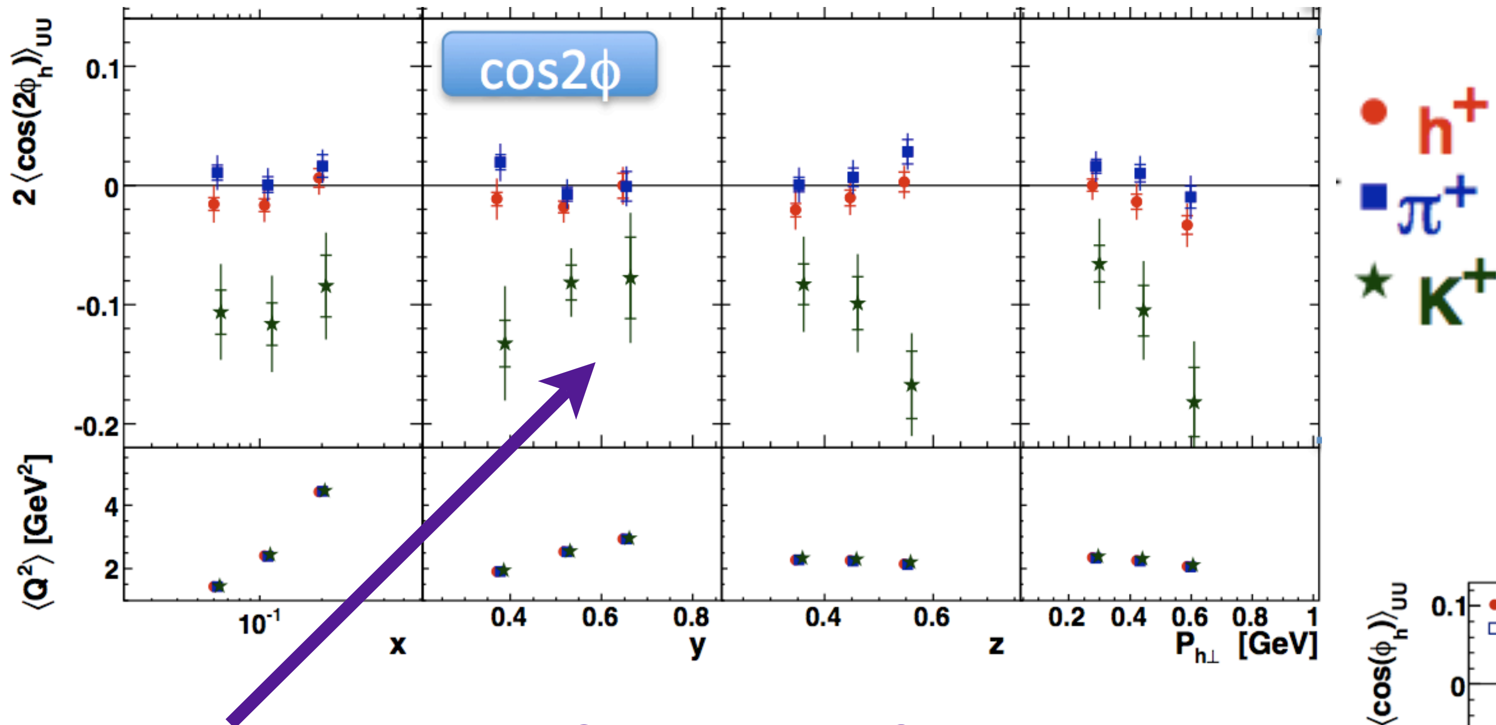


I. Savin, DIS2010

Boer-Mulders

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right]$$

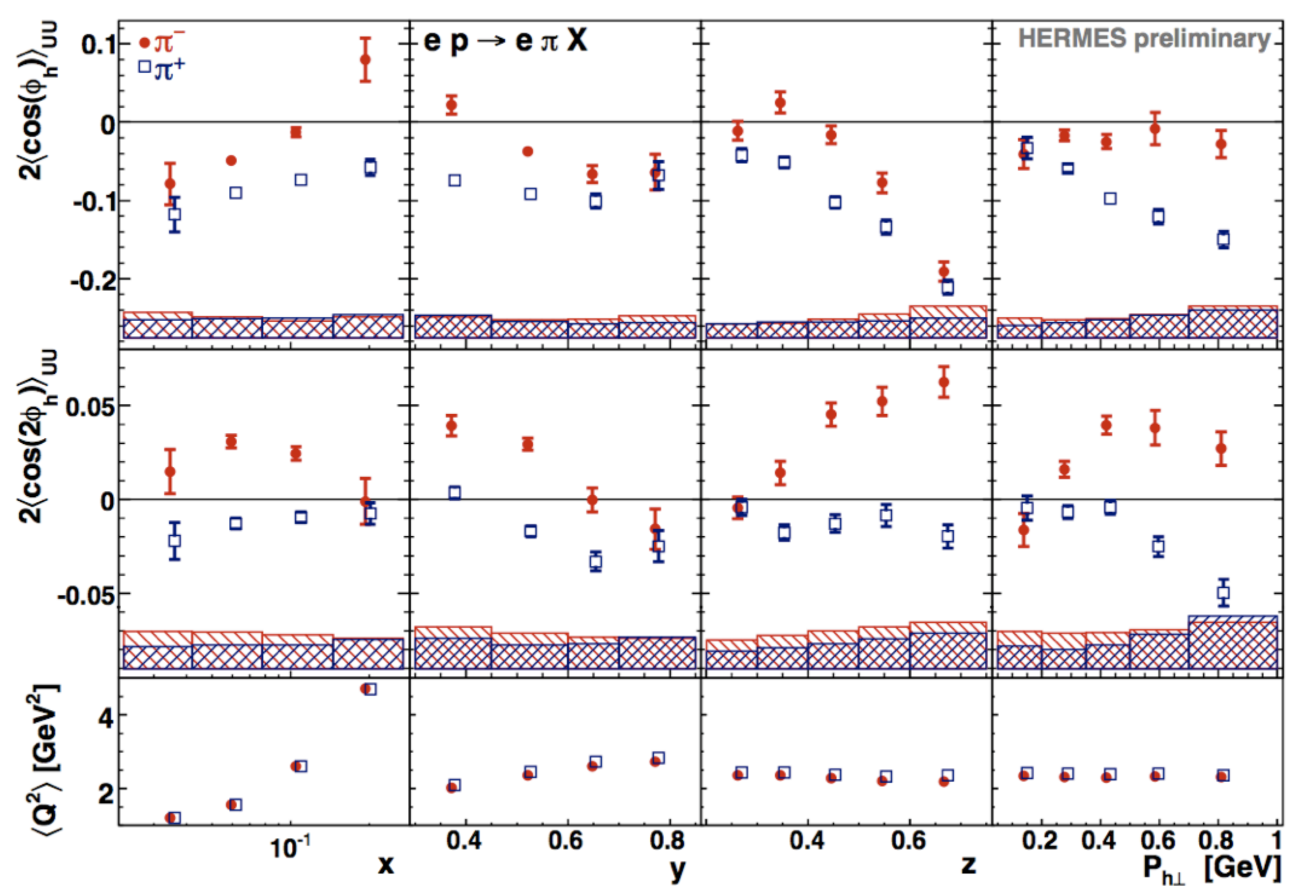
f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$



Kaons are different

Contalbrigo, DIS2011
HERMES

Rith PoS(ICHEP 2010)169

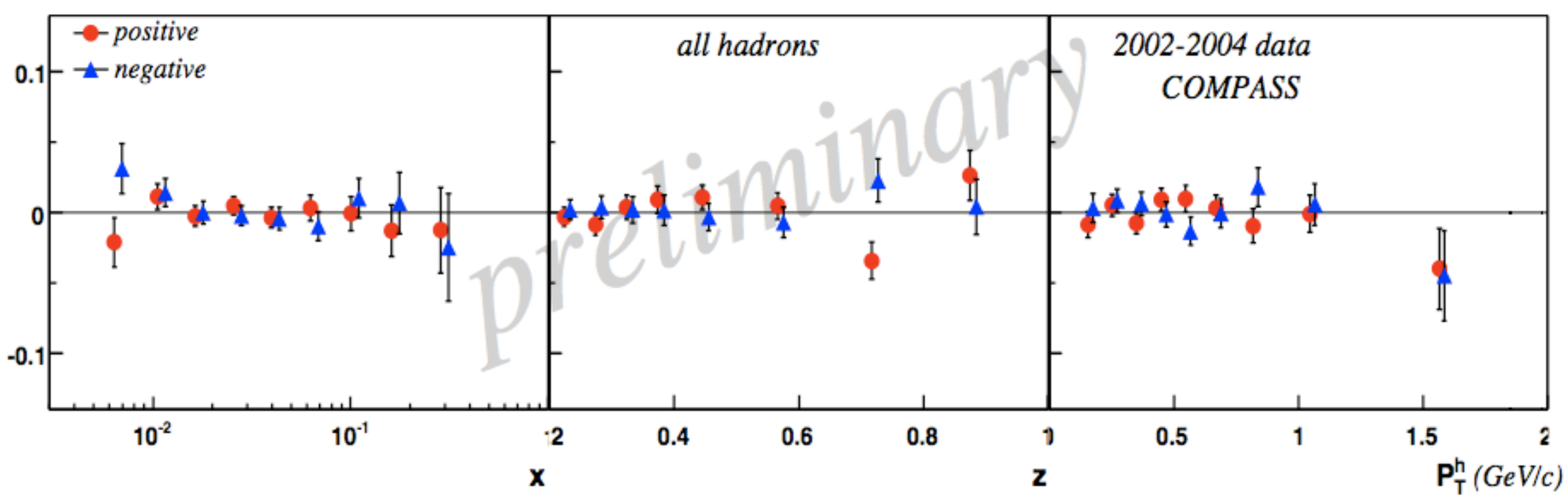


Pretzelocity

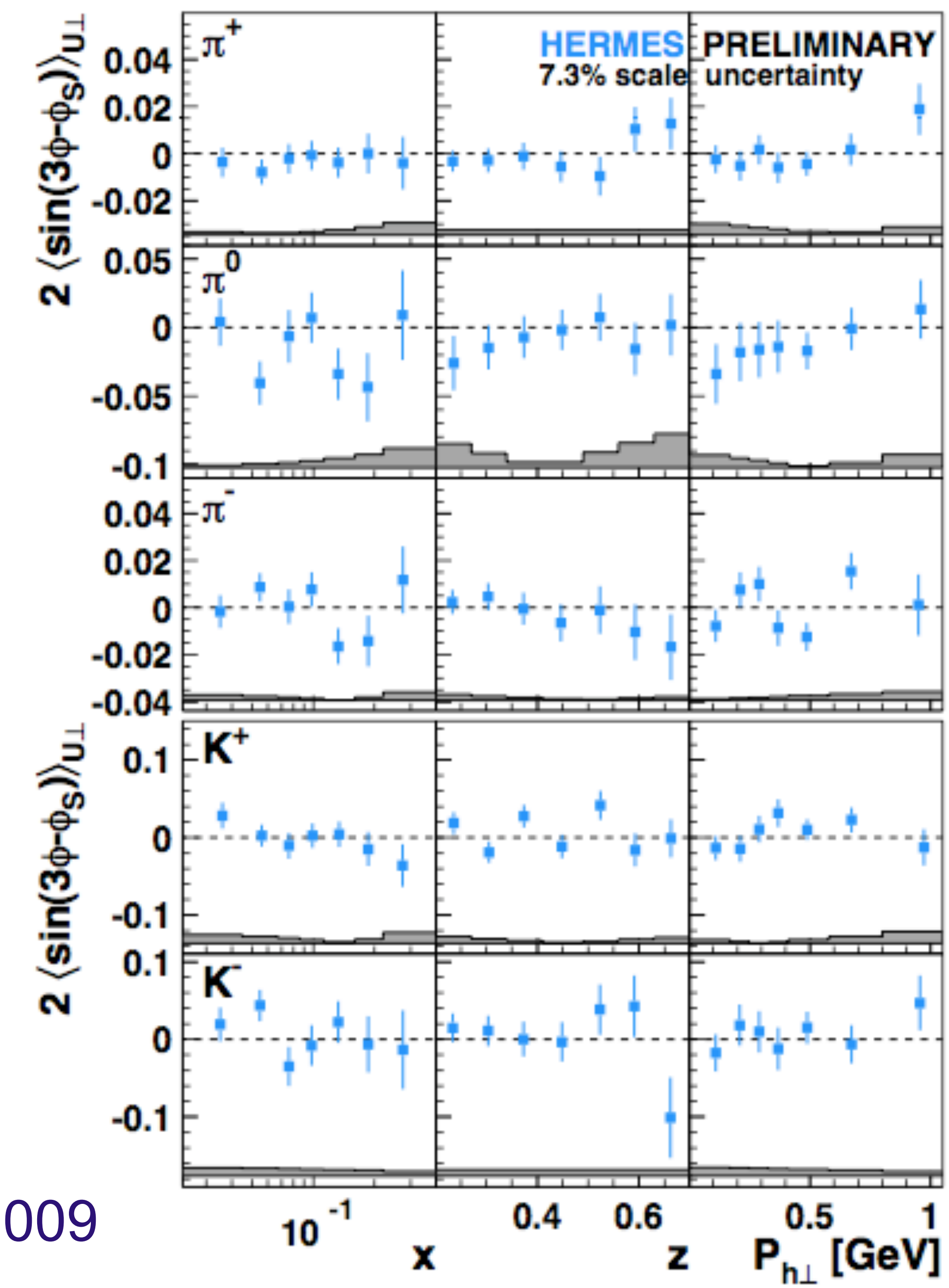
$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h}\, h_{1T}^{\perp}H_1^{\perp}\right]$$

f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$

Asymmetry is consistent with zero



A. Bressan, IWSS10



S. Gliske, APS2009

A1 ≈ g1/F1

$$f_1^q(x,k_T) = f_1(x)\frac{1}{\pi\mu_0^2}\exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x,k_T) = g_1(x)\frac{1}{\pi\mu_2^2}\exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

$$D_1^q(z,p_T) = D_1(z)\frac{1}{\pi\mu_D^2}\exp\left(-\frac{p_T^2}{\mu_D^2}\right),$$

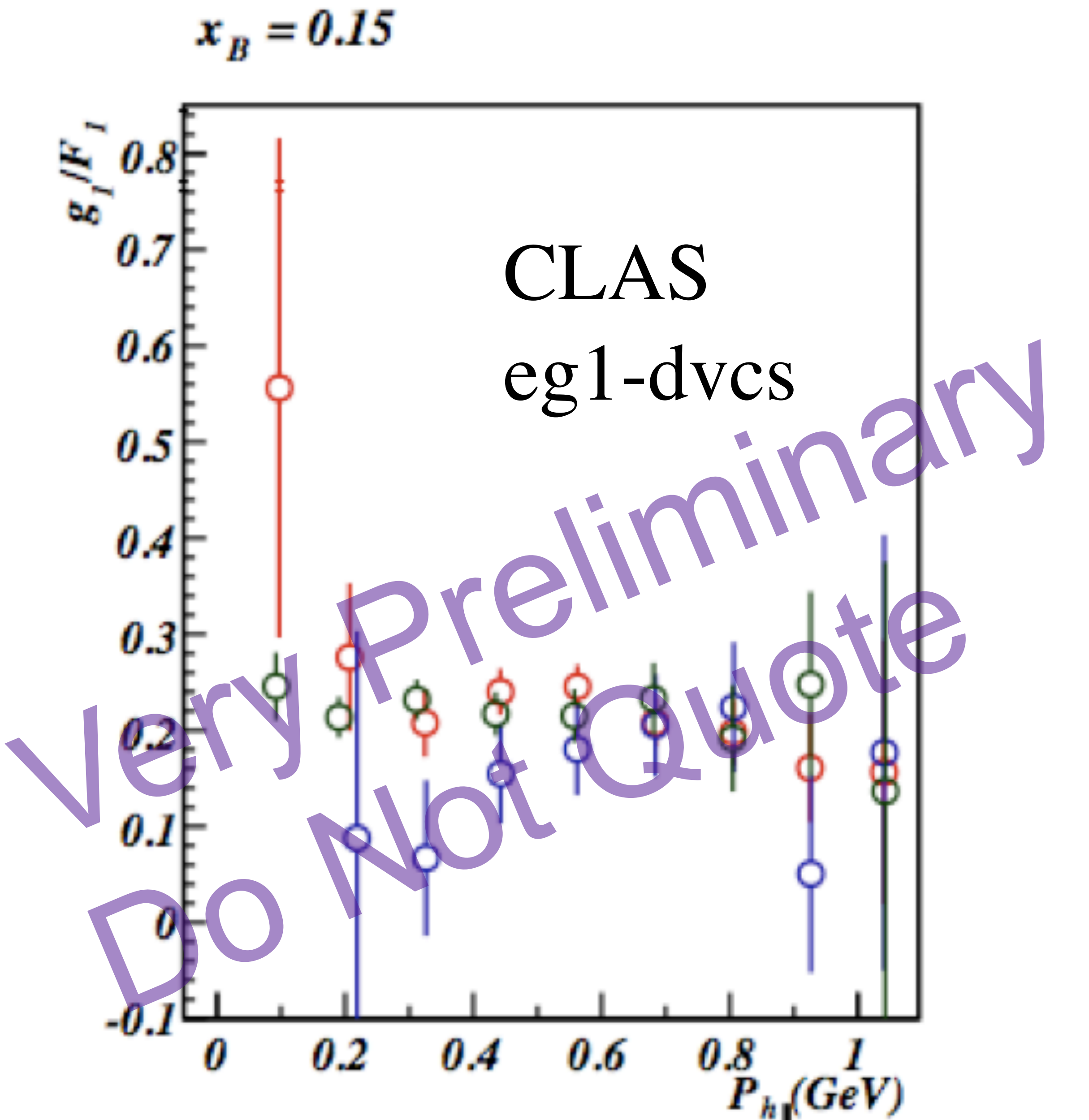
$$\frac{g_1}{F_1} \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q\to\pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q\to\pi}(z)} e^{-z^2 P_T^2 \frac{(\mu_0^2-\mu_2^2)}{(\mu_D^2+z^2\mu_0^2)(\mu_D^2+z^2\mu_2^2)}}$$

eg1-dvcs data

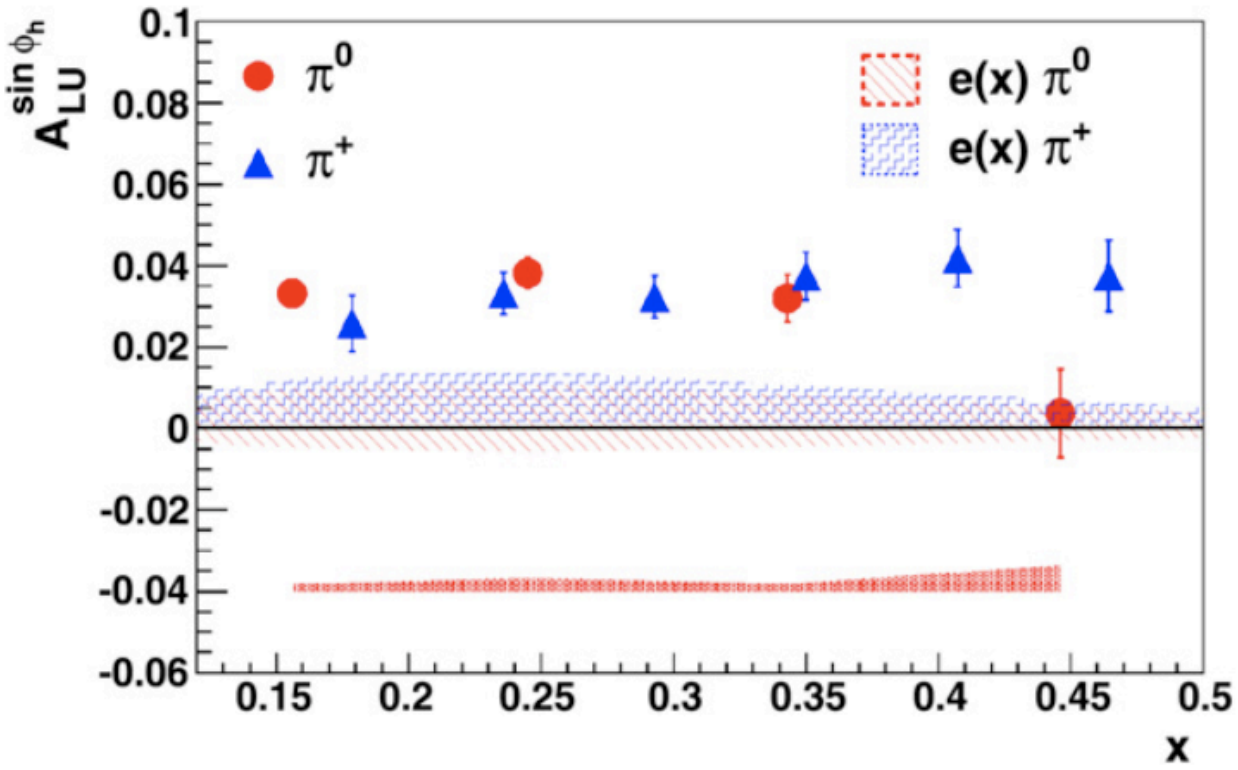
P_T dependence → $\mu_0 \neq \mu_2$

- π^+ (red)
- π^- (blue)
- π^0 (green)

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

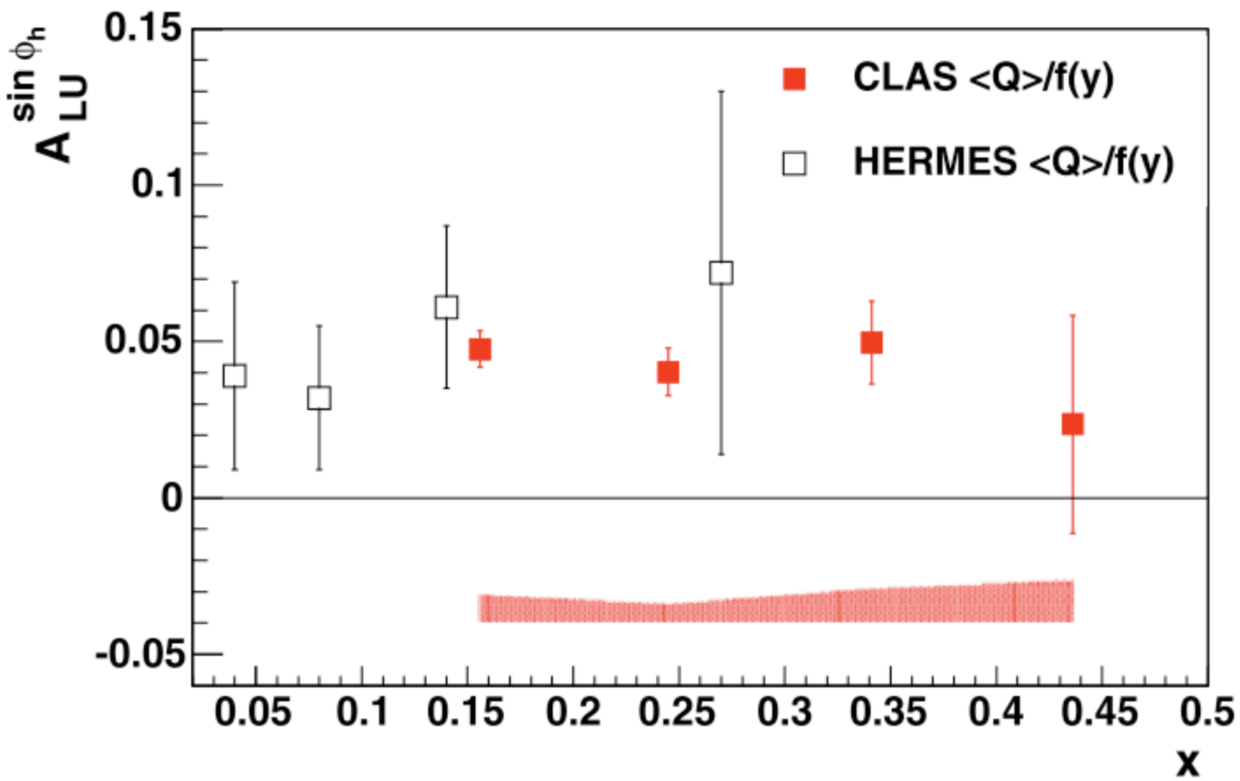


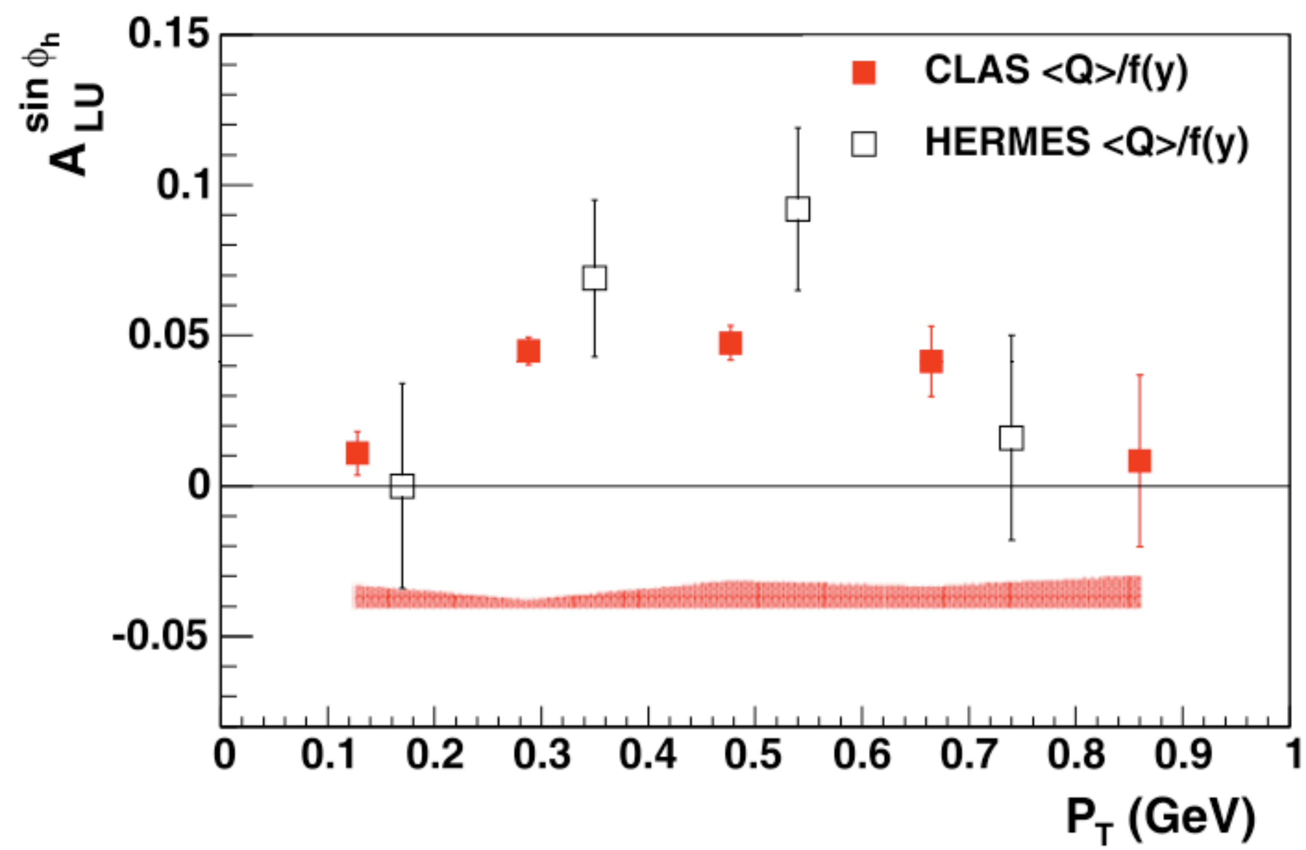
Sub-Leading Twist



- Aghasyan, PLB704(11)397 (CLAS)
  Airapetian, PLB648(07)164 (HERMES)
  Avagyan, SpinPhysProc(03)239 (π^+)
- $A_{UL}^{\pi^\circ(\pi+)}$ for an unpolarized H_2 target
- Beam: 5.8 (CLAS) & 28 GeV (HERMES)
- $Q^2>1$; $0.4 < z < 0.7$
- Similar results at different Q^2s

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{E}}{z}\right)\right]$$





Cahn and More

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z}\right)\right]$$

N/q	U	L	T
U	$\mathbf{f_1}$		$h_1^\perp$
L		$\mathbf{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\mathbf{h_1}$ $h_{1T}^\perp$

q/h	U	L	T
U	$\mathbf{D_1}$		$D_{1T}^\perp$
L		$\mathbf{G_{1L}}$	$G_{1T}^\perp$
T	$H_1^\perp$	$H_{1L}^\perp$	$\mathbf{H_1}$ $H_{1T}^\perp$

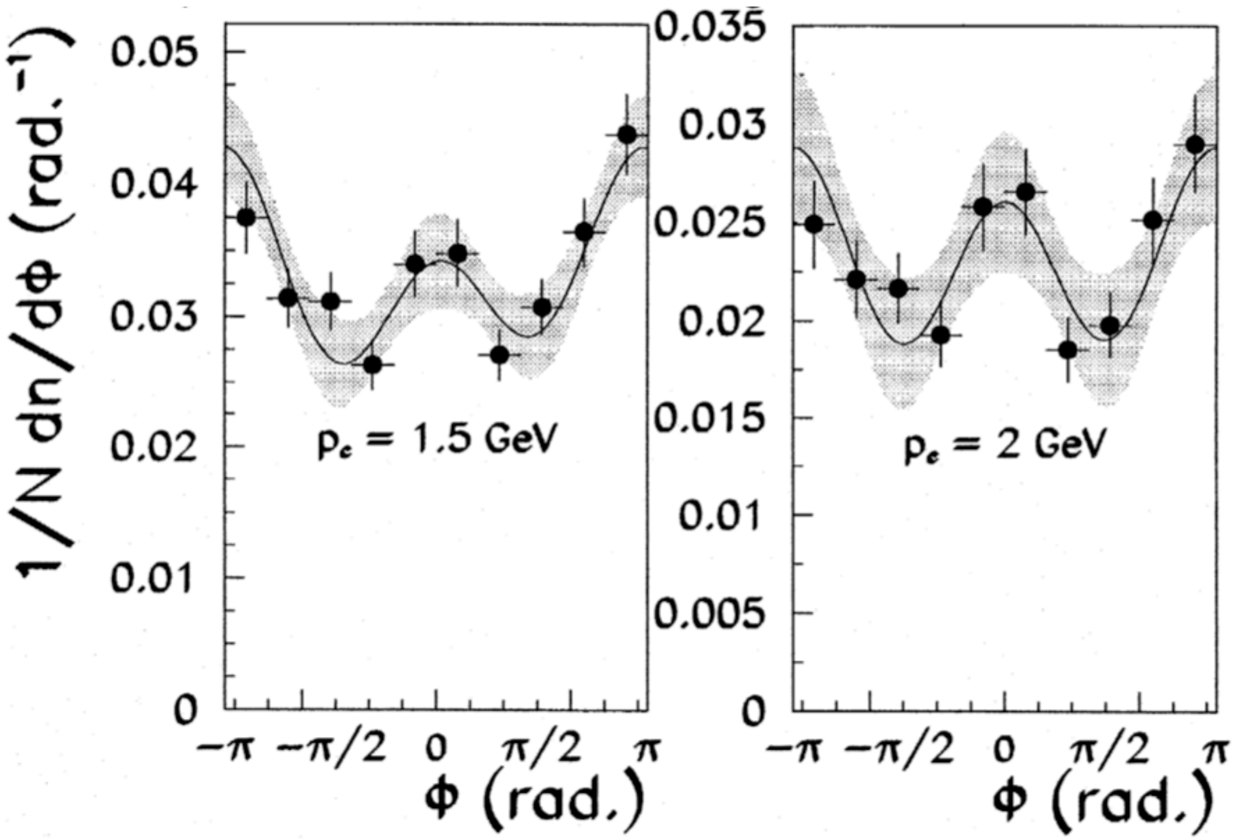
N/q	U	L	T
U	$f^\perp$	$g^\perp$	h, $\mathbf{e}$
L	$f_L^\perp$	$g_L^\perp$	$\mathbf{h_L}$, e_L
T	f_T, $f_T^\perp$	$\mathbf{g_T}$, $g_T^\perp$	h_T, e_T, $h_T^\perp$, $e_T^\perp$

q/h	U	L	T
U	$D^\perp$	$D_L^\perp$	D_T, $D_T^\perp$
L	$G^\perp$	$\mathbf{G_L^\perp}$	$\mathbf{G_T}$, $G_T^\perp$
T	H, $\mathbf{E}$	$\mathbf{H_L}$, E_L	H_T, E_T, $H_T^\perp$, $E_T^\perp$

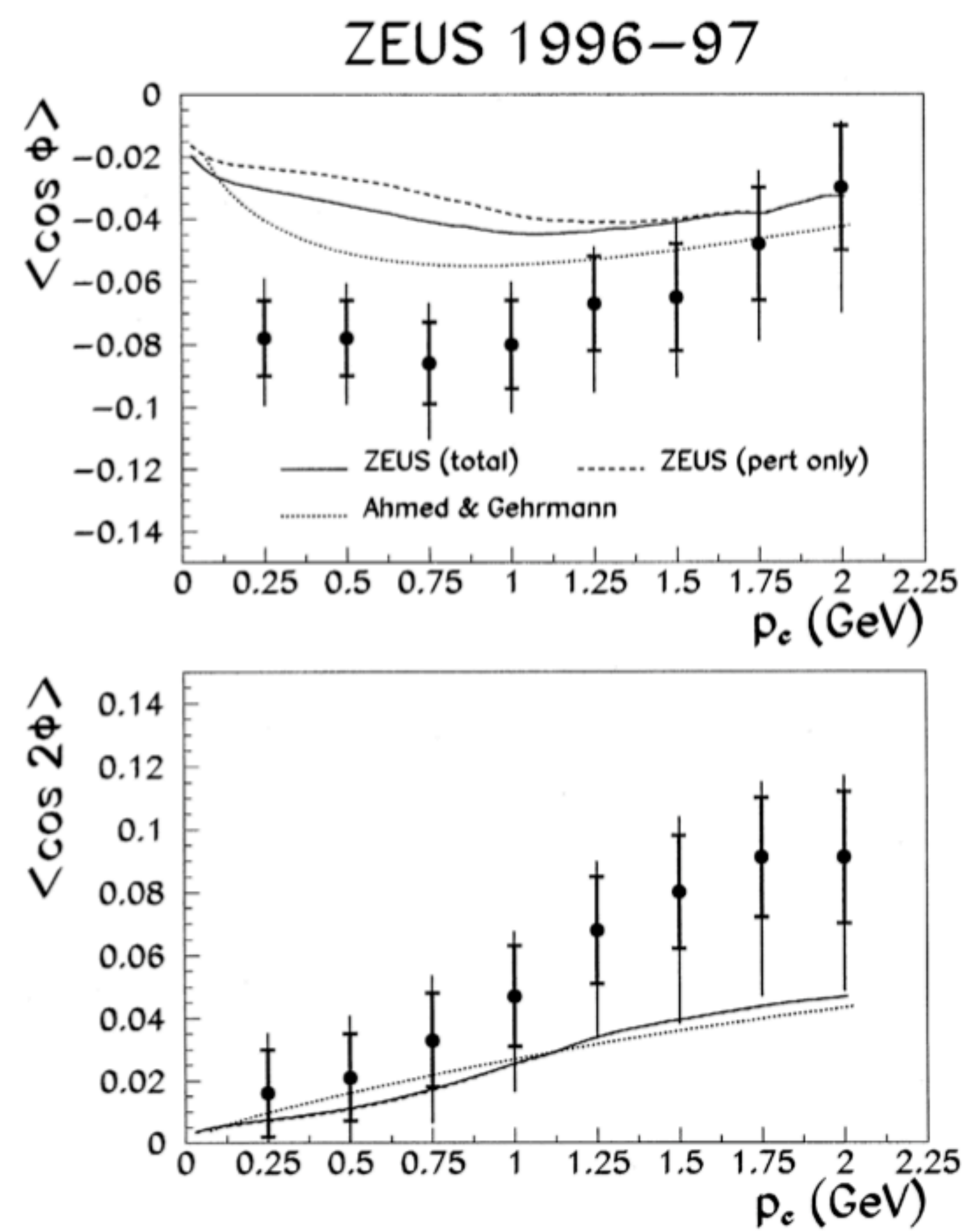
Cahn Effect

Helicity

Unpolarized SIDIS cosφ

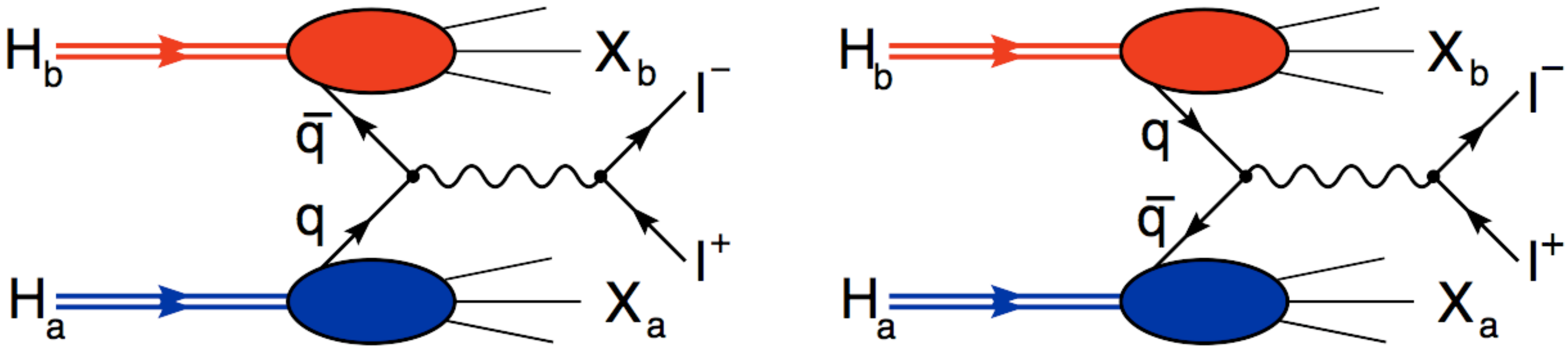


Breitweg, PL**B481**(2000)199 (ZEUS)



- F_{UU} for charged-hadron SIDIS
- $\langle x \rangle = 0.022$; $\langle Q^2 \rangle = 750$ GeV^2; $0.2<y<0.8$; $0.2<z<1$
- Data fit well to a + b cosφ + c cos2φ
- Both <cosφ> and <cos2φ> ≠ 0 at high Q^2
- <cosφ> at leading-order comes from QCD Compton scattering (γ*q →qg)
- <cos2φ> at leading-order comes from photon-gluon fusion (γ*g → qq)
- Non-pert. <cosφ> from LT interference

Drell-Yan



P_1, P_2	4 momenta of hadrons
$q = Q$	4 momenta of virtual gamma quanta
$Q^2 \equiv M^2_{l^+l^-}$	squared invariant mass of the lepton pair
$s = (P_1 + P_2)^2 \simeq 2P_1P_2$	squared energy of colliding hadrons in the center of mass system
$x_1 = \frac{Q^2}{2P_1q}, \quad x_2 = \frac{Q^2}{2P_2q}$	Bjorken variables of colliding hadrons
$y = \frac{1}{2}\ln\frac{x_1}{x_2}$	rapidity
$x_F = x_1 - x_2$	Feynman variable
$x_{1,2} = \frac{\sqrt{x_F^2 + 4\tau} \pm x_F}{2} = \sqrt{\tau}e^{\pm y}$	relation between $x_{1,2}$, x_F and y

Drell-Yan

Arnold, PR**D79**(09)034005

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2}{Fq^2}\{&((1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta)F_{UU}^2 + \sin2\theta\cos\phi F_{UU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UU}^{\cos2\phi})\\
&+ S_{aL}(\sin2\theta\sin\phi F_{LU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LU}^{\sin2\phi}) + S_{bL}(\sin2\theta\sin\phi F_{UL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UL}^{\sin2\phi})\\
&+ |\vec{S}_{aT}|[\sin\phi_a((1+\cos^2\theta)F_{TU}^1 + (1-\cos^2\theta)F_{TU}^2 + \sin2\theta\cos\phi F_{TU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TU}^{\cos2\phi})\\
&+ \cos\phi_a(\sin2\theta\sin\phi F_{TU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TU}^{\sin2\phi})] + |\vec{S}_{bT}|[\sin\phi_b((1+\cos^2\theta)F_{UT}^1 + (1-\cos^2\theta)F_{UT}^2\\
&+ \sin2\theta\cos\phi F_{UT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UT}^{\cos2\phi}) + \cos\phi_b(\sin2\theta\sin\phi F_{UT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UT}^{\sin2\phi})]\\
&+ S_{aL}S_{bL}((1+\cos^2\theta)F_{LL}^1 + (1-\cos^2\theta)F_{LL}^2 + \sin2\theta\cos\phi F_{LL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LL}^{\cos2\phi})\\
&+ S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1+\cos^2\theta)F_{LT}^1 + (1-\cos^2\theta)F_{LT}^2 + \sin2\theta\cos\phi F_{LT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LT}^{\cos2\phi})\\
&+ \sin\phi_b(\sin2\theta\sin\phi F_{LT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LT}^{\sin2\phi})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)F_{TL}^1 + (1-\cos^2\theta)F_{TL}^2\\
&+ \sin2\theta\cos\phi F_{TL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TL}^{\cos2\phi}) + \sin\phi_a(\sin2\theta\sin\phi F_{TL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TL}^{\sin2\phi})]\\
&+ |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a+\phi_b)((1+\cos^2\theta)F_{TT}^1 + (1-\cos^2\theta)F_{TT}^2 + \sin2\theta\cos\phi F_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TT}^{\cos2\phi})\\
&+ \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}_{TT}^1 + (1-\cos^2\theta)\bar{F}_{TT}^2 + \sin2\theta\cos\phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi \bar{F}_{TT}^{\cos2\phi})\\
&+ \sin(\phi_a+\phi_b)(\sin2\theta\sin\phi F_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TT}^{\sin2\phi})\\
&+ \sin(\phi_a-\phi_b)(\sin2\theta\sin\phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi \bar{F}_{TT}^{\sin2\phi})]\}.
\end{aligned}
$$

What PANDA can measure with a transversely polarized proton target

Drell-Yan

$$F_{UT}^{1} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} f_1 \bar{f}_{1T}^{\perp}\right]$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} h_1^{\perp} \bar{h}_1\right]$$

$$f_{1T}^{\perp q}(x,k_T)\big|_{DY} = -f_{1T}^{\perp}(x,k_T)\big|_{SIDIS}$$

$$h_1^{\perp}(x,k_T)\big|_{DY} = -h_1^{\perp}(x,k_T)\big|_{SIDIS}$$

$$F_{UT}^{\sin(2\phi+\phi_b)} = -\mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^{2}(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2} h_1^{\perp}\bar{h}_{1T}^{\perp}\right]$$

$$F_{UT}^{\sin(2\phi-\phi_b)} \equiv -\frac{1}{2}(F_{UT}^{\cos 2\phi} - F_{UT}^{\sin 2\phi})$$

$$F_{UT}^{\sin(2\phi+\phi_b)} \equiv \frac{1}{2}(F_{UT}^{\cos 2\phi} + F_{UT}^{\sin 2\phi})$$

quark pol. (columns) / nucleon pol. (rows)

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

$$\mathcal{C}[w(\vec{k}_{aT},\vec{k}_{bT})f_1\bar{f}_2] \equiv \frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}d^2\vec{k}_{bT} \times \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})w(\vec{k}_{aT},\vec{k}_{bT}) \times [f_1^q(x_a,\vec{k}_{aT}^2)f_2^{\bar{q}}(x_b,\vec{k}_{bT}^2) + f_1^{\bar{q}}(x_a,\vec{k}_{aT}^2)f_2^q(x_b,\vec{k}_{bT}^2)].$$

Dihadron SIDIS

Bacchetta, PR**D69**(2004)074026

$$d^7\sigma_{OO} = \frac{\alpha^2}{2\pi Q^2 y}\sum_a e_a^2\left\{A(y)f_1(x)D_1(z,\zeta,M_h^2) - V(y)\cos\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{1}{z}f_1(x)\tilde{D}^{\sphericalangle}(z,\zeta,M_h^2) + \frac{M}{M_h}xh(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\right]\right\}$$

$$d^7\sigma_{LO} = \frac{\alpha^2}{2\pi Q^2 y}\lambda\sum_a e_a^2 W(y)\sin\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{M}{M_h}xe(x)H_1^{\sphericalangle}(z,\zeta,M_h^2) + \frac{1}{z}f_1(x)\tilde{G}^{\sphericalangle}(z,\zeta,M_h^2)\right]$$

$$d^7\sigma_{OL} = \frac{\alpha^2}{2\pi Q^2 y}S_L\sum_a e_a^2 V(y)\sin\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{M}{M_h}xh_L(x)H_1^{\sphericalangle}(z,\zeta,M_h^2) + \frac{1}{z}g_1(x)\tilde{G}^{\sphericalangle}(z,\zeta,M_h^2)\right]$$

$$d^7\sigma_{OT} = \frac{\alpha^2}{2\pi Q^2 y}|\vec{S}_\perp|\sum_a e_a^2\left\{B(y)\sin(\phi_R+\phi_S)\frac{|\vec{R}_T|}{M_h}h_1(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\right.$$

$$\left. + V(y)\sin\phi_S\frac{M_h}{Q}\left[h_1(x)\left(\frac{1}{z}\tilde{H}(z,\zeta,M_h^2) + \frac{|\vec{R}_T|^2}{M_h^2}H_1^{\sphericalangle\ o(1)}(z,\zeta,M_h^2)\right) - \frac{M}{M_h}xf_T(x)D_1(z,\zeta,M_h^2)\right]\right\}$$

$$d^7\sigma_{LT} = \frac{\alpha^2}{2\pi Q^2 y}\lambda|\vec{S}_\perp|\sum_a e_a^2 W(y)\cos\phi_S\frac{M_h}{Q}\left[-\frac{M}{M_h}xg_T(x)D_1(z,\zeta,M_h^2) - \frac{1}{z}h_1(x)\tilde{E}(z,\zeta,M_h^2)\right]$$

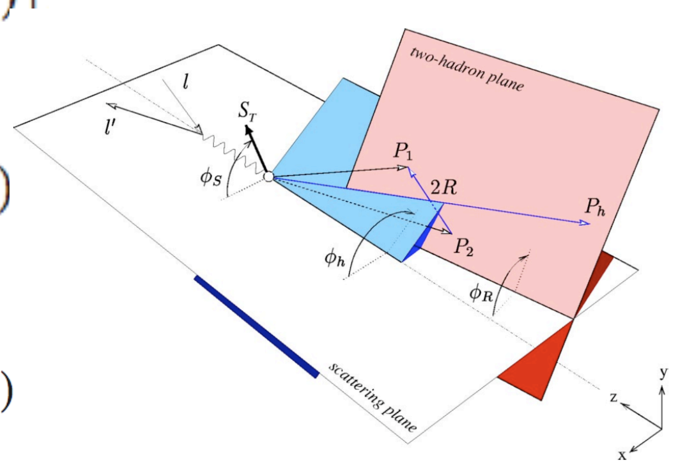
$$d^7\sigma_{LL} = \frac{\alpha^2}{2\pi Q^2 y}\lambda S_L\sum_a e_a^2\left\{C(y)g_1(x)D_1(z,\zeta,M_h^2) - W(y)\cos\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{1}{z}g_1(x)\tilde{D}^{\sphericalangle}(z,\zeta,M_h^2)\right.\right.$$

$$\left.\left. - \frac{M}{M_h}xe_L(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\right]\right\},$$

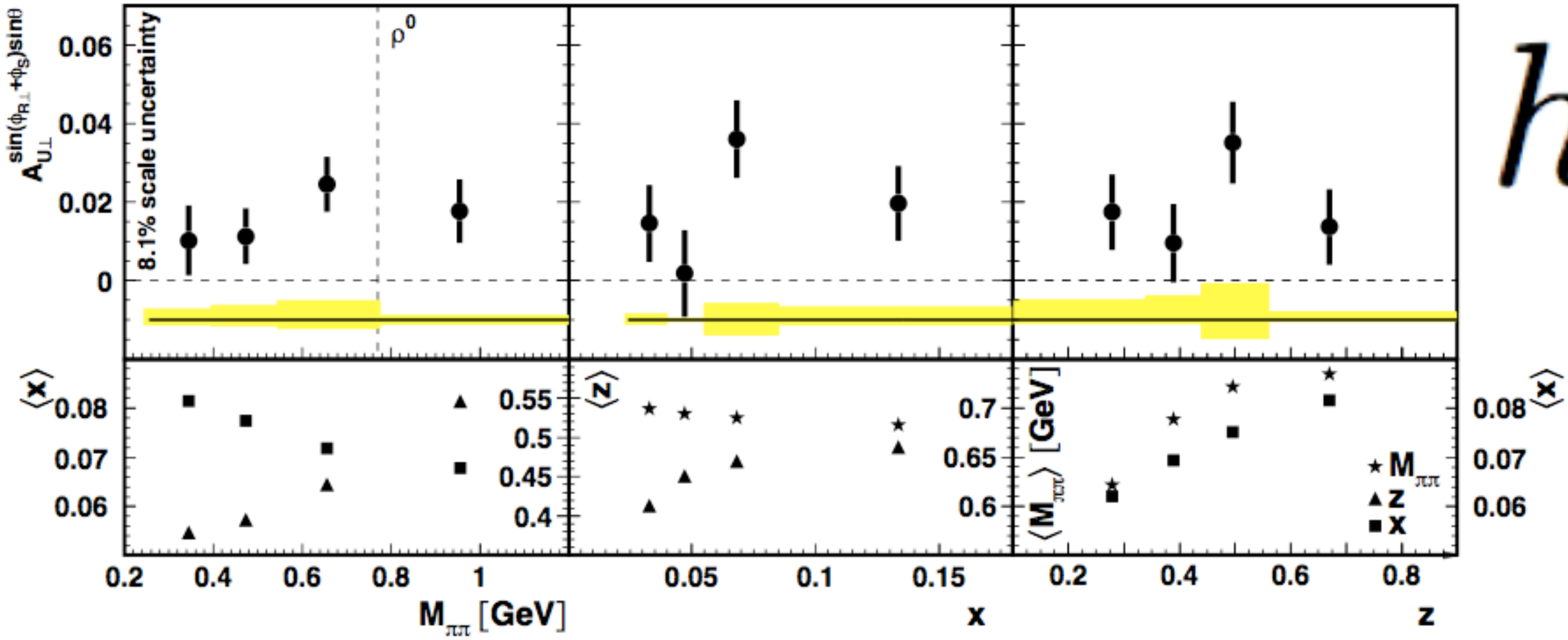
$$\zeta = \frac{1}{M_h}\left(\sqrt{M_1^2 - |\vec{R}|^2} - \sqrt{M_2^2 - |\vec{R}|^2 - 2|\vec{R}|\cos\theta}\right)$$

Leading Twist

Sub-Leading Twist (extra factor of 1/Q)



Dihadron SIDIS



h_1

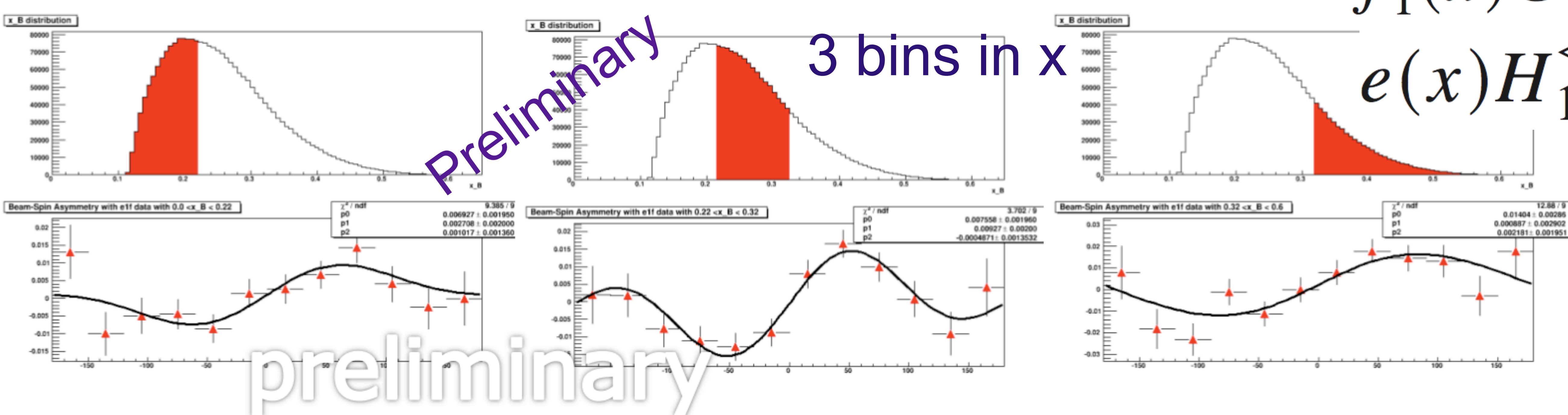
HERMES
JHEP**06**(08)017

A$_{UL}$ is positive but small

$f_1(x)\tilde{G}^{\sphericalangle}$

$e(x)H_1^{\sphericalangle}$

3 bins in x



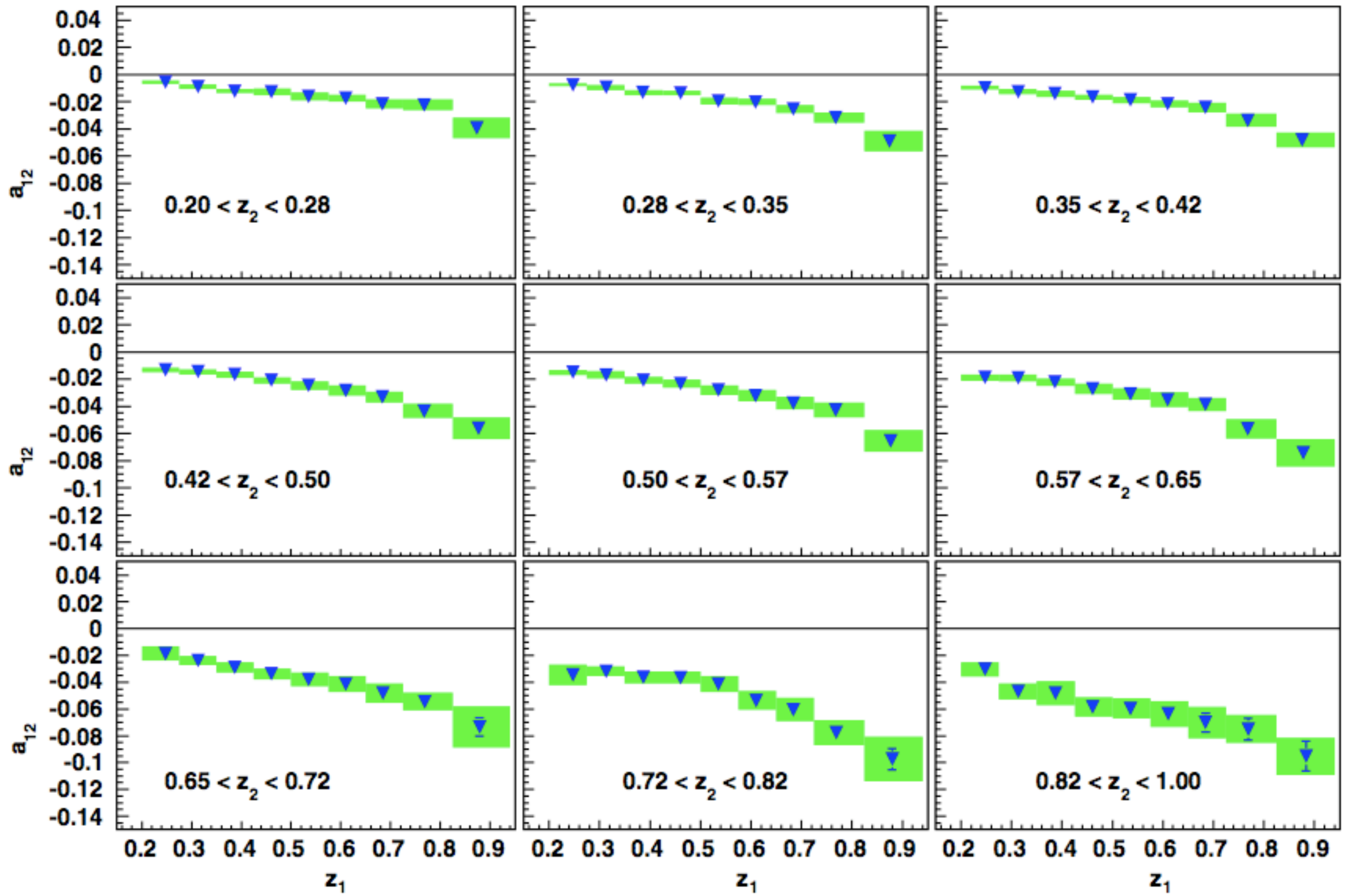
A$_{LU}$ for $\pi^+\pi^-$ in the CLAS e1f experiment. Analysis by Silvia Pisano (Frascati/Roma Tor Vergata). Clear sinϕ and sin2ϕ asymmetries.

Interference Fragmentation

Vossen, PRL107(11)072004 (Belle)

$$a_{12R}(z_1, z_2, m_1^2, m_2^2) \propto \frac{1}{2}\frac{\sin^2\theta}{1+\cos^2\theta} \cdot \frac{\sum_{q,\bar{q}} e_q^2 z_1^2 z_2^2 H_1^{\sphericalangle,q}(z_1, m_1^2) H_1^{\sphericalangle,\bar{q}}(z_2, m_2^2)}{\sum_{q,\bar{q}} e_q^2 z_1^2 z_2^2 D_1^{q}(z_1, m_1^2) D_1^{\bar{q}}(z_2, m_2^2)}$$

$e^+e^- \rightarrow \pi^+\pi^-$

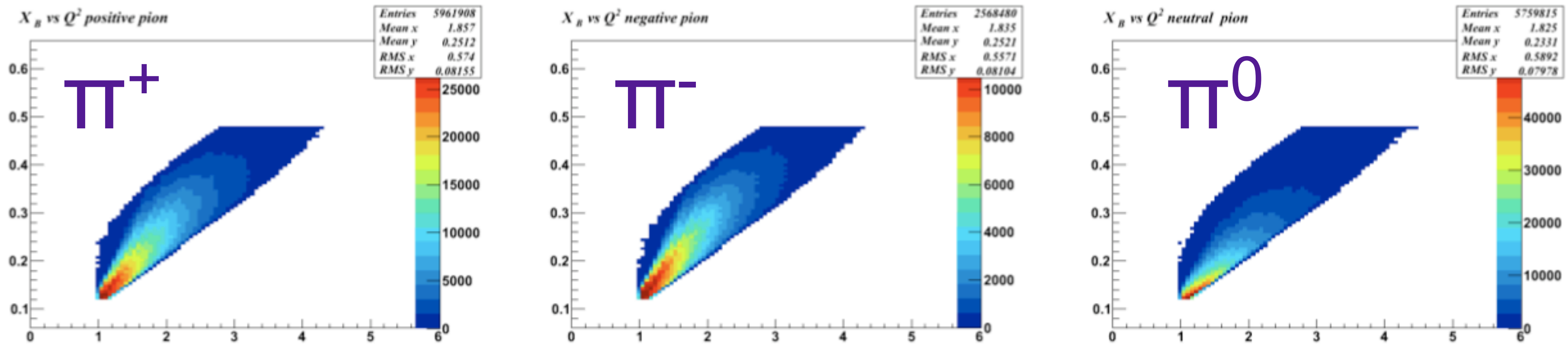


EG1-DVCS at JLab

0.6

x

0.1



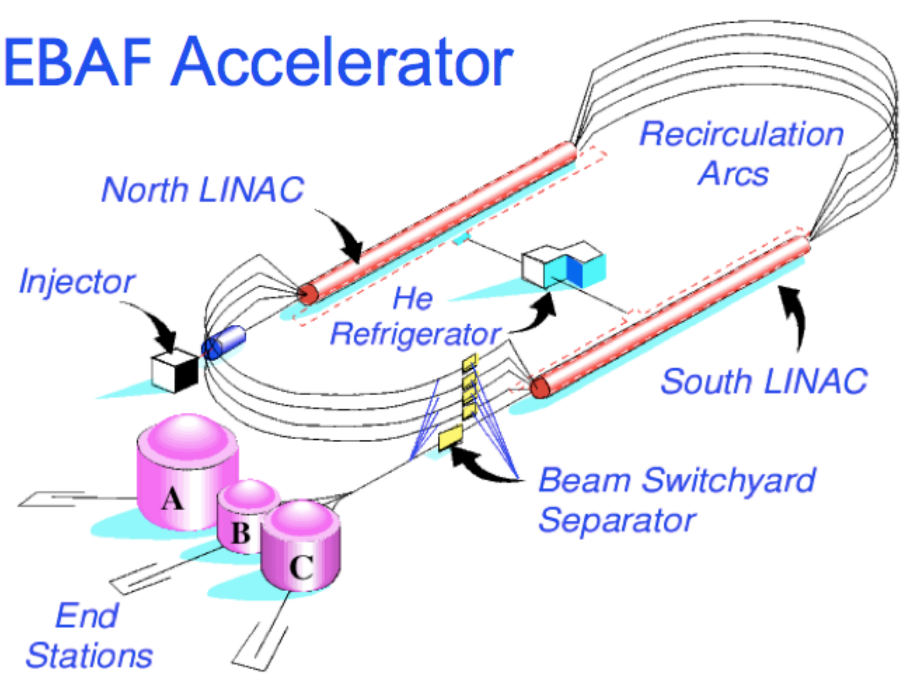
Q^2 (GeV^2) 0 6

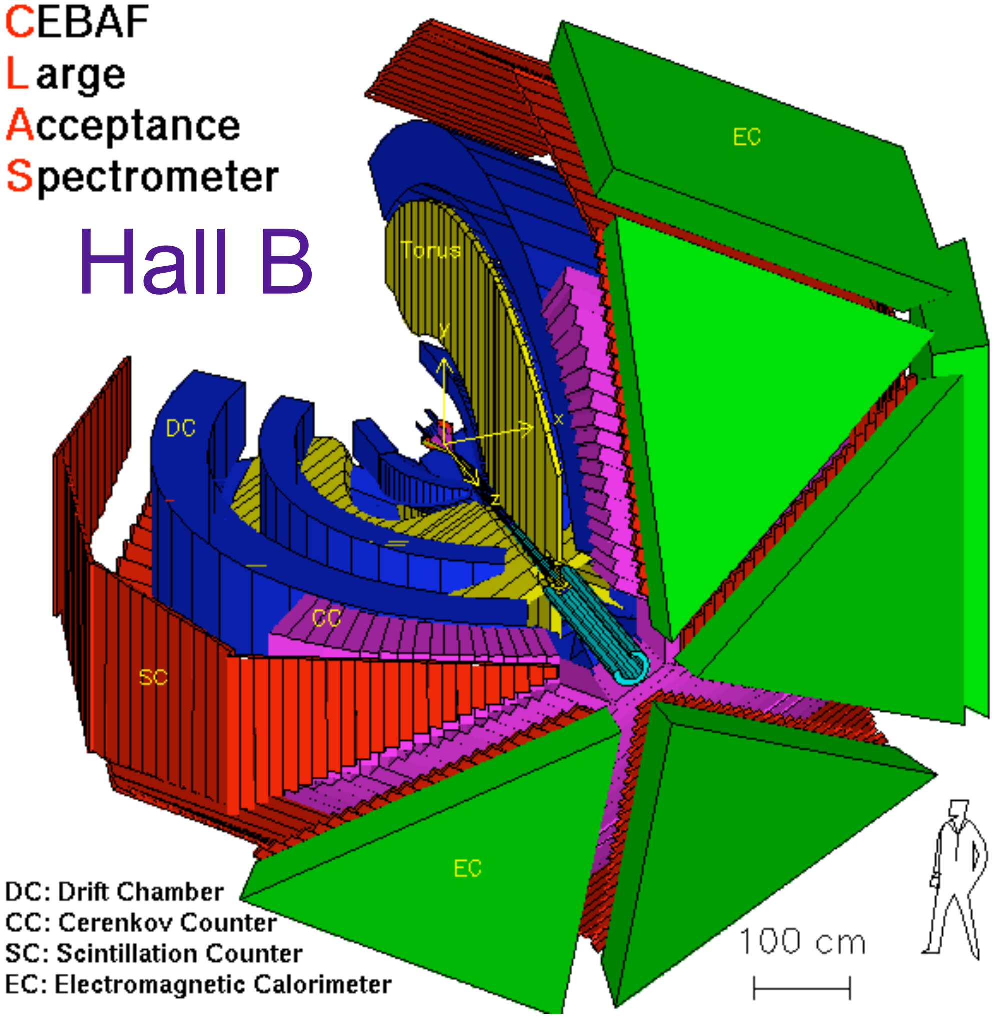
- Beam time: February to September 2009
- ~6 GeV Polarized electron beam (P_b~85%)
- Frozen $^{15}NH_3$ target (P_t~75%)
- CEBAF large acceptance spectrometer (CLAS) plus Inner Calorimeter
- ~19 billion electron triggers on NH3 target

CEBAF
Large
Acceptance
Spectrometer

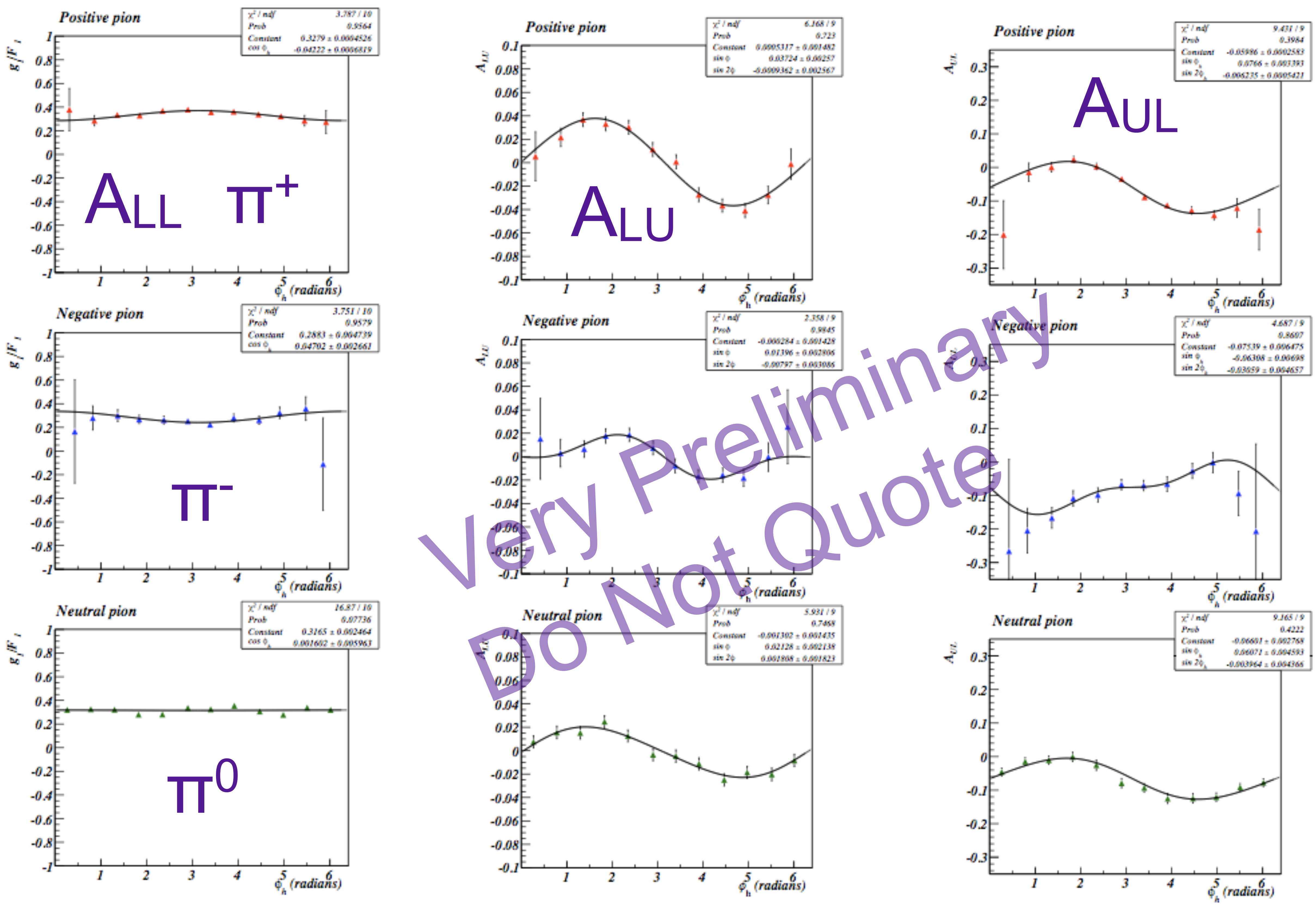
Hall B

The CEBAF Accelerator



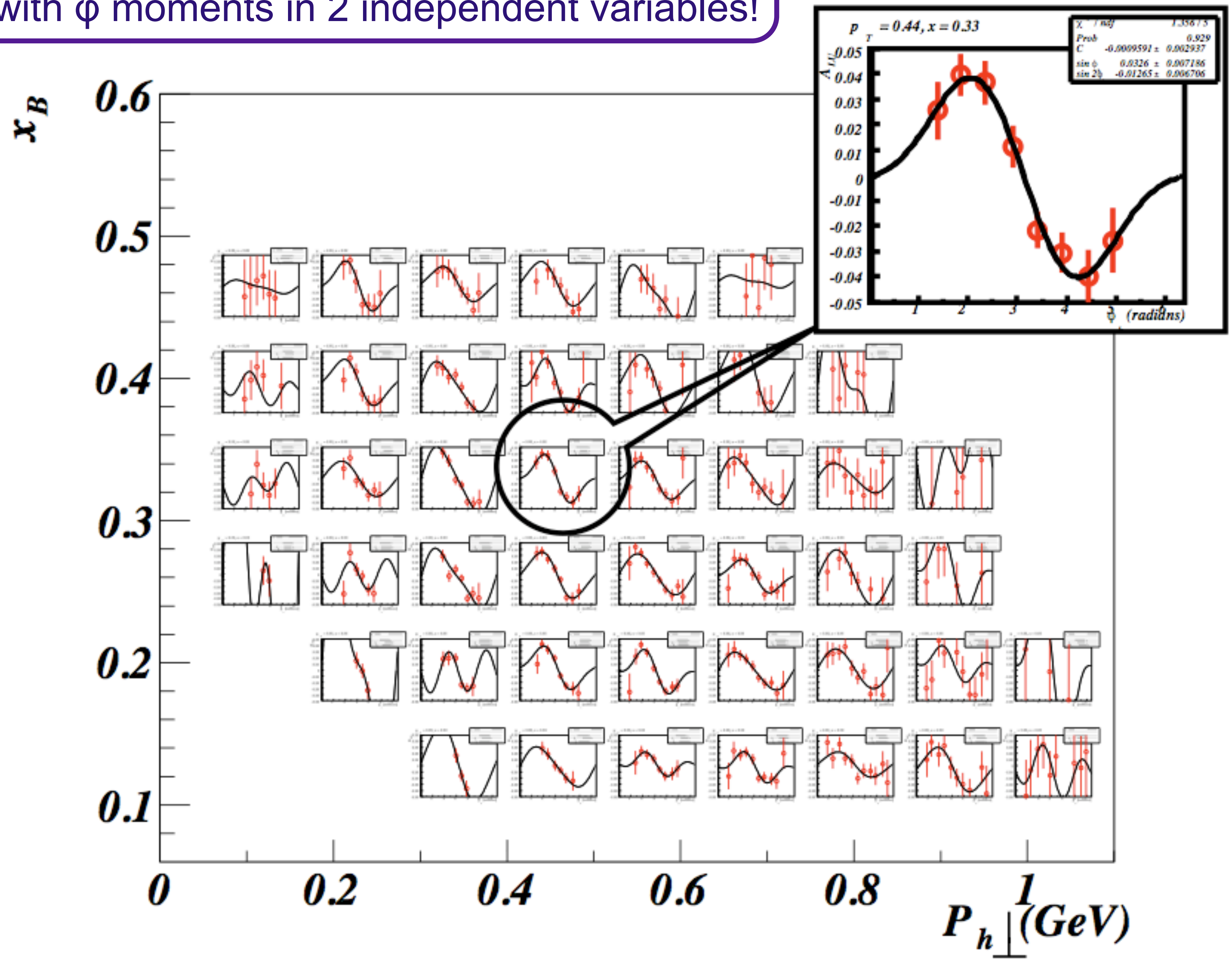


CLAS Eg1-dvcs



CLAS Eg1-dvcs

Measurements with φ moments in 2 independent variables!

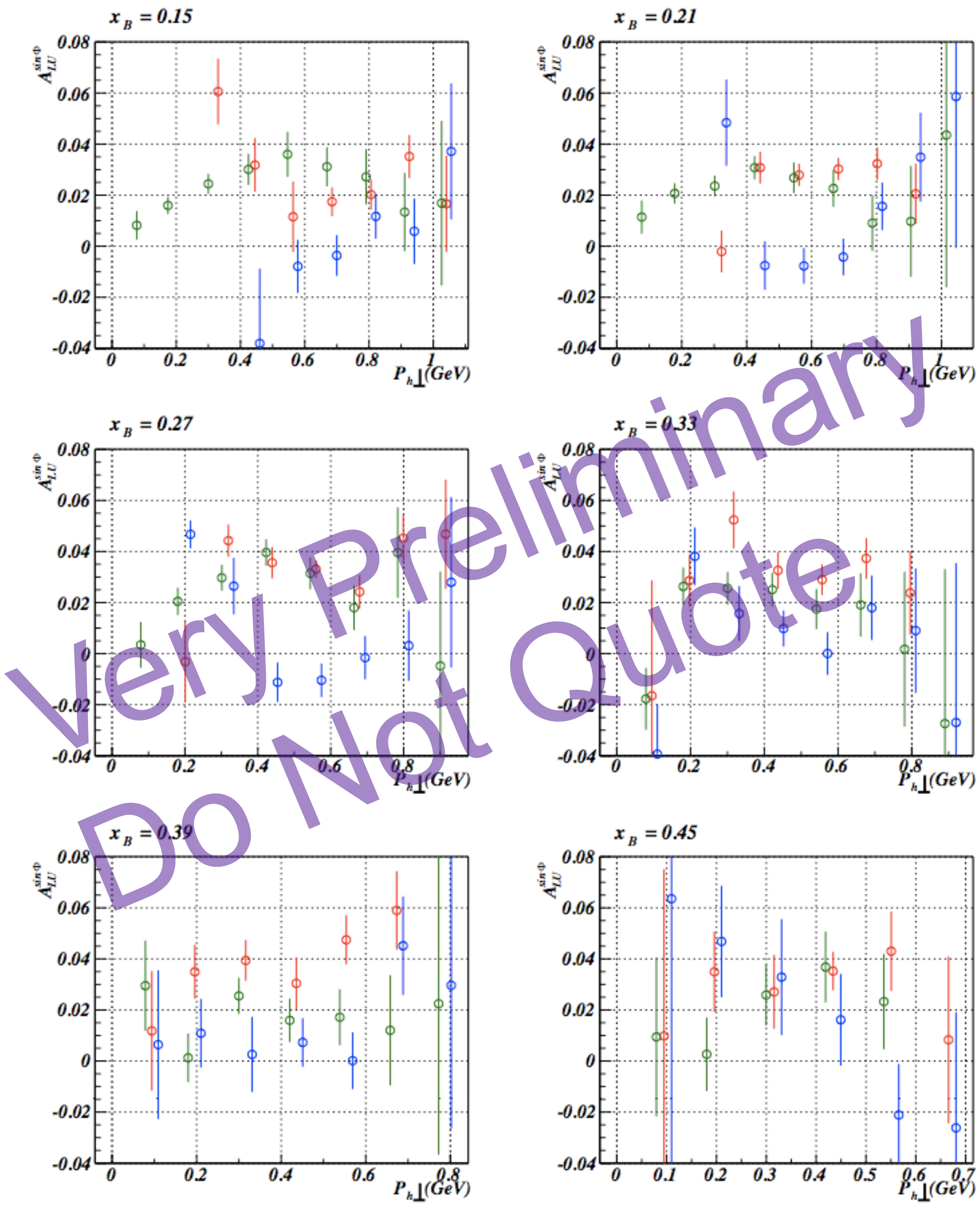


$\pi^+ A_{LU}$ vs. ϕ

CLAS Eg1-dvcs

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{E}}{z}\right)\right]$$

- π^+ (red)
- π^- (blue)
- π^0 (green)

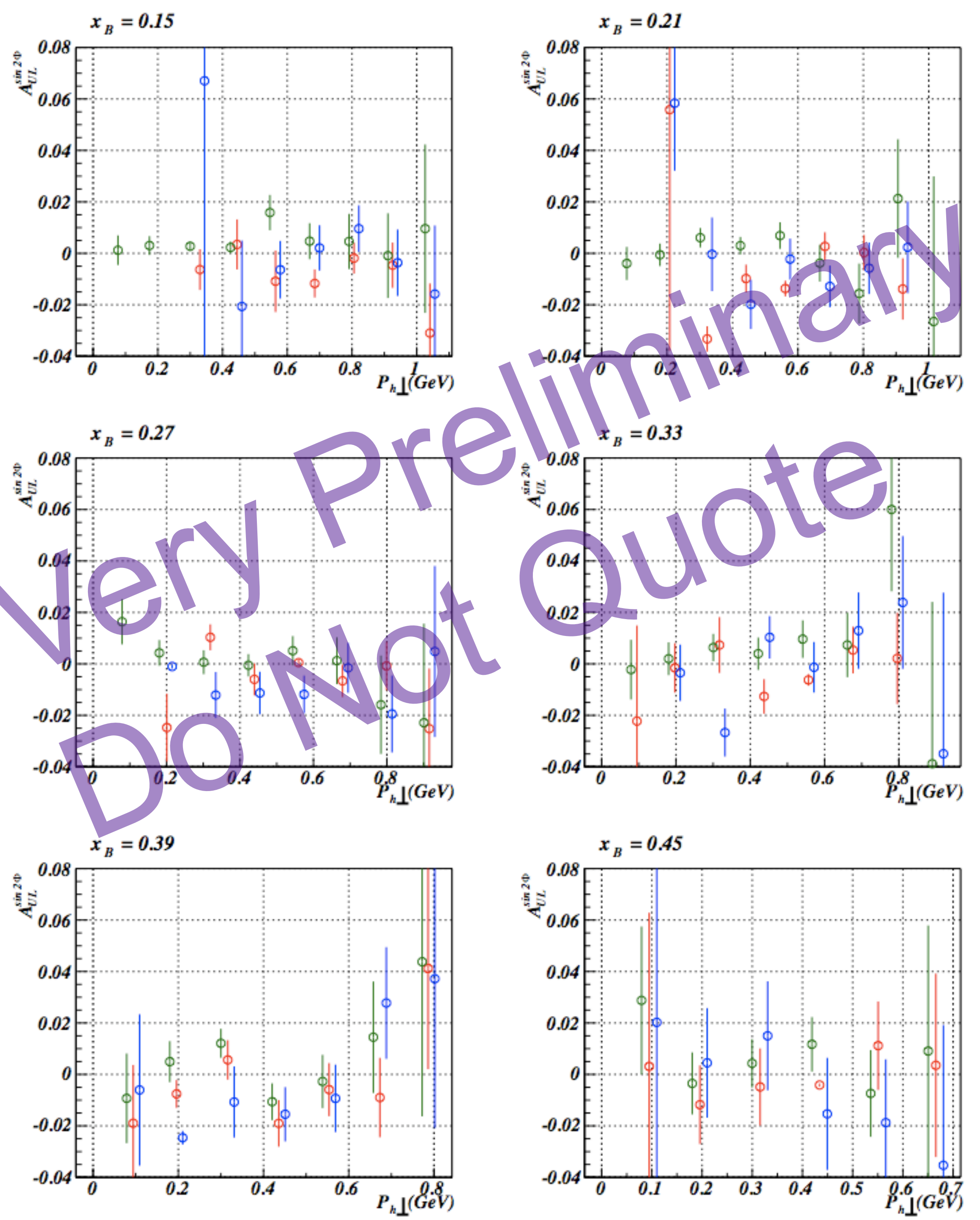


CLAS Eg1-dvcs

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$

- π^+ (red)
- π^- (blue)
- π^0 (green)

f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$



CLAS Eg1-dvcs

$$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right]$$

- π^+ (red)
- π^- (blue)
- π^0 (green)

f_1		$h_1^\perp$
	g_{1L}	$h_{1L}^\perp$
$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

CLAS Eg1-dvcs

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(x h_L H_1^{\perp} + \frac{M_h}{M}\, g_{1L}\,\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(x f_L^{\perp} D_1 - \frac{M_h}{M}\, h_{1L}^{\perp}\,\frac{\tilde{H}}{z}\right)\right]$$

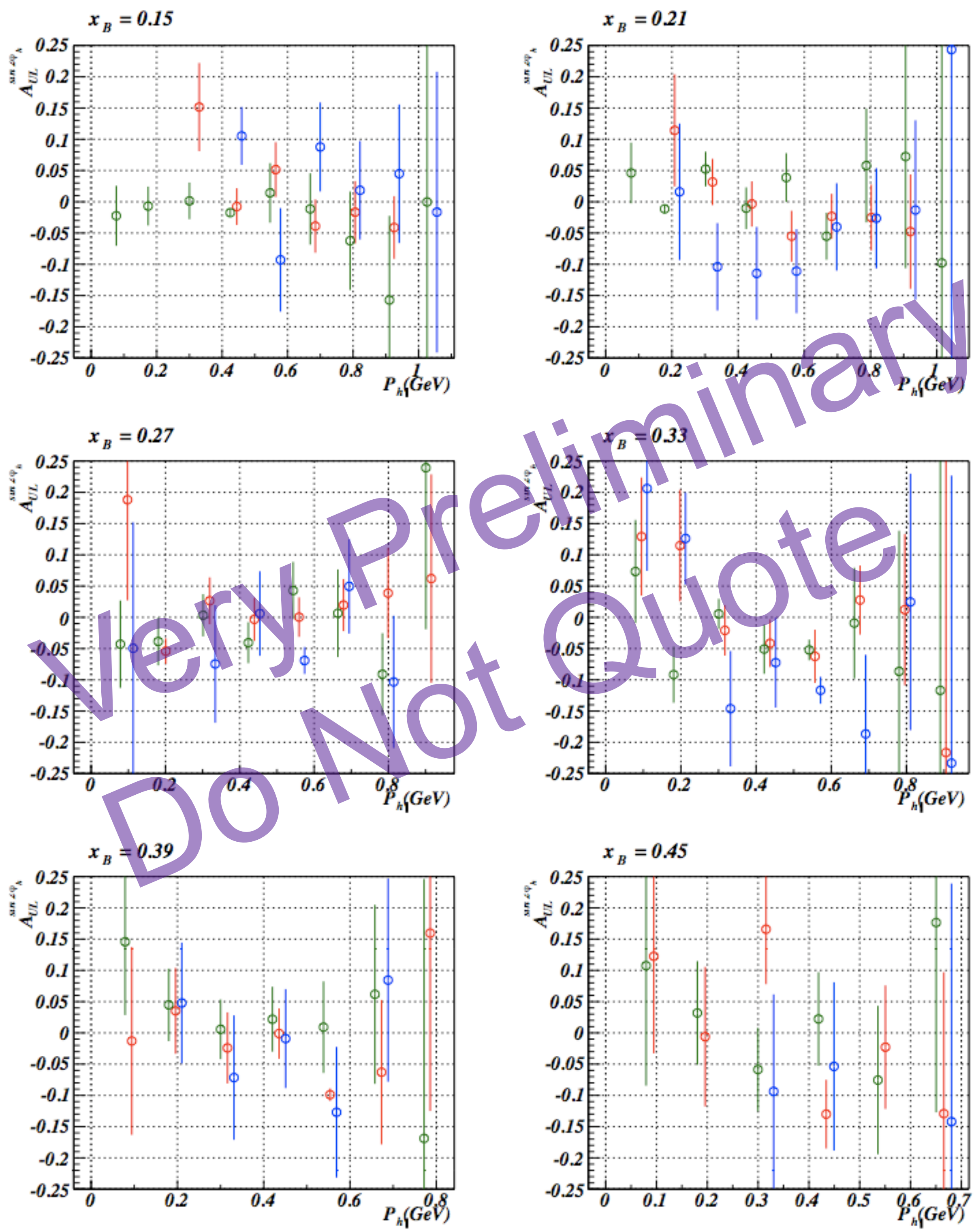
- π^+ (red)
- π^- (blue)
- π^0 (green)

CLAS Eg1-dvcs

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$

f_1		$h_1^{\perp}$
	g_{1L}	$h_{1L}^{\perp}$
$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

- π^+ (red)
- π^- (blue)
- π^0 (green)



Conclusions

- In order to truly understand the nucleon, we will need to explore transverse momentum distributions (TMDs)
- The formalism is complicated and the number of useful objects many
- However, objects like the Collins Fragmentation function show up in many different measurements.
- If these objects prove to be universal, we will have a bright future in the next decade measuring them using eN, NN, and e^+e^- reactions.
- Although TMDs are sensitive to spin-orbit correlations, there is no easy, intuitive way to connect them to L_z.