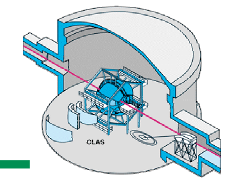
The College of WILLIAM & MARY



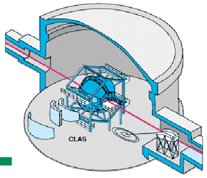
Experimental Moments of Nucleon Structure Functions at Low Q^2

K. Griffioen
College of William & Mary
griff@physics.wm.edu

Jefferson Lab Users Meeting
18-20 June 2007

The College of WILLIAM & MARY

Introduction



Our tendency is to go from inclusive to exclusive reactions

$\boxed{f(x,Q^2)}$ $\boxed{f(x,Q^2,p_\perp)}$

Our tendency is to go from low resolution (Q^2) to high

Σ of constituents resolved constituents

Our tendency is to go from holism to reductionism

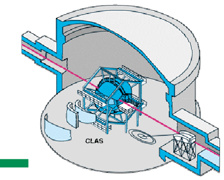
This talk is about going in the other direction:

$$\int f(x,Q^2,p_\perp)dp_\perp \rightarrow f(x,Q^2)$$

$$\int f(x,Q^2)dx \rightarrow f(Q^2)$$

$$\int f(Q^2)dQ^2 \rightarrow \text{hyperfine splitting}$$

Global Properties



Energy-Weighted Sum Rule

$$S(F) = \sum_a (E_a - E_0) |\langle a|F|0\rangle|^2 = \langle 0|[F,[H,F]]|0\rangle$$

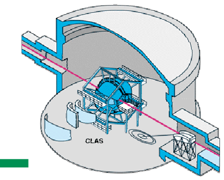
GDH Sum Rule

$$\int_{k_\pi}^{\infty} \frac{dk}{k} \Delta\sigma^{\gamma N}(k) = \frac{2\pi^2 \alpha \kappa^2}{M^2}$$

$$\Delta\sigma^{\gamma N} = \sigma_{3/2}^{\gamma N} - \sigma_{1/2}^{\gamma N}$$

Sum over excited states is tied to property of ground state

High Energy Diversion



Gottfried Sum Rule

0.235(26) at Q^2=4 GeV^2

$$\Phi_1^{p,n}(Q^2) = \int_0^1 F_1^{p,n}(x,Q^2)dx$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

$$\Phi_1^p - \Phi_1^n = \frac{1}{6}[u_v - d_v + 2\bar{u} - 2\bar{d}]$$

Bjorken Sum Rule

0.176(7) at Q^2=5 GeV^2

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 g_1^{p,n}(x,Q^2)dx$$

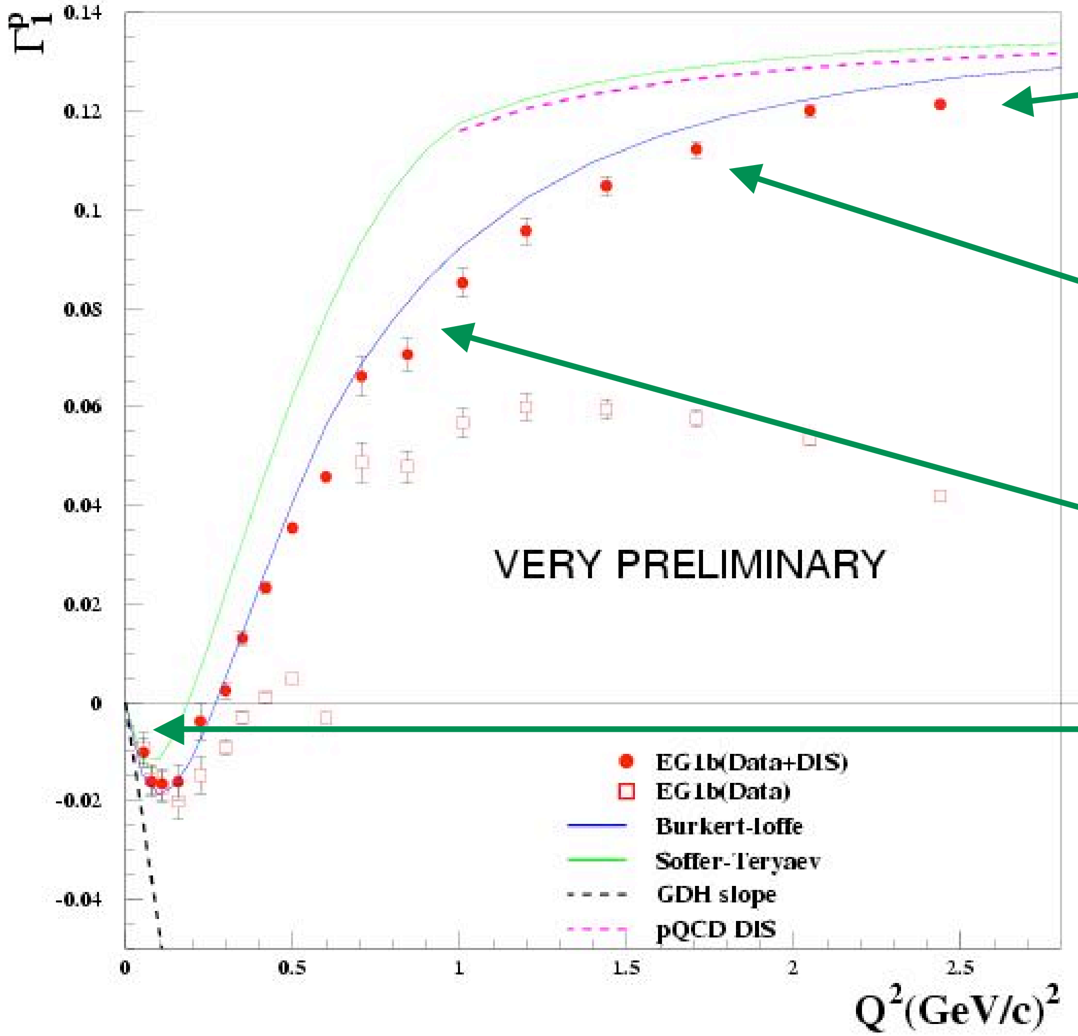
$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x)$$

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}[\Delta u_v - \Delta d_v + 2\Delta\bar{u} - 2\Delta\bar{d}]$$

Complicating Factor

$$\Delta C_{NS}^{\bar{M}S} = 1 - \frac{\alpha_S}{\pi} - 3.583\left(\frac{\alpha_S}{\pi}\right)^2 - 20.215\left(\frac{\alpha_S}{\pi}\right)^3 + \ldots$$

Regions of Q^2



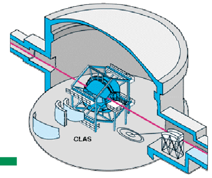
scaling: $\ln Q^2$

higher twist: $(1/Q^2)^n$

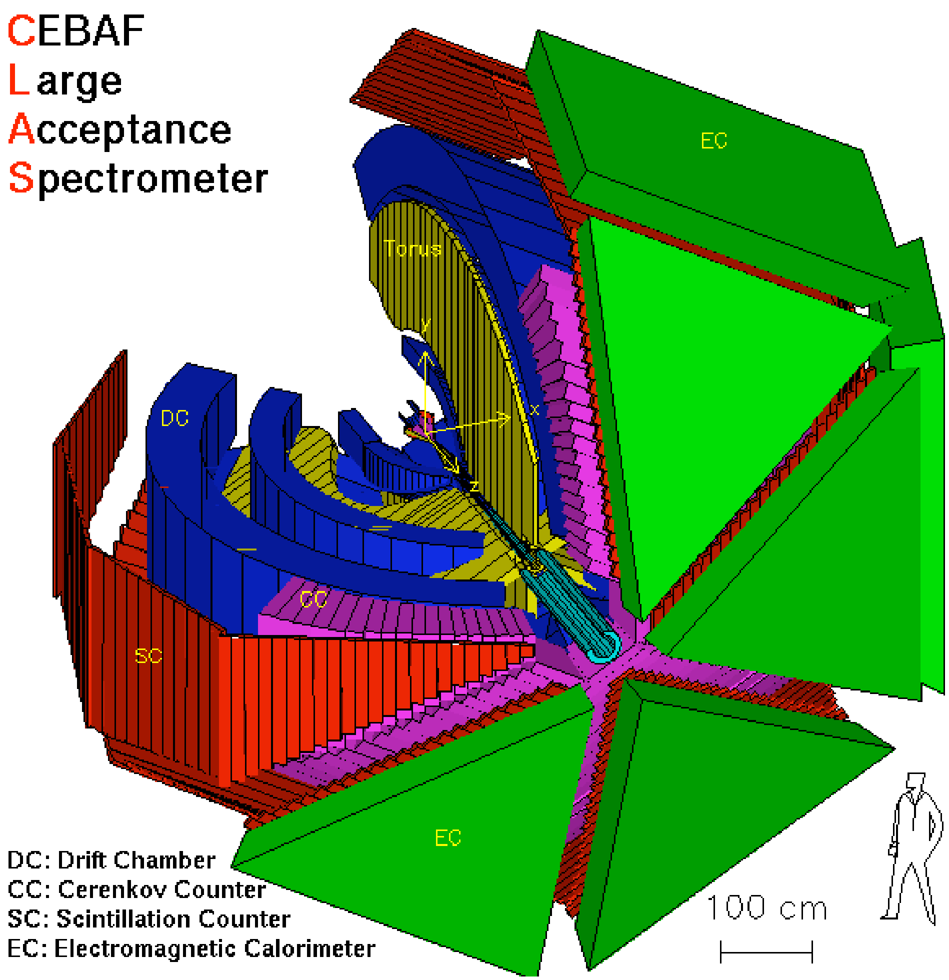
no nice expansion

χPT: $(Q^2)^n$

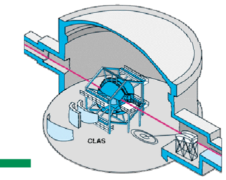
Spin Structure with CLAS



- Long-standing program in Hall-B at JLab to measure longitudinal double spin asymmetries $A_{||}$ on $^{15}NH_3$ and $^{15}ND_3$
- EG1: $0.05<Q^2<3.5$ GeV^2
  - data (2001); anal (2007)
- EG4: $0.01<Q^2<1$ GeV^2
  - data (2006); anal (2008)
- EG12: $0.5<Q^2<7$ GeV^2
  - data (2012?); anal (2014)



Formalism



$$A_{\parallel} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}}$$

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}}$$

$$A_{\parallel} = D(A_1 + \eta A_2)$$

$$= \frac{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)}{F_1(x,Q^2)}$$

We can extract A_1 using a model for A_2 (small), or g_1 using a model for g_2 (small)

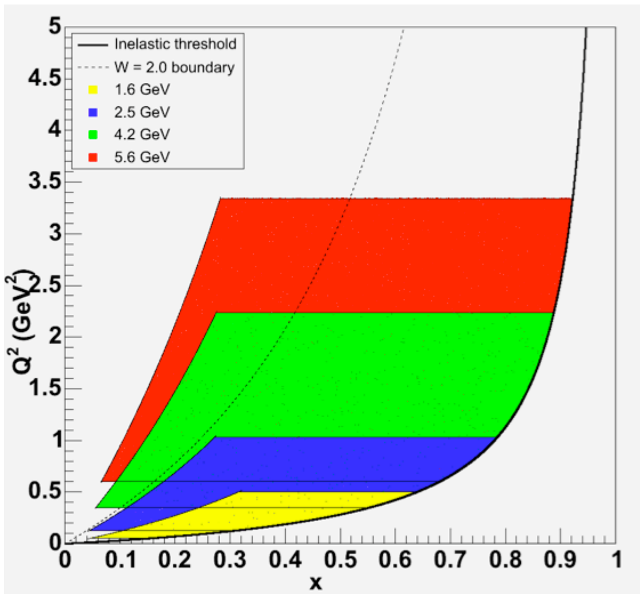
$$A_2 = \frac{2\sigma_{LT}}{\sigma^T_{1/2} + \sigma^T_{3/2}}$$

We can extract A_1 and A_2 from $A_{\parallel}$ at multiple values of $\eta(E_{beam})$

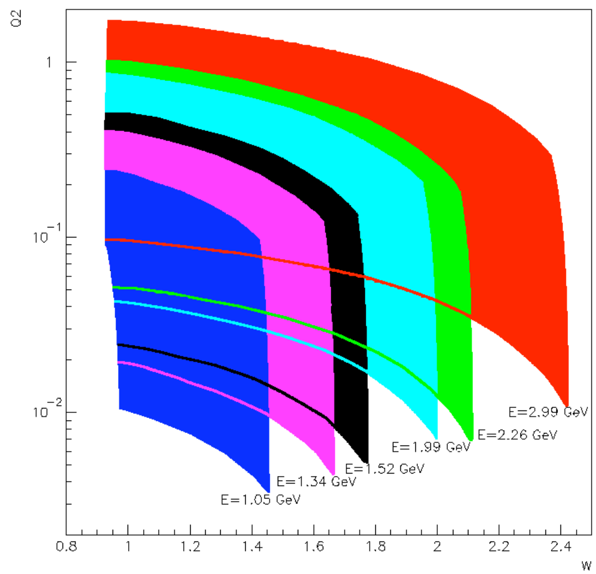
$$= \frac{\gamma[g_1(x,Q^2) + g_2(x,Q^2)]}{F_1(x,Q^2)}$$

Kinematics

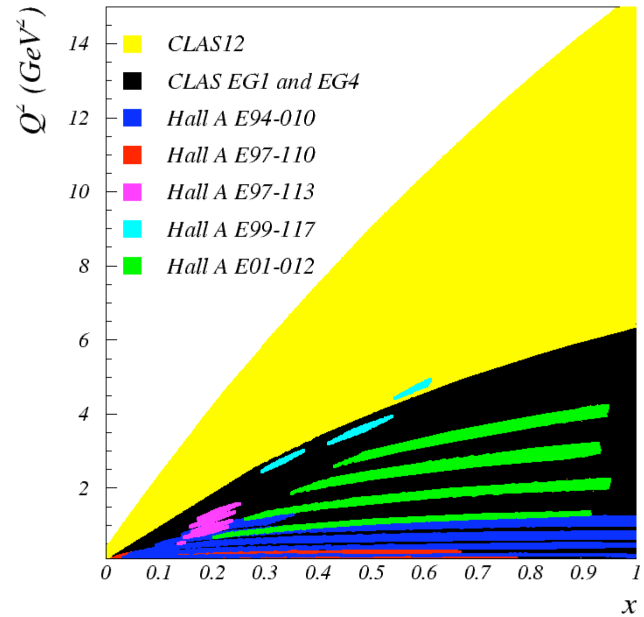
EG1



EG4

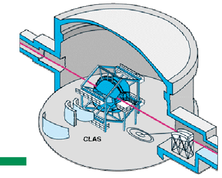


EG12



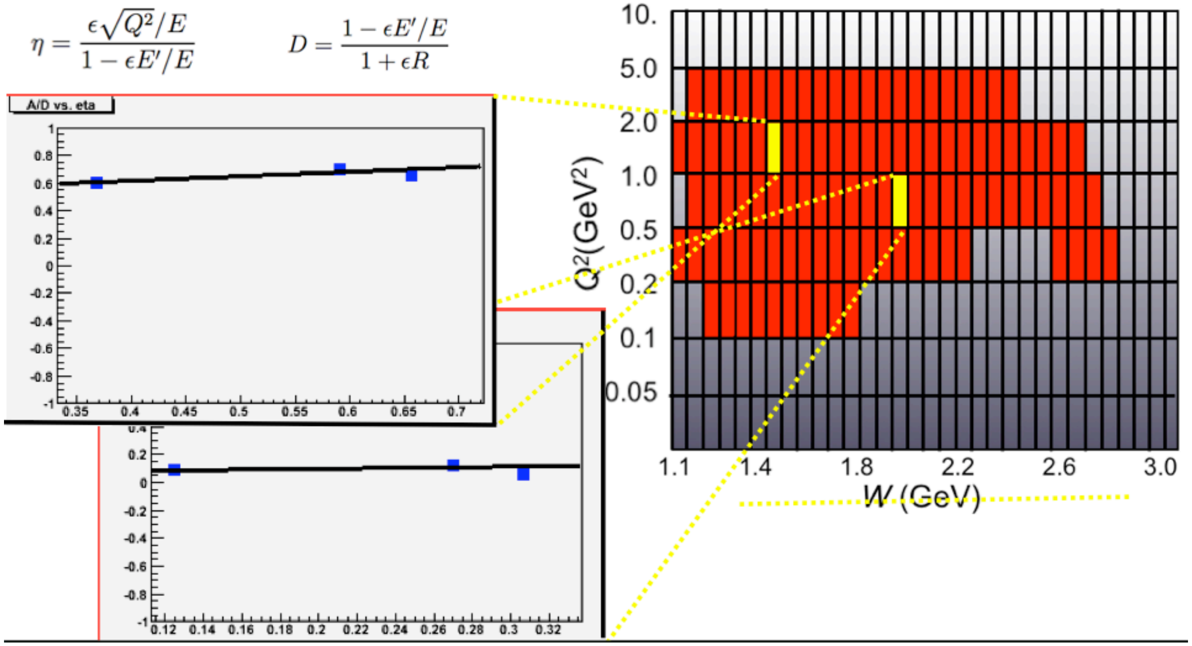
- Overlapping colors correspond to different beam energies
- CLAS measures a large range in x at each fixed Q^2
- Different E_{beam} for fixed (x,Q^2) allows separation of A_1 & A_2

EG1 Extraction of A_2

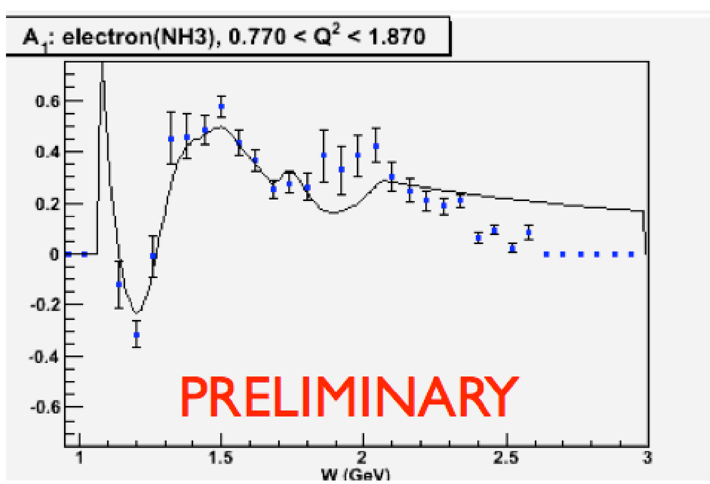


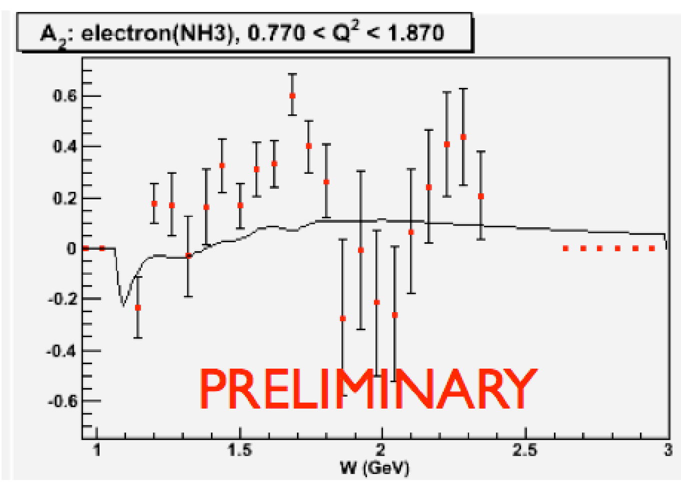
$$A_{||} = D(A_1 + \eta A_2)$$

$$\eta = \frac{\epsilon\sqrt{Q^2}/E}{1-\epsilon E'/E} \qquad D = \frac{1-\epsilon E'/E}{1+\epsilon R}$$

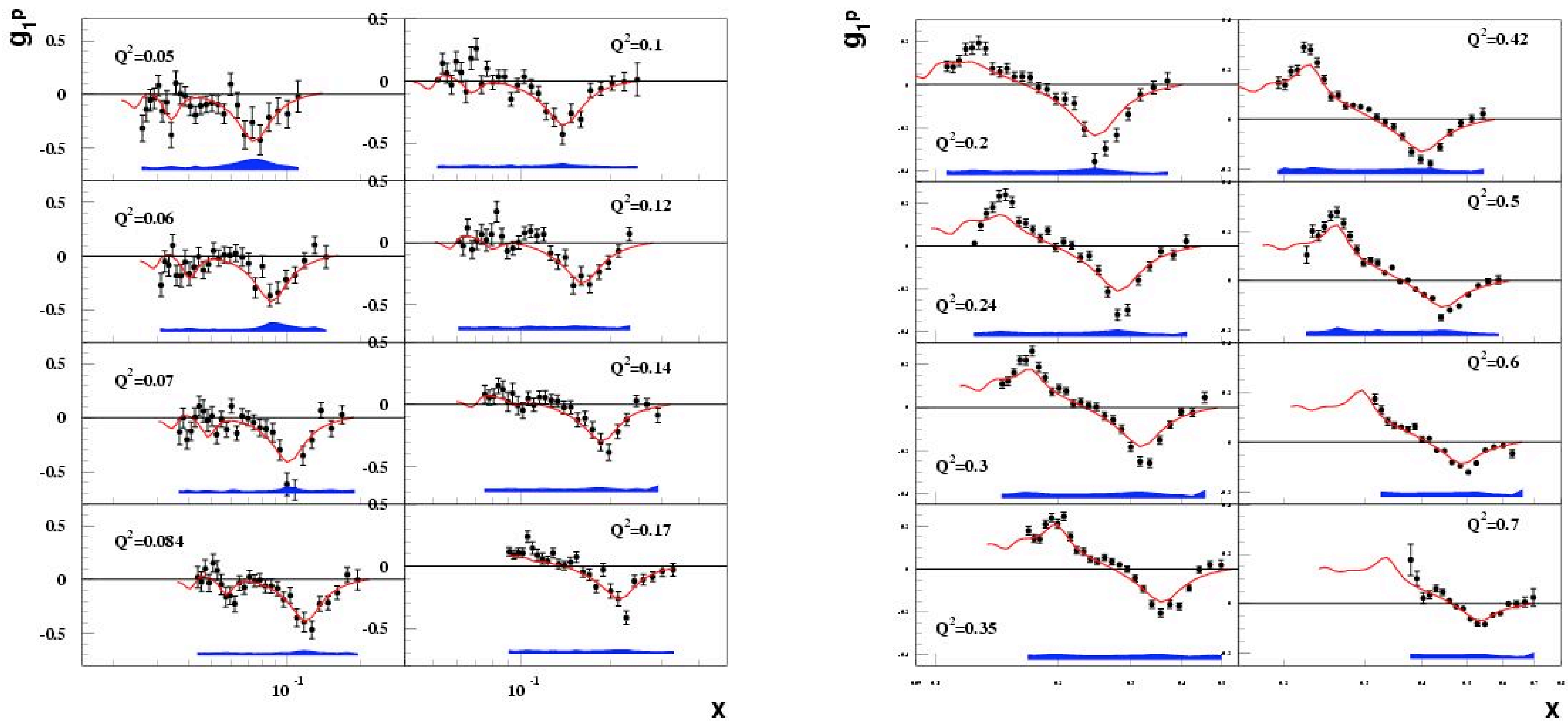


- Analysis is in progress to obtain both A_1 and A_2 from the EG1 data
- Intercept gives A_1
- Slope gives A_2
- A_2 is larger than EG1 model (MAID, AO) as is Hall C RSS experiment





EG1 g_1^p (Q^2<0.7)

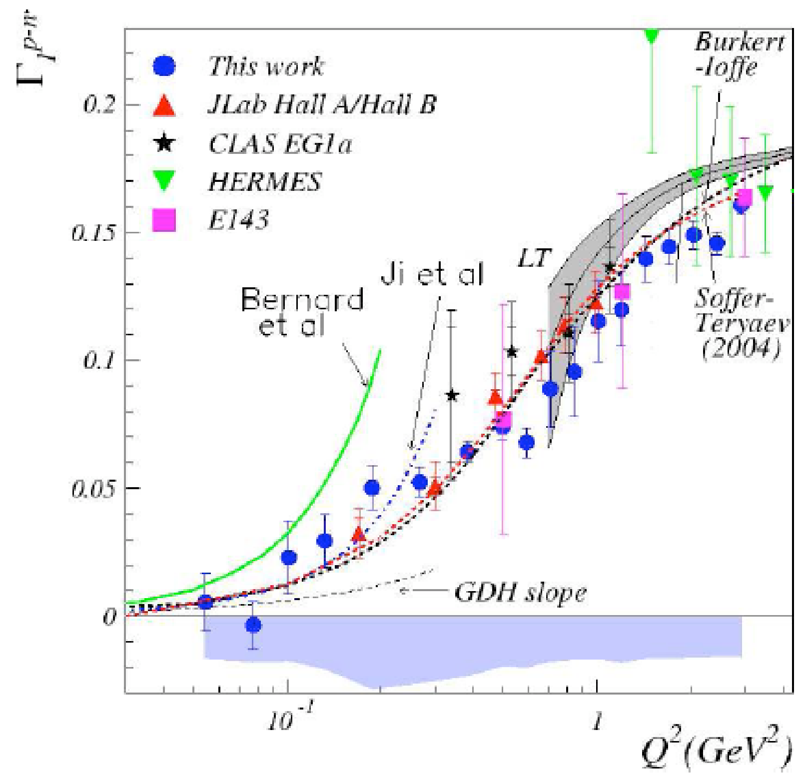


- At low Q^2 the Δ resonance drives g_1 negative
- Extensive x-range at fixed Q^2 allows integration over x
- Red curve is the EG1 model used for radiative corrections

Bjorken Sum & Higher Twist

$$\Gamma_1^{(n)} = \int_0^1 x^n g_1(x,Q^2)\,dx = \frac{a_n}{2}, \quad n = 0,2,4,\ldots,$$

$$\Gamma_2^{(n)} = \int_0^1 x^n g_2(x,Q^2)\,dx = \frac{1}{2}\frac{n}{n+1}(d_n - a_n), \quad n = 2,4,\ldots,$$



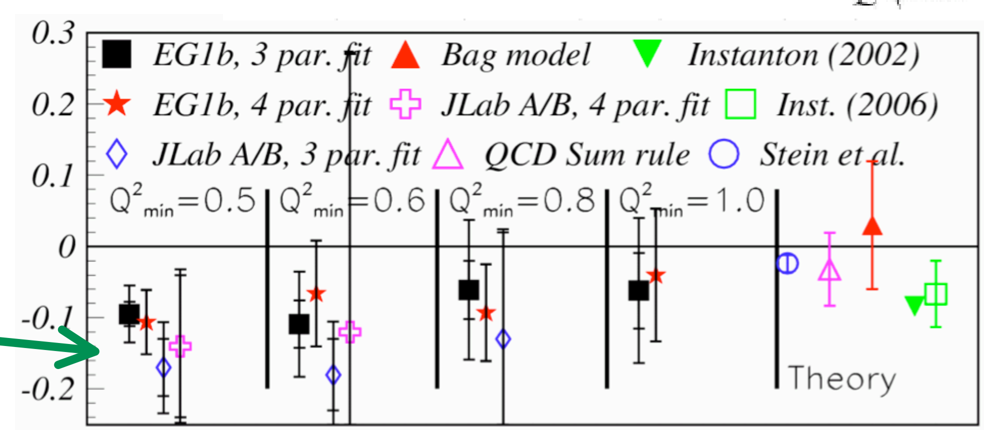
Bjorken Sum Rule:

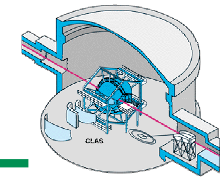
$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3\right] + \frac{\mu_4^{p-n}}{Q^2} + \ldots$$

$$\mu_4^{p-n} = \frac{M^2}{9}\left(a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n}\right)$$

$$d_2^{p-n} = \int_0^1 dx\, x^2 \left(2g_1^{p-n} + 3g_2^{p-n}\right)$$

Fit Γ_1^{p-n} to powers of $1/Q^2$ and extract f_2^{p-n}





Moments $\Gamma_1^{p,d}$

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x,Q^2)dx \qquad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

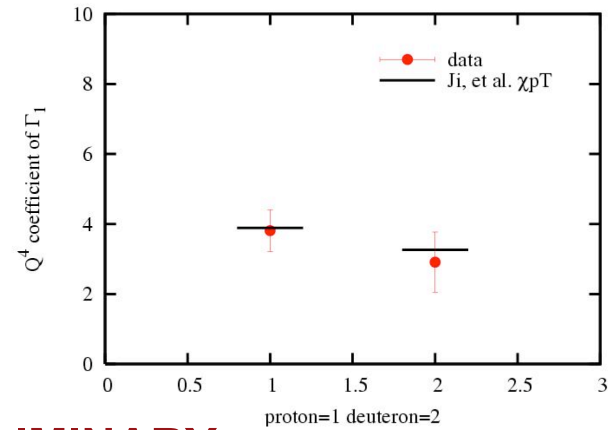
low Q^2 fit

$$\Gamma_1^d(Q^2) = (1 - 1.5\omega_D)\left\{\Gamma_1^p(Q^2) + \Gamma_1^n(Q^2)\right\}$$

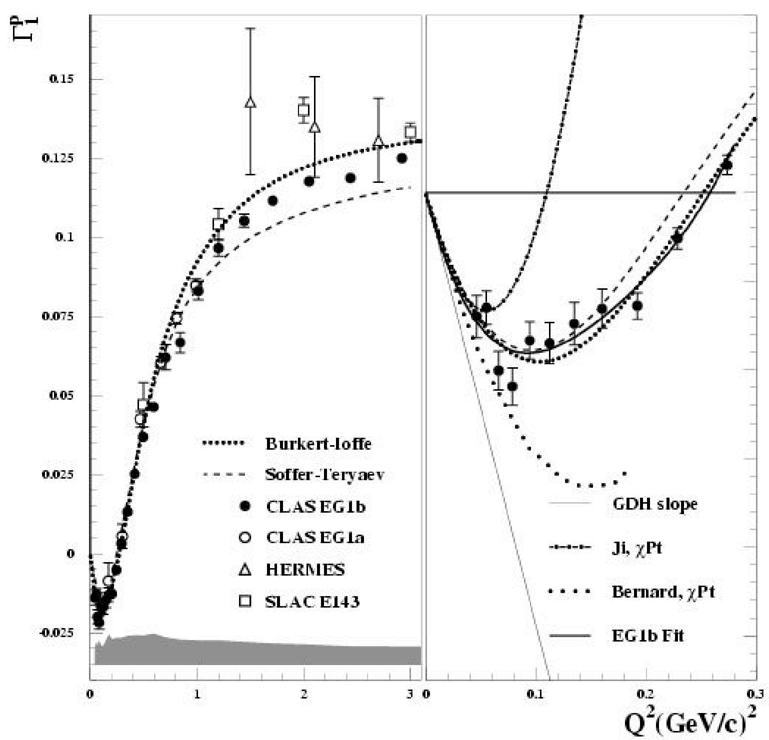
$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^2}{8M^2}Q^2 + 3.89Q^4 + \ldots$$

$$\Gamma_1^n(Q^2) = -\frac{\kappa_n^2}{8M^2}Q^2 + 3.15Q^4 + \ldots.$$

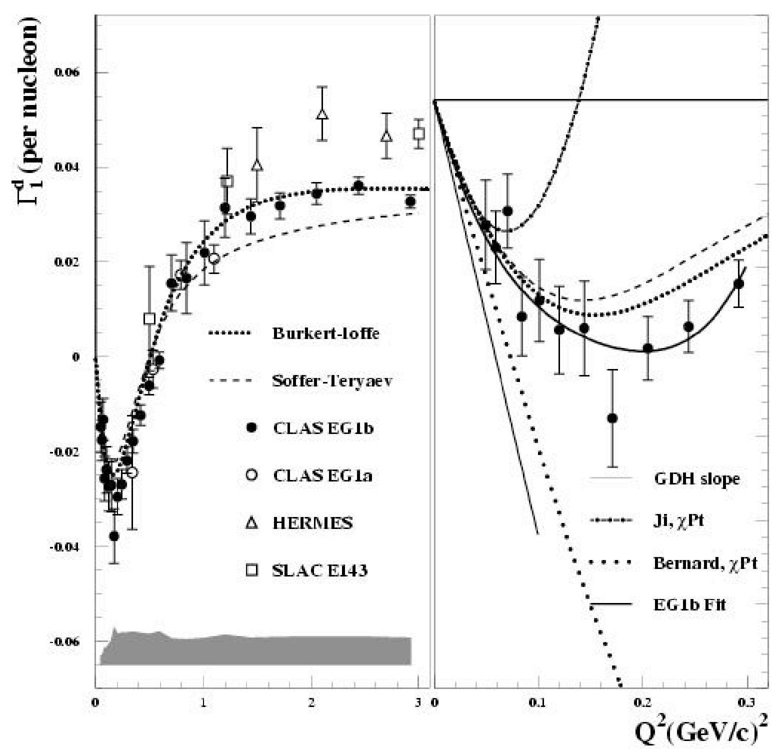
GDH + χpT



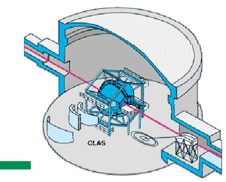
PRELIMINARY

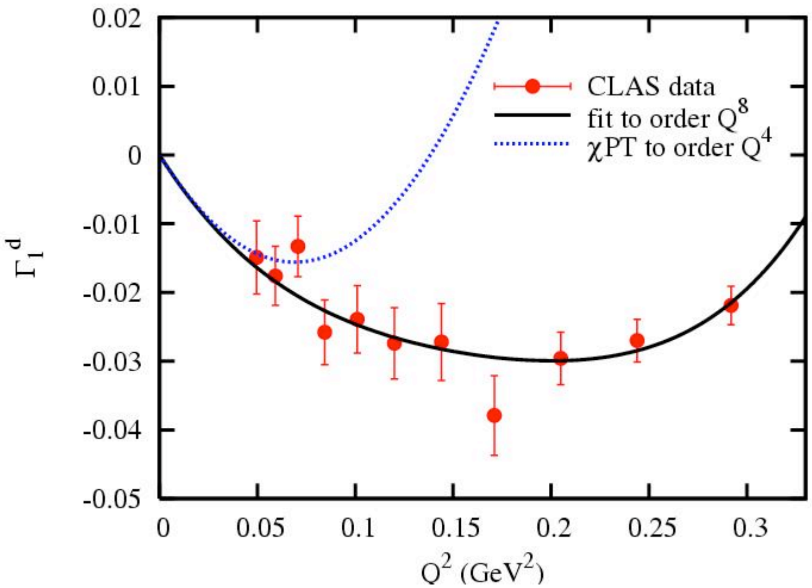


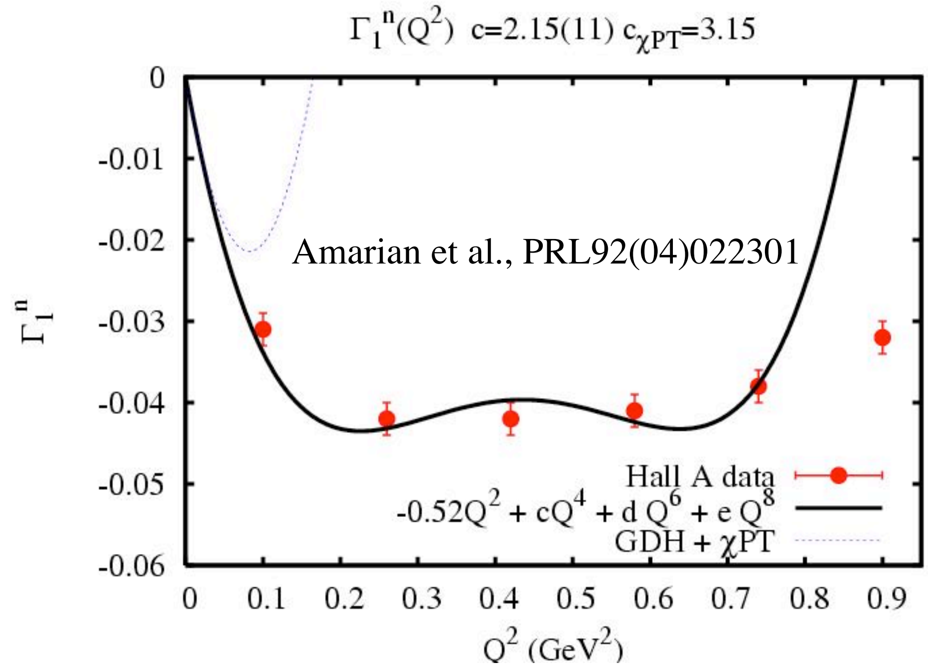
PRELIMINARY

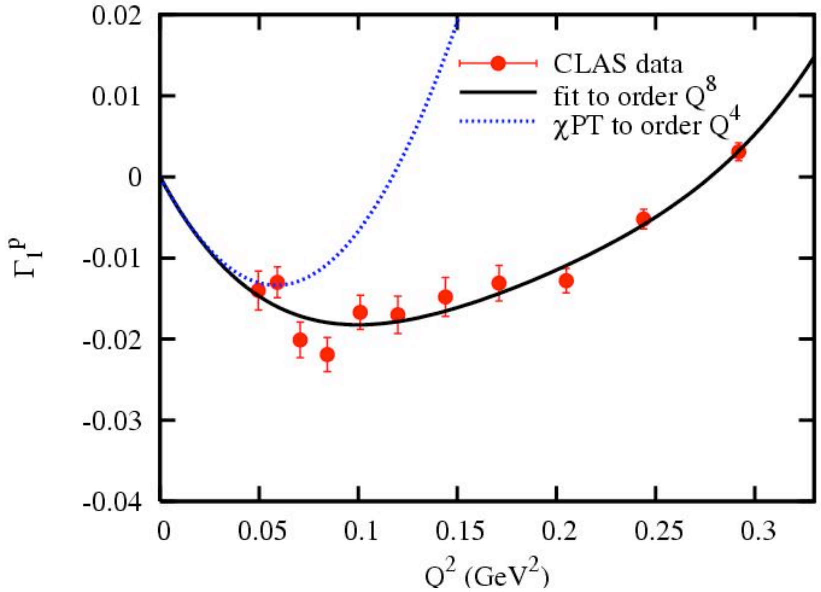


Low Q^2 Fits of Γ_1









χPT X. D. Ji, C. W. Kao and J. Osborne, Phys. Lett. B **472**, 1 (2000) [arXiv:hep-ph/9910256].

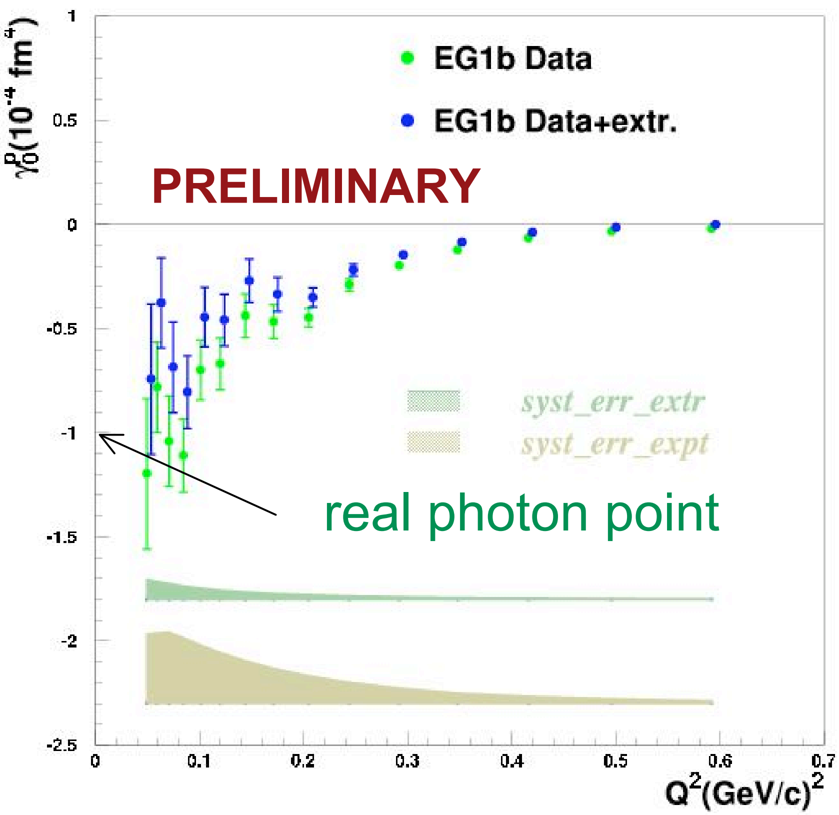
fit to $aQ^2 + bQ^4 + cQ^6 + dQ^8$

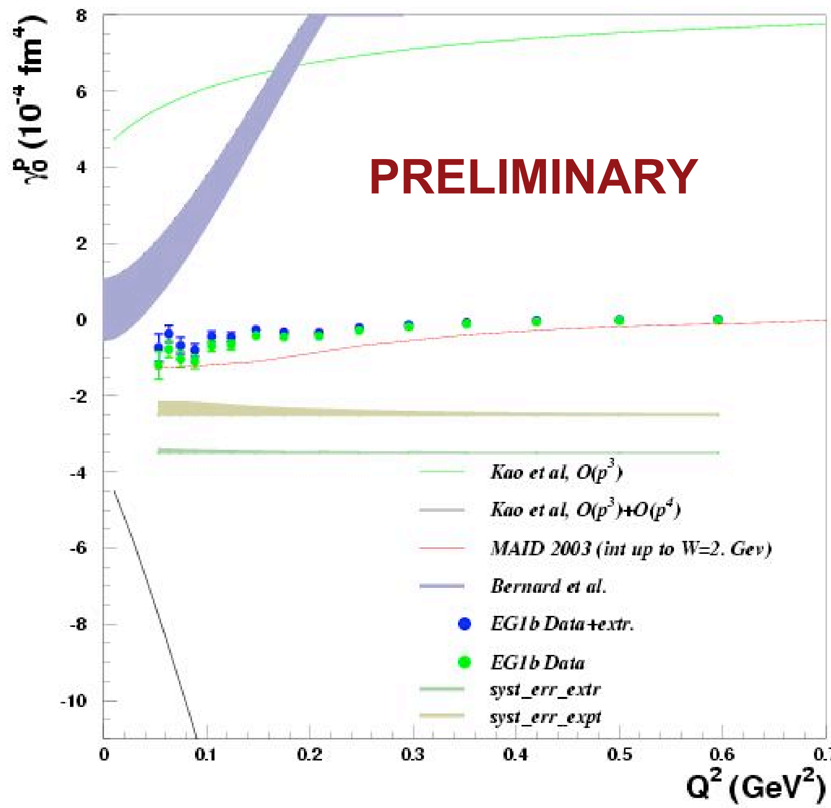
a fixed by GDH
b compared to χPT

The College of WILLIAM & MARY

Forward Spin Polarizability

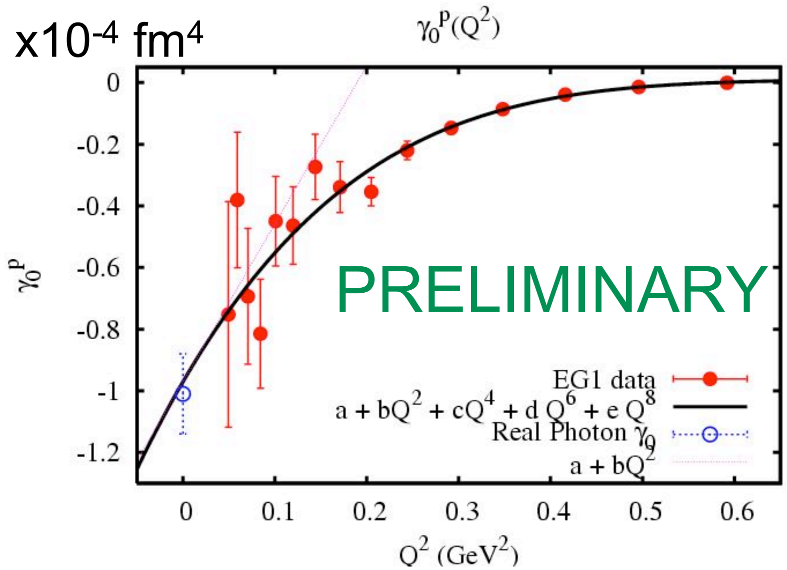
$$\gamma_0(Q^2) = \frac{4e^2M^2}{\pi Q^6}\int_0^{x_0} dx\, x^2\left\{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)\right\}$$





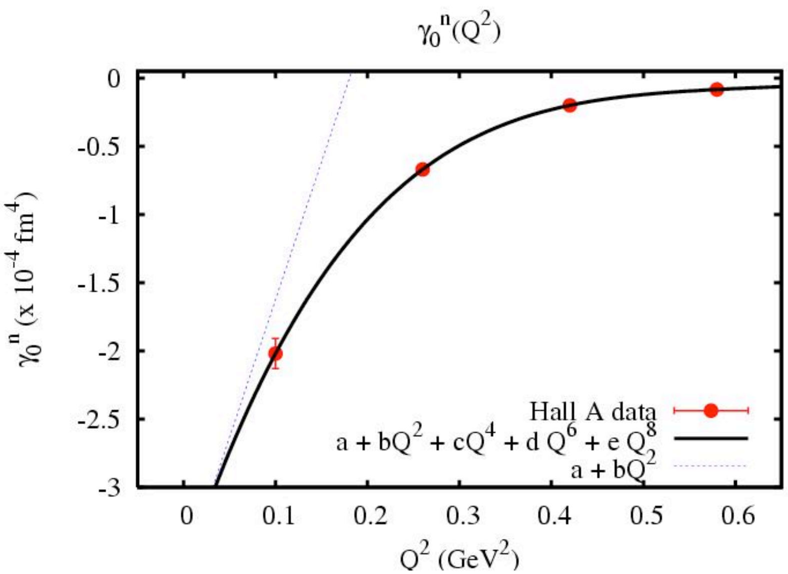
The College of WILLIAM & MARY

γ_0 Fits at Low Q^2



$a = -0.97(11)$
$b = 5.13(94)$

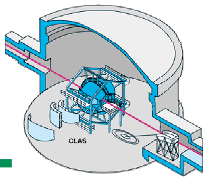
Prok et al., CLAS EG1



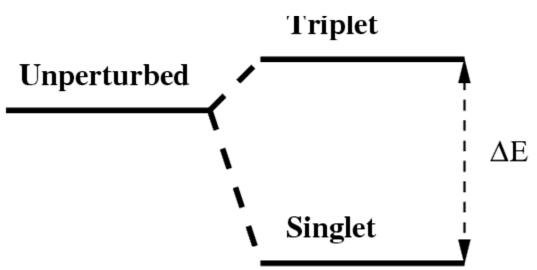
$a = -3.643(1)$
$b = 20.180(8)$

Amarian et al., PRL93(04)152301

Hydrogen Hyperfine Splitting



$$E_{\rm HFS}(e^-p) = 1.4204057517667(9)\,{\rm GHz} = (1+\Delta_{QED}+\Delta_R^p+\Delta_S)E_F^p$$



$$\Delta_S = \Delta_Z + \Delta_{\rm pol} \qquad \delta_Z^{\rm rad} = \frac{\alpha}{3\pi}\left[2\ln\frac{\Lambda^2}{m^2} - \frac{4111}{420}\right]$$

Zemach: $\Delta_Z = -2\alpha m_e \langle r\rangle_Z (1+\delta_Z^{\rm rad})$

$$\langle r\rangle_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa} - 1\right]$$

$\Delta_S = -38.62(16)$ ppm $\Delta_Z = -41.0(5)$ ppm $\Delta_{\rm pol} = 2.38(58)$ ppm

$$\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi(1+\kappa)M}(\Delta_1+\Delta_2) = (0.2264798\ {\rm ppm})(\Delta_1+\Delta_2)$$

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\}$$

$$\Delta_2 = -24m_p^2\int_0^\infty \frac{dQ^2}{Q^4}B_2(Q^2).$$

$$\tau = \nu^2/Q^2$$

$$B_1 = \int_0^{x_{\rm th}} dx\,\beta(\tau)g_1(x,Q^2)\,,$$

$$B_2 = \int_0^{x_{\rm th}} dx\,\beta_2(\tau)g_2(x,Q^2)\,,$$

$$\beta(\tau) = \frac{4}{9}\left(-3\tau+2\tau^2+2(2-\tau)\sqrt{\tau(\tau+1)}\right)$$

$$\beta_2(\tau) = 1+2\tau-2\sqrt{\tau(\tau+1)}\,,$$

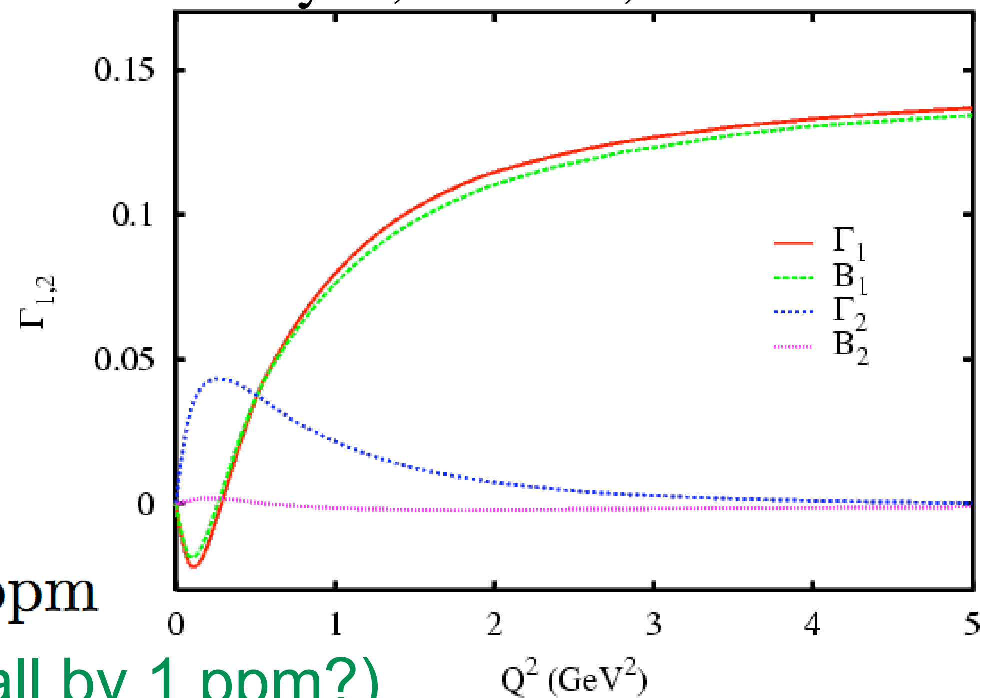
$\Delta_{1,2}$ from $g_{1,2}$

Comparisons between $\Gamma_1 = \int g_1 dx$ and $B_1 = \int \beta_1 g_1 dx$
and between $\Gamma_2 = \int g_2 dx$ and $B_2 = \int \beta_2 g_2 dx$

PRL**96**,163001

Nazaryan, Carlson, KG

- $B_1 \approx \Gamma_1$
- $B_2 \approx 0$
- Experimentally, errors on Γ_1 are understood; we exploit this fact.
- $\Gamma_2 = \int g_2 dx \neq 0$ at low Q^2.

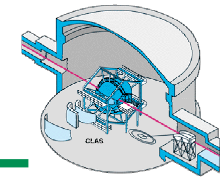


$$\Delta_{pol} = (1.3 \pm 0.3) \text{ ppm}$$

(from EG1: too small by 1 ppm?)

Nucleon structure is the largest uncertainty in calculating HFS.
Better g_1, g_2, G_M, G_E data at low Q^2 required to resolve discrepancy.

Moments at Low Q^2



$$\Gamma_{1,2}^{(N)}(Q^2) = \int_0^{x_{\rm th}} x^N g_{1,2}(x, Q^2) dx$$

$$\Gamma_{1,2}^{(N)} \sim (Q^2)^{N+1}$$

$$\Gamma_1^{(2)} \rightarrow \gamma_0 Q^6/(16\alpha m_p^2)$$

$$\gamma_0(Q^2) = \frac{16\alpha m_p^2}{Q^6} \int_0^{x_{\rm th}} x^2 \left(g_1 - \frac{4m_p^2 x^2}{Q^2} g_2 \right) dx$$

$$\Gamma_1^{(0)} = -\kappa_p^2 Q^2/(8m_p^2) + c_1 Q^4 + \ldots$$

$$B_1 = \Gamma_1^{(0)} - 10 m_p^2 \Gamma_1^{(2)}/(9Q^2) + \ldots.$$

$$\Delta_1[0, Q_1^2] = \left\{ -\frac{3}{4} r_P^2 \kappa_p^2 + 18 m_p^2 c_1 - \frac{5m_p^2}{4\alpha}\gamma_0 \right\} Q_1^2$$

$$\Delta_2[0, Q_1^2] = 3 m_p^2 Q_1^2 (\gamma_0 - \delta_{LT})/2\alpha$$

$$\delta_{LT}(Q^2) = \left(\frac{1}{2\pi^2}\right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu$$
$$= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1(x, Q^2) + g_2(x, Q^2)] dx$$

Δ_{pol}

$$\Delta_2[0, Q_1^2] = 3m_p^2 Q_1^2(\gamma_0 - \delta_{LT})/2\alpha$$

$$\Delta_1[0, Q_1^2] = \left\{ -\frac{3}{4} r_P^2 \kappa_p^2 + 18 m_p^2 c_1 - \frac{5m_p^2}{4\alpha}\gamma_0 \right\} Q_1^2$$

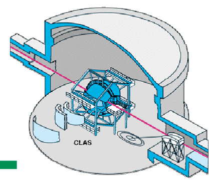
γ_0 = -1.01x10^{-4} fm^4 (photons)
r_P = 0.878(15) fm (Kelly)
c_1 = 2.95-3.89 (fits/χPT)
δ_{LT} = 1.35x10^{-4} fm^4 (MAID)

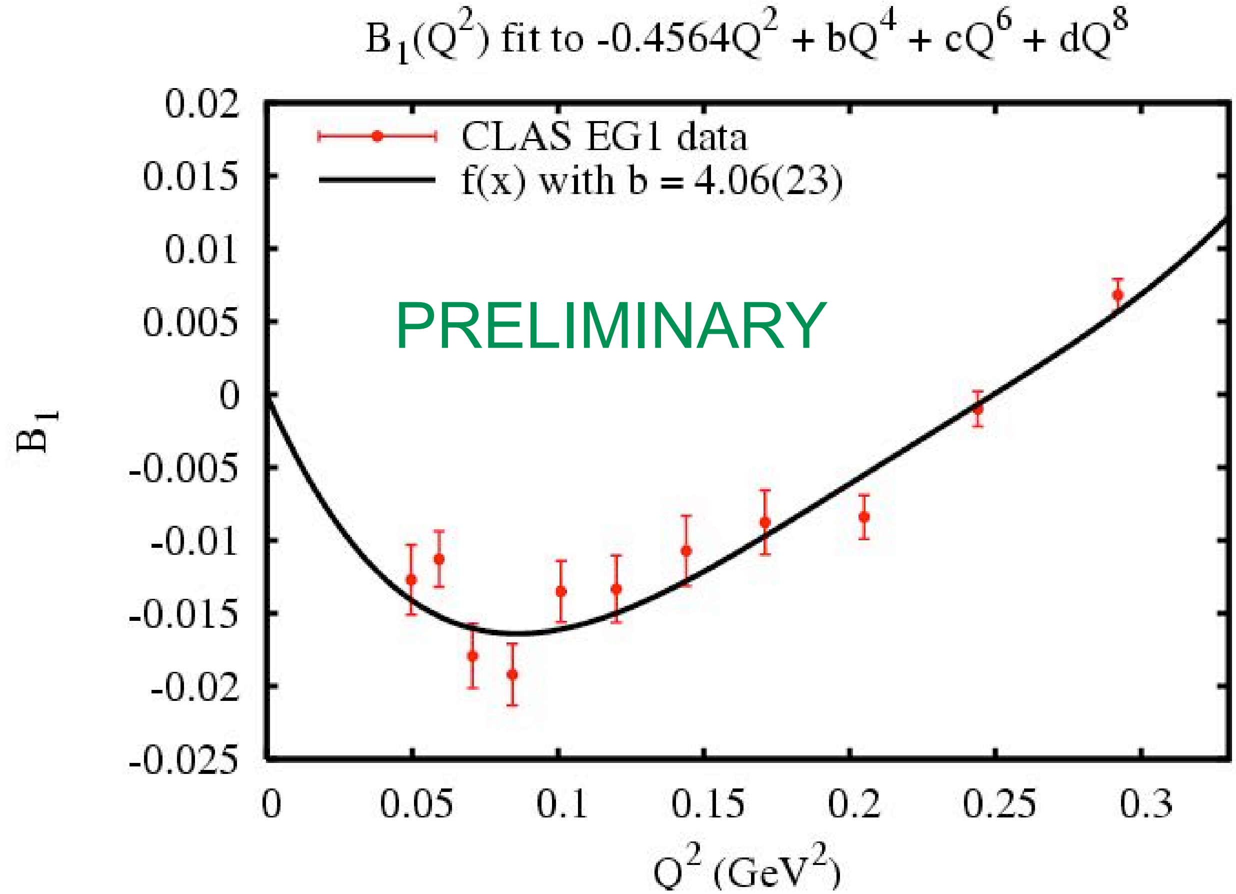
$$g_2 = g_2^{WW} \rightarrow \Gamma_2^{(N)} = -N\Gamma_1^{(N)}/(N+1)$$

Δ_2[0,0.05] =
-0.40(05) [g_2^{WW}]
-1.4 [MAID]
-0.24 [EG1 Model]

term	Q^2 (GeV^2)	from	Kelly's F_2
Δ_1	[0,0.05]	F_2 and g_1	0.45 ± 0.30
	[0.05,20]	F_2	7.01 ± 0.22
		g_1	-1.10 ± 0.55
	[20,∞]	F_2	0.00
		g_1	0.12 ± 0.01
total Δ_1			6.48 ± 0.89
Δ_2	[0,0.05]	g_2	-0.24 ± 0.24
	[0.05,20]	g_2	-0.33 ± 0.33
	[20,∞]	g_2	0.00
total Δ_2			-0.57 ± 0.57
$\Delta_1 + \Delta_2$			5.91 ± 1.06
Δ_{pol}			1.34 ± 0.24 ppm

B_1 from CLAS



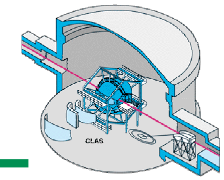


Various estimates change Δ_{pol} up or down within the quoted errors. New data at low Q^2 are needed to improve this.

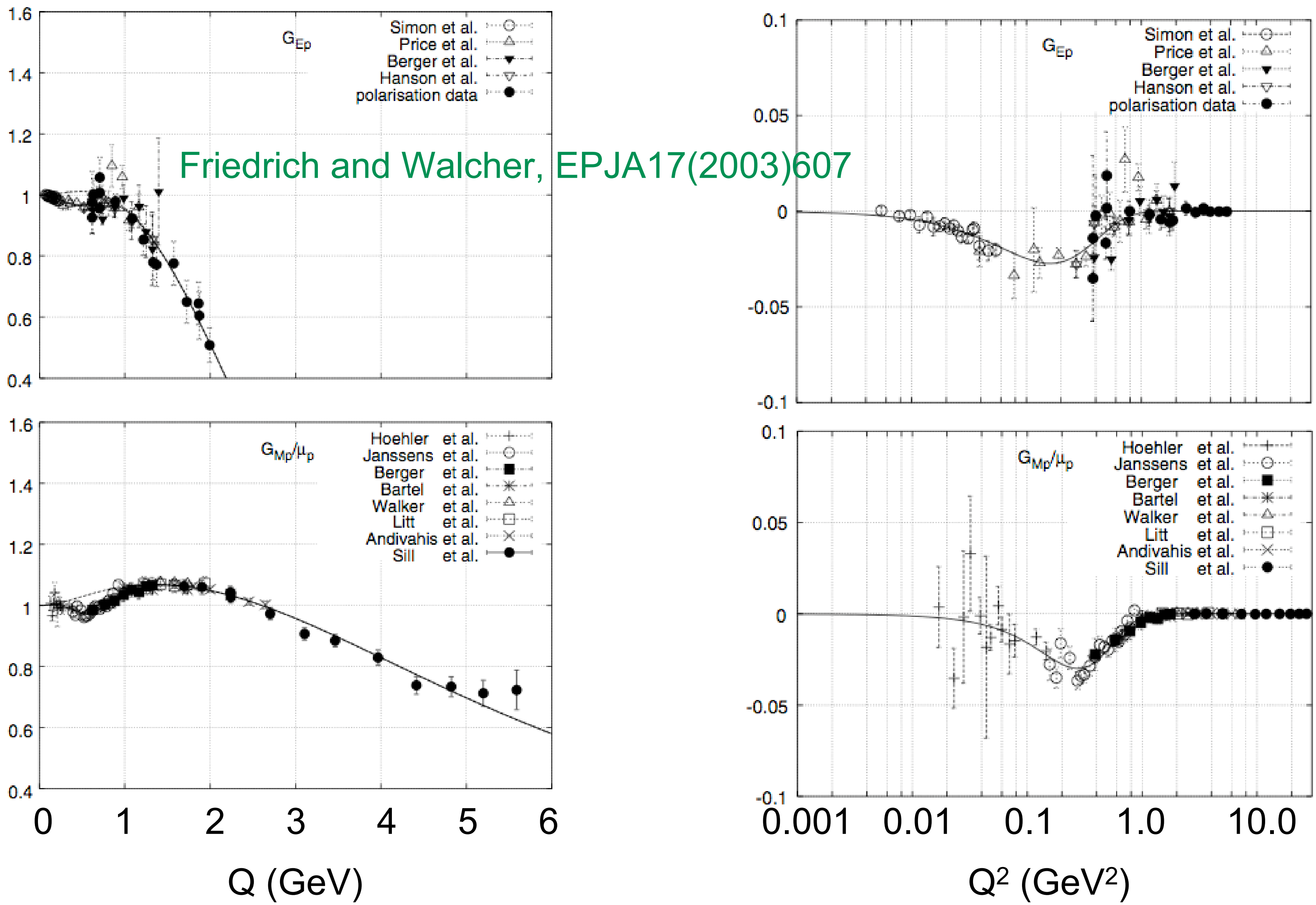
$$\Delta_1[0,0.05] = [-0.75 r_P^2 \kappa_p^2 + 18 m_p^2 b]0.05 = 0.32$$

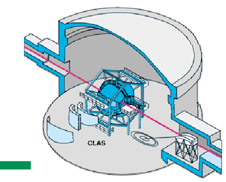
The College of WILLIAM & MARY

G_{Ep} & G_{Mp}



Friedrich and Walcher, EPJA17(2003)607

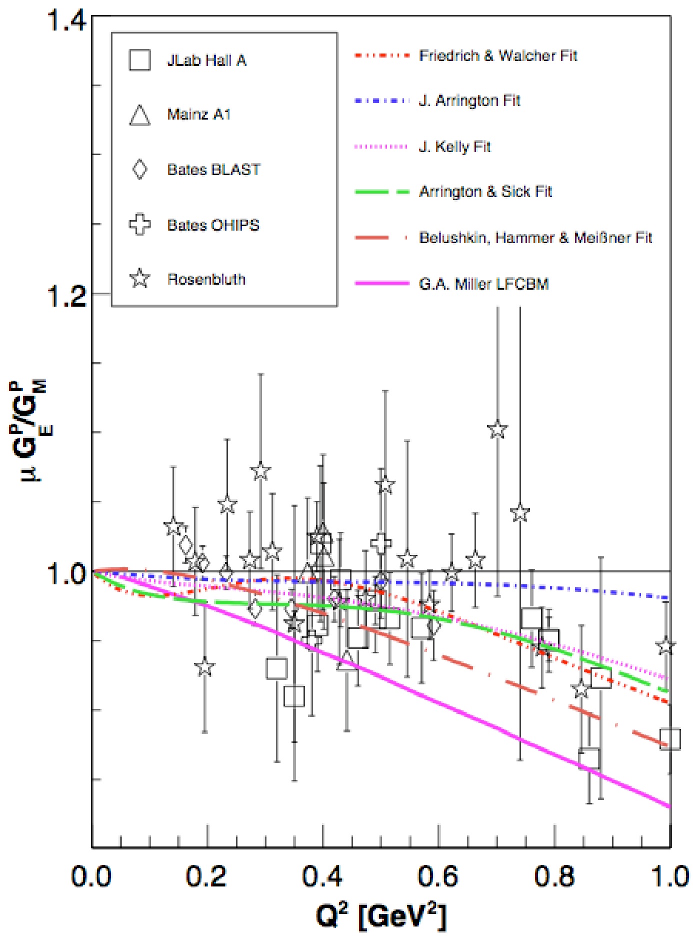


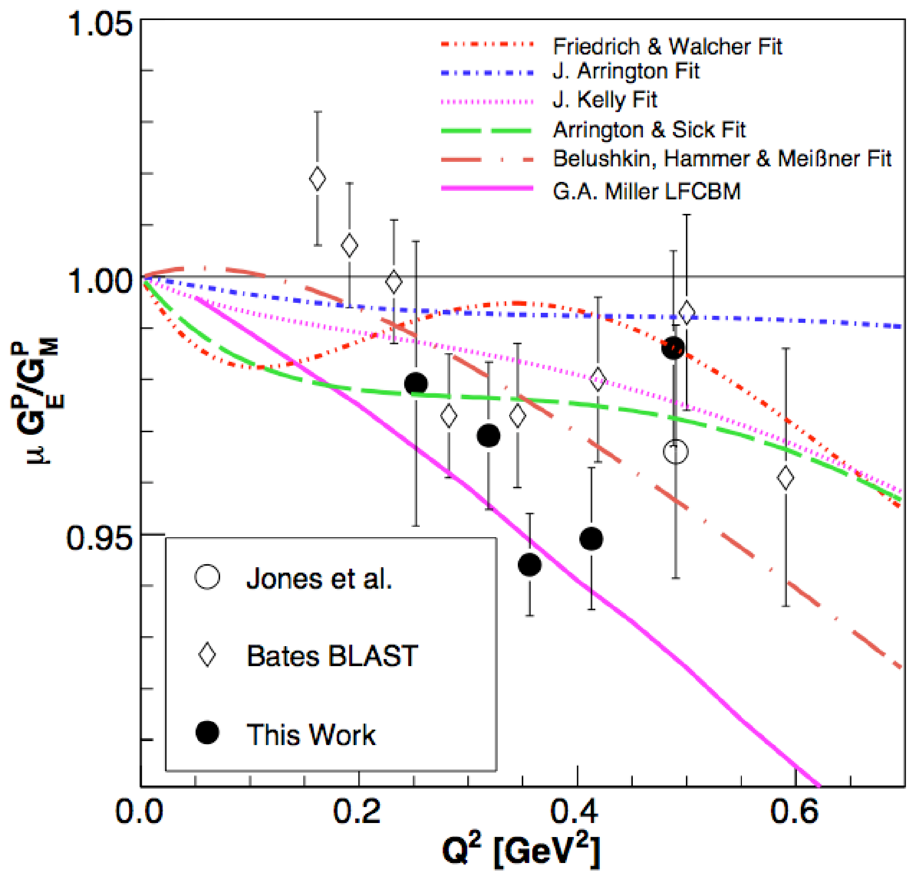


$\mu G_{Ep}/G_{Mp}$

G. Ron et al., nucl-ex 0706.0128

Hall A

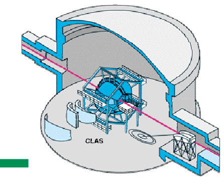




The diversity of fits reflects an inaccurate knowledge of the form factors at low Q^2

The College of WILLIAM & MARY

Zemach Radius



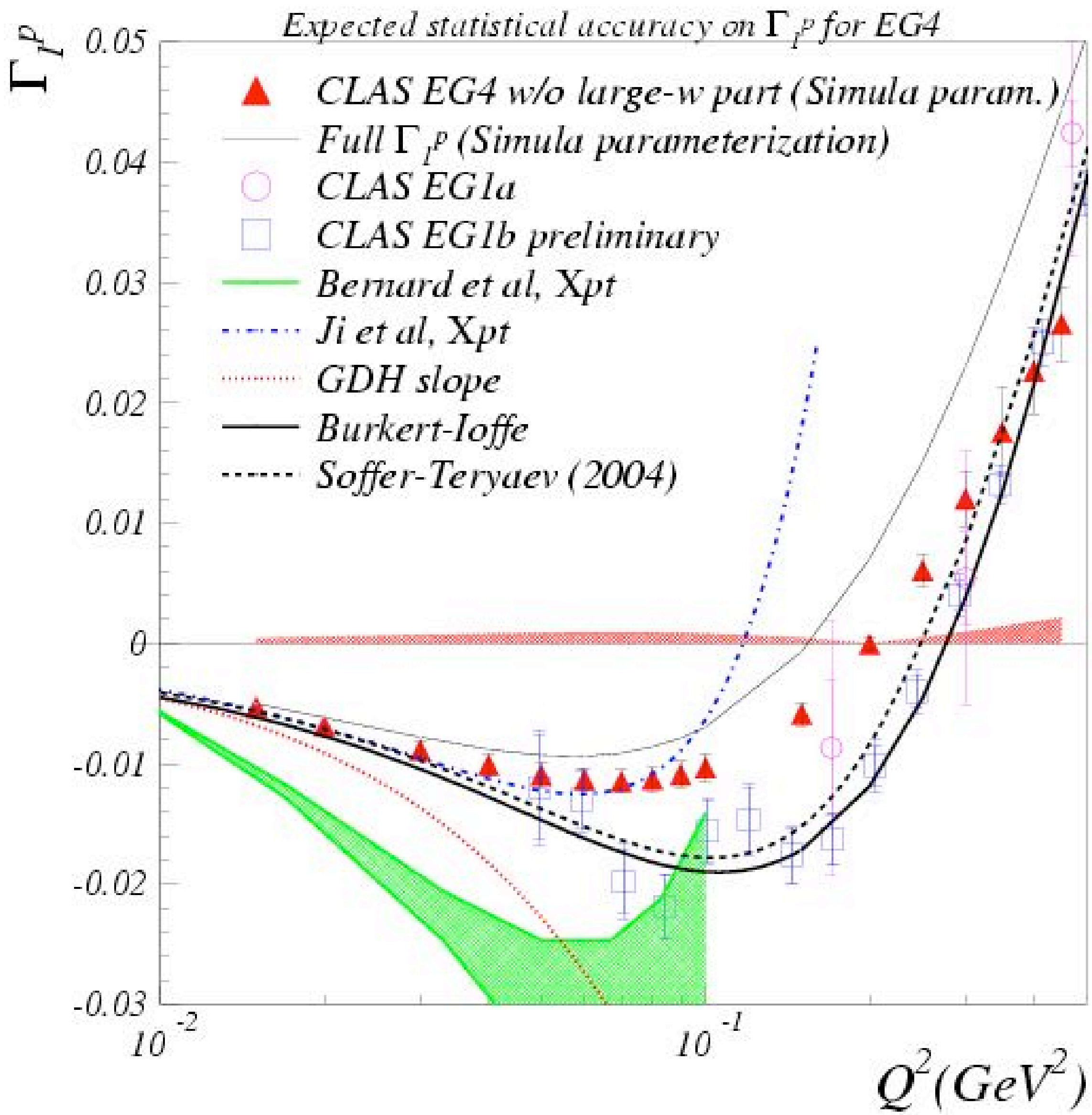
Zemach: $\Delta_Z = -2\alpha m_e \langle r \rangle_Z (1 + \delta_Z^{\mathrm{rad}})$

$$\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa} - 1 \right]$$

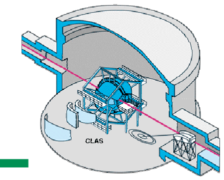
Reference	r_Z (fm)	Δ_Z (ppm)	Δ_S - Δ_Z - Δ_{pol} (ppm)
Kelly	1.069(13)	-41.01	1.11
Sick	1.086(12)	-41.67	1.77
Friedrich	1.048	-40.20	0.30
Dipole	1.025	-39.32	-0.58

Quoted errors on S,Z and pol are 0.16, 0.49, and 0.24 ppm respectively.
Quoted error on S-Z-pol is 0.57 ppm.
Largest uncertainty in hyperfine splitting comes from low Q^2 form factors!

EG4 Expectations



In Conclusion



Jefferson Lab, past, present and future, provides high-quality structure function and form factor data that make the experimental determination of moments possible.

- rigorous χPT calculations are often lost on us because our Q^2 is too high
- atomic physics with 14-digit accuracy, in which the nuclear physics enters at the ppm level, is often lost on us because our Q^2 is too high
- more structure function data at low Q^2 are on the way
- before the 12 GeV upgrade, Jefferson Lab should do more precise measurements at low Q^2 including G_E and G_M