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A Decade of Structure Function Measurements at Jefferson Lab

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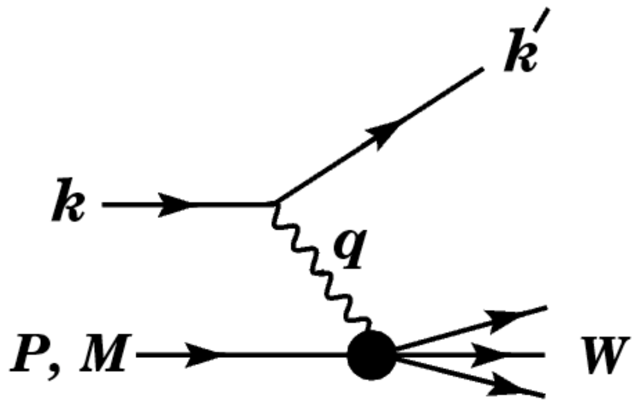
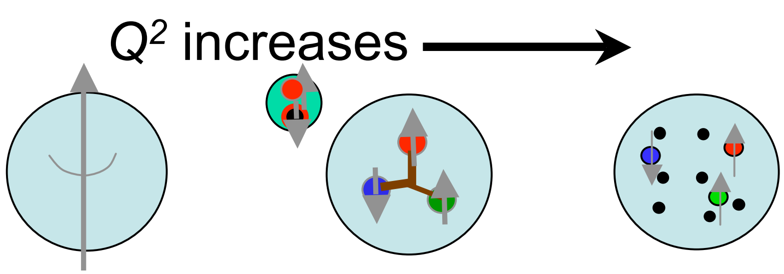
Inelastic Scattering



Lorentz invariants:

$\nu = p \cdot q/M = (E - E')_{\rm lab}$
$Q^2 = -q \cdot q = (4EE' \sin^2 \frac{\theta}{2})_{\rm lab}$
$x = -q \cdot q/2p \cdot q = (Q^2/2M\nu)_{\rm lab}$
$y = p \cdot q/p \cdot k = (\nu/E)_{\rm lab}$
$W^2 = (p+q)^2 = (M^2 + 2M\nu - Q^2)_{\rm lab}$
$s = (k+p)^2 = (2EM + M^2)_{\rm lab}$



Unpolarized Cross Section:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{8\pi\alpha^2 y}{Q^4}\left[\frac{y}{2}F_1 + \frac{\xi}{2xy}F_2\right]$$

Polarized Cross Section:

$$\frac{d^2\Delta\sigma}{dx dQ^2} = \frac{8\pi\alpha^2 y}{Q^4}[\cos\alpha\{(\xi + \frac{y}{2})g_1 - \frac{\gamma y}{2}g_2\} - \sin\alpha\cos\phi\{\frac{y}{2}g_1 + g_2\}]$$

α = polar angle of target spin wrt the beam axis
ϕ = azimuthal spin angle wrt the scattering plane
$\alpha = 0°$ (longitudinal); $\alpha = 90°, \phi = 0°$ (transverse).
$\gamma^2 = 4M^2x^2/Q^2 = Q^2/\nu^2$
$\xi = 1 - y - \gamma y^2/4$

Parton Model:

$F_1(x,Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) + q^\downarrow(x) + \bar{q}^\uparrow(x) + \bar{q}^\downarrow(x))$
$F_2(x,Q^2) = 2xF_1(x,Q^2)$
$g_1(x,Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x))$

$F_2(x,Q^2)$ and $g_1(x,Q^2)$

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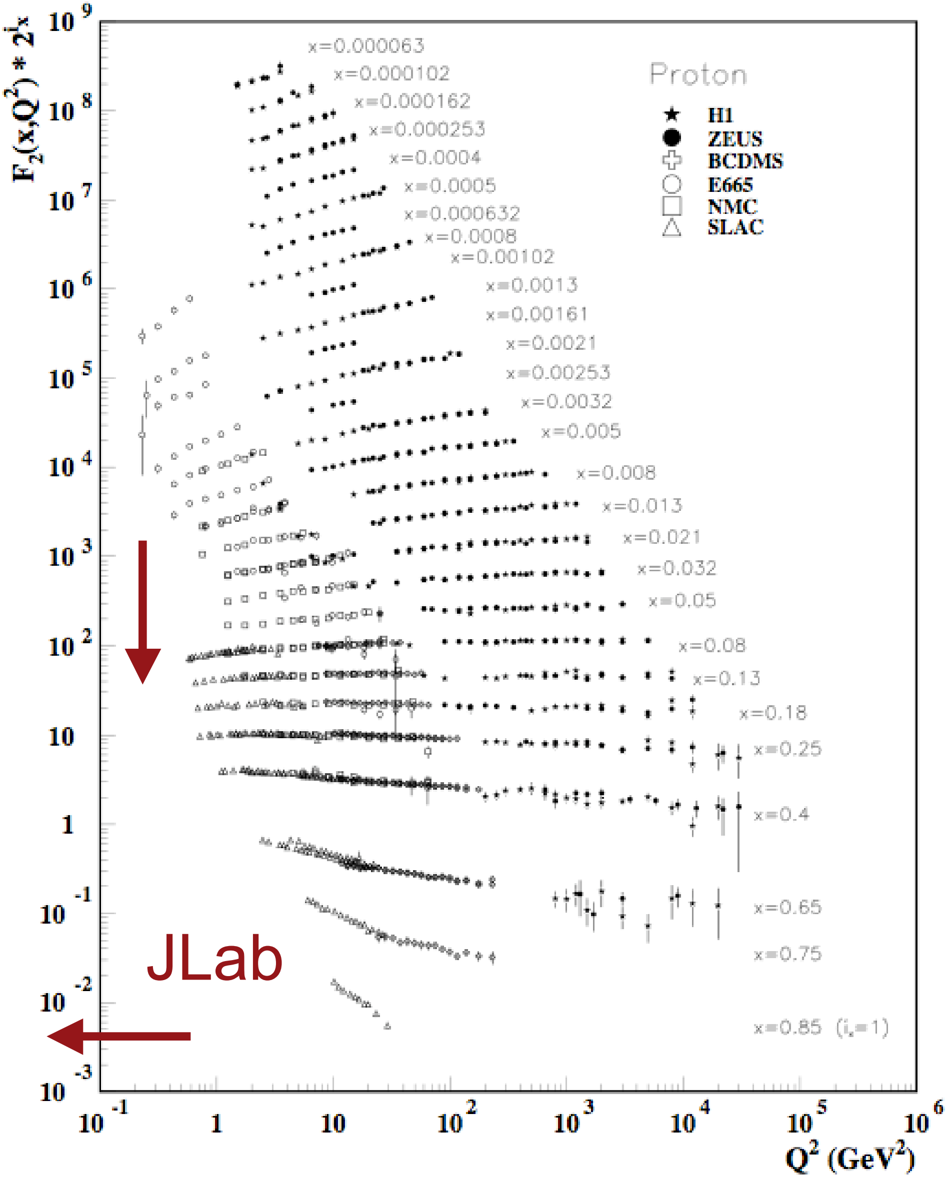
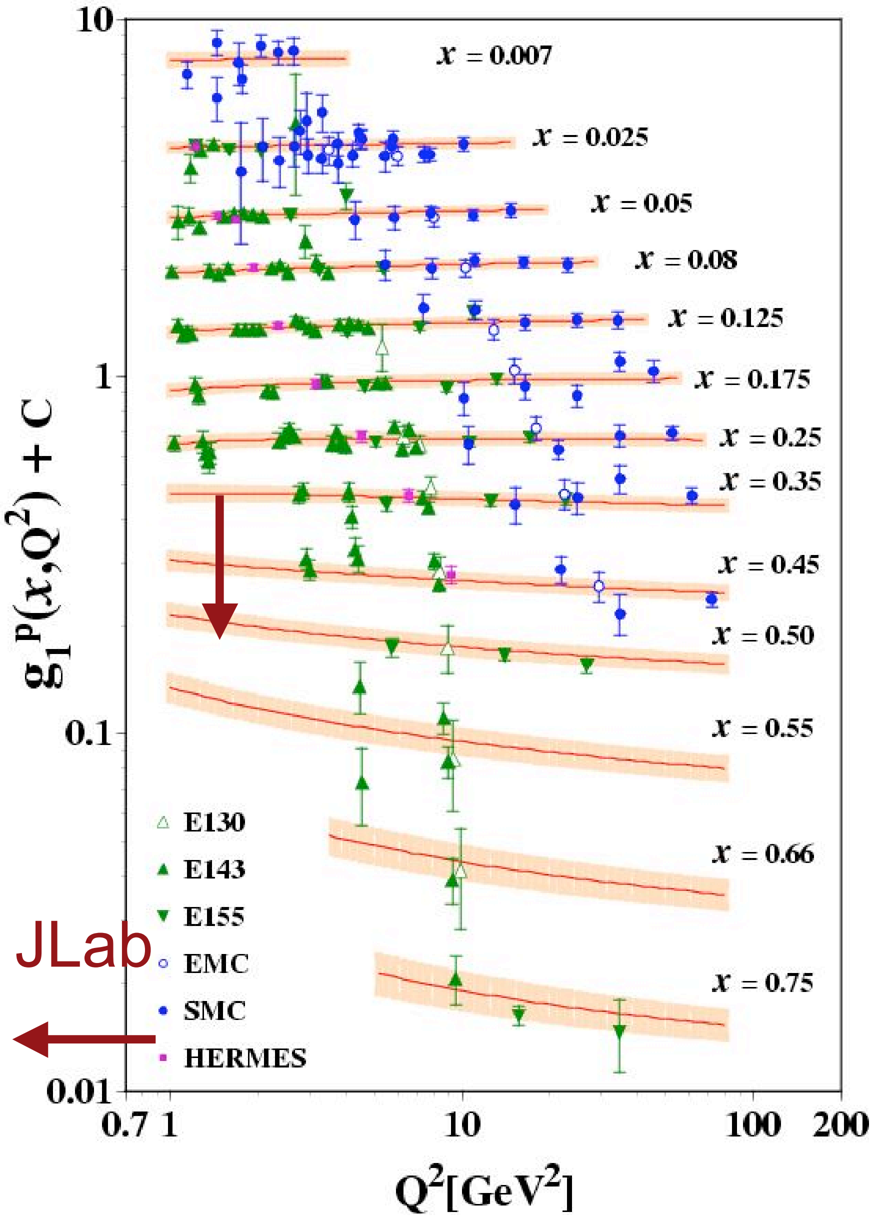
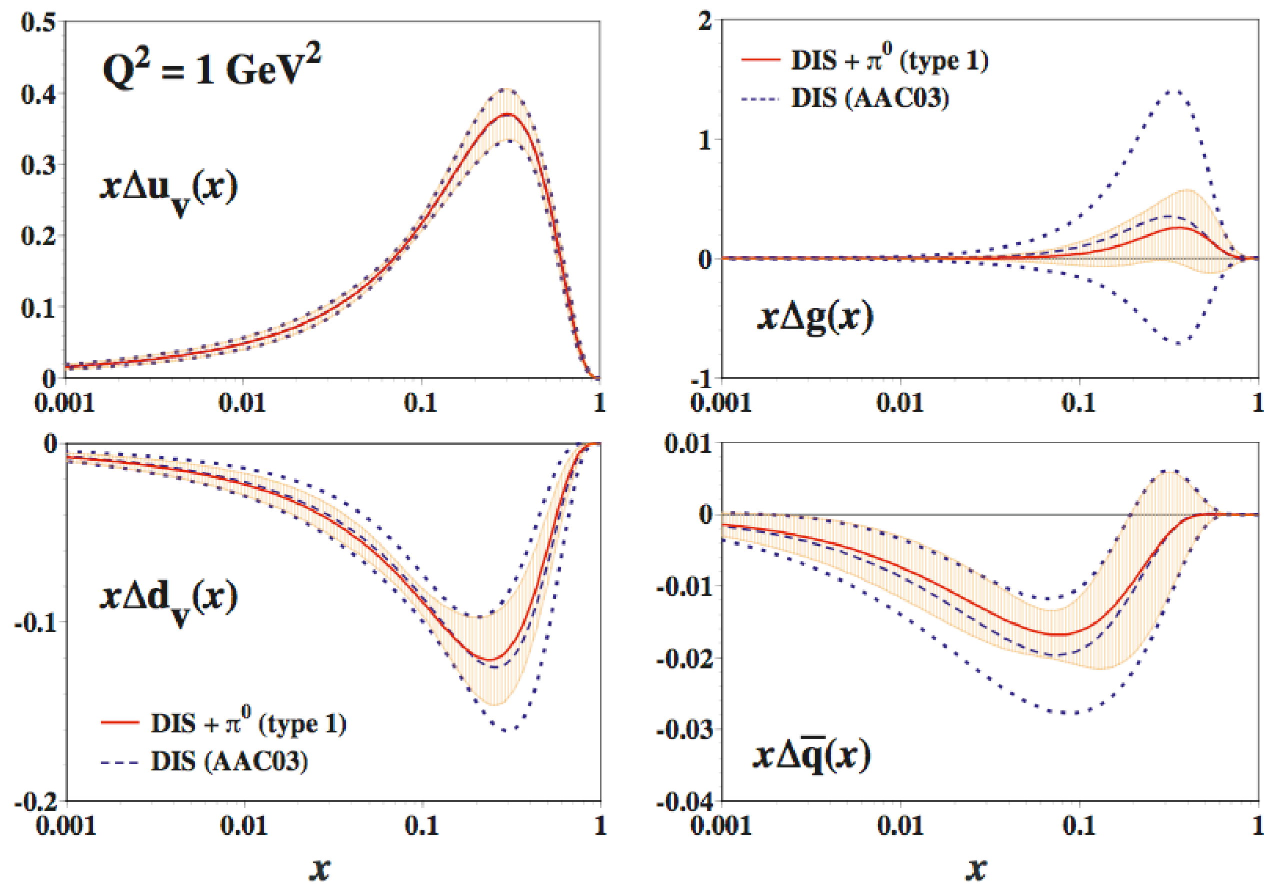




NLO Fits

$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

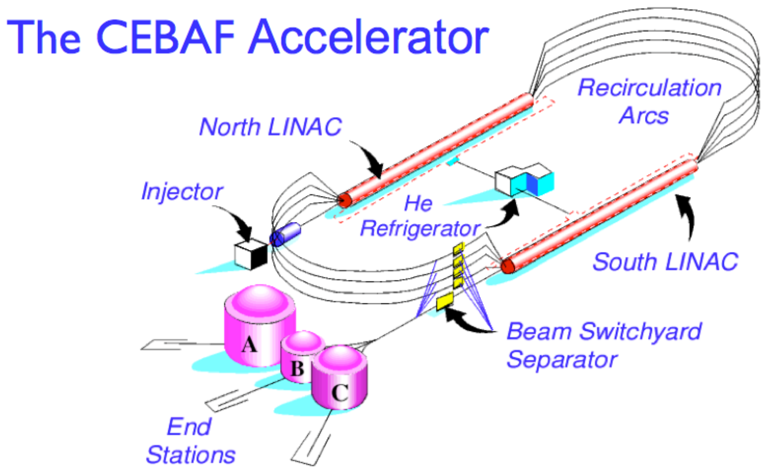
	$\Delta\bar{q}$	Δg	$\Delta\Sigma$
Type 1	-0.05 ± 0.01	0.31 ± 0.32	0.27 ± 0.07
Type 2	-0.06 ± 0.02	0.47 ± 1.08	0.25 ± 0.10
AAC03	-0.06 ± 0.02	0.50 ± 1.27	0.21 ± 0.14

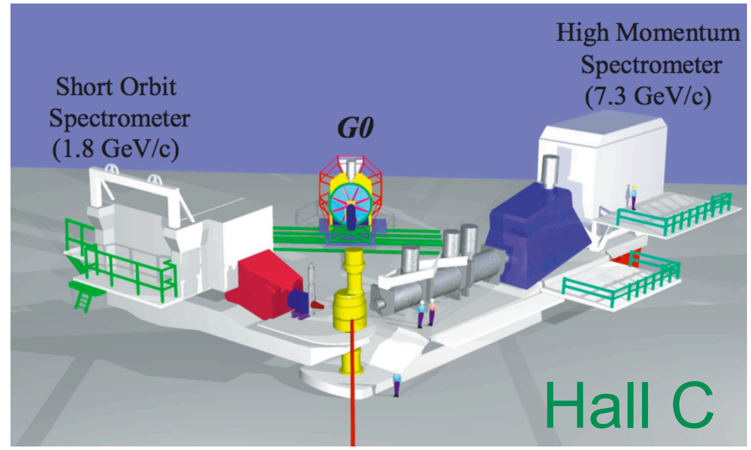


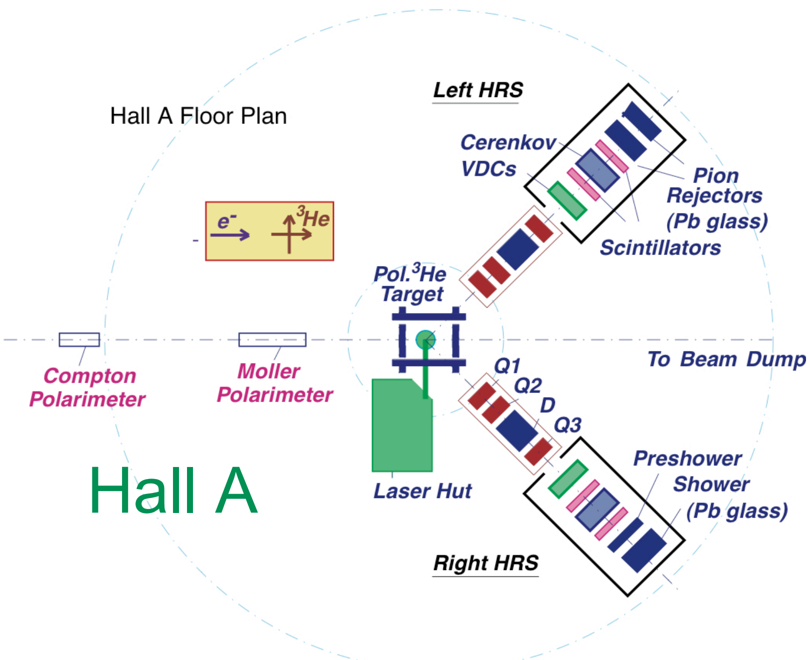
The Q^2 Map

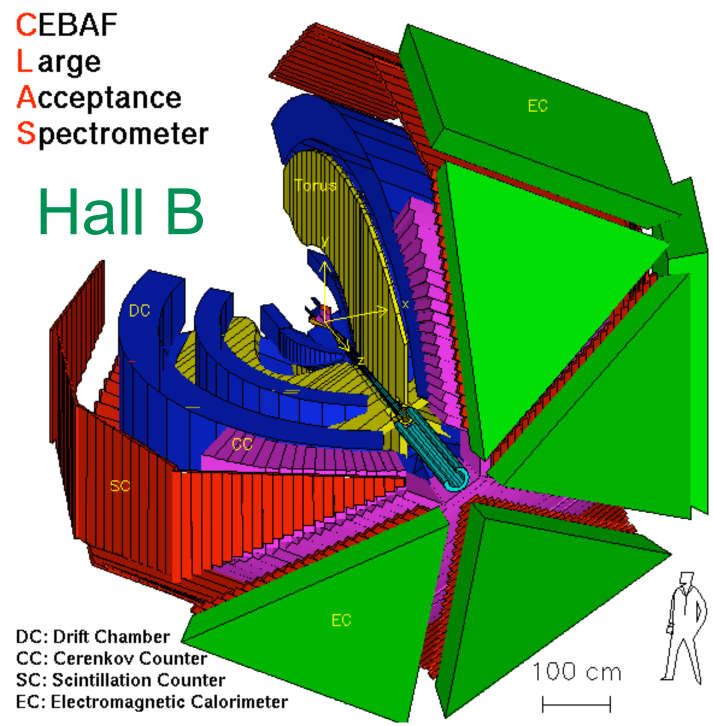
- Infinite Q^2 — Parton Model, PDF(x)
- Large Q^2 — pQCD, PDF(x,log(Q^2))
- Medium Q^2 — Higher twist, target mass correct.
- Low Q^2 — Resonances (complexity)
- Tiny Q^2 — Chiral perturbation theory
- Zero Q^2 — Real photons
- Complexity, as measured by γ_0, δ_{LT}, d_2 and Γ_1 disappears rapidly at high and low Q^2

JLab


The CEBAF Accelerator
North LINAC
Recirculation Arcs
Injector
He Refrigerator
South LINAC
Beam Switchyard Separator
A
B
C
End Stations


Short Orbit Spectrometer (1.8 GeV/c)
G0
High Momentum Spectrometer (7.3 GeV/c)
Hall C


Hall A Floor Plan
Left HRS
Cerenkov
VDCs
Pion Rejectors (Pb glass)
Scintillators
Pol.³He Target
Compton Polarimeter
Moller Polarimeter
Q1
Q2
D
Q3
To Beam Dump
Preshower
Shower (Pb glass)
Laser Hut
Right HRS
Hall A


CEBAF
Large
Acceptance
Spectrometer
Hall B
EC
Torus
DC
CC
SC
DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter
100 cm

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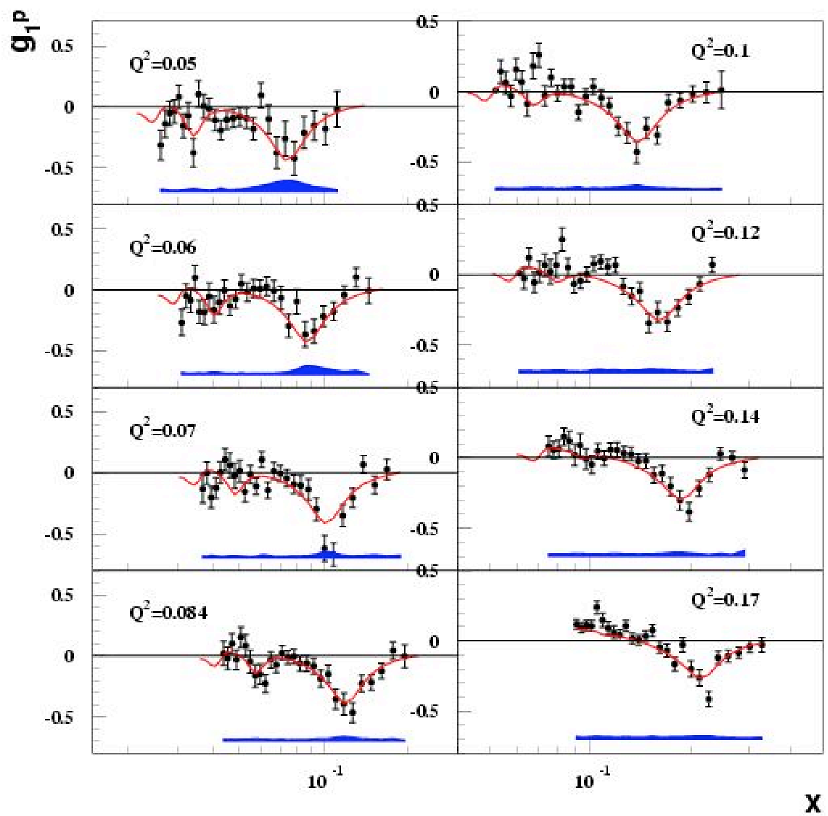
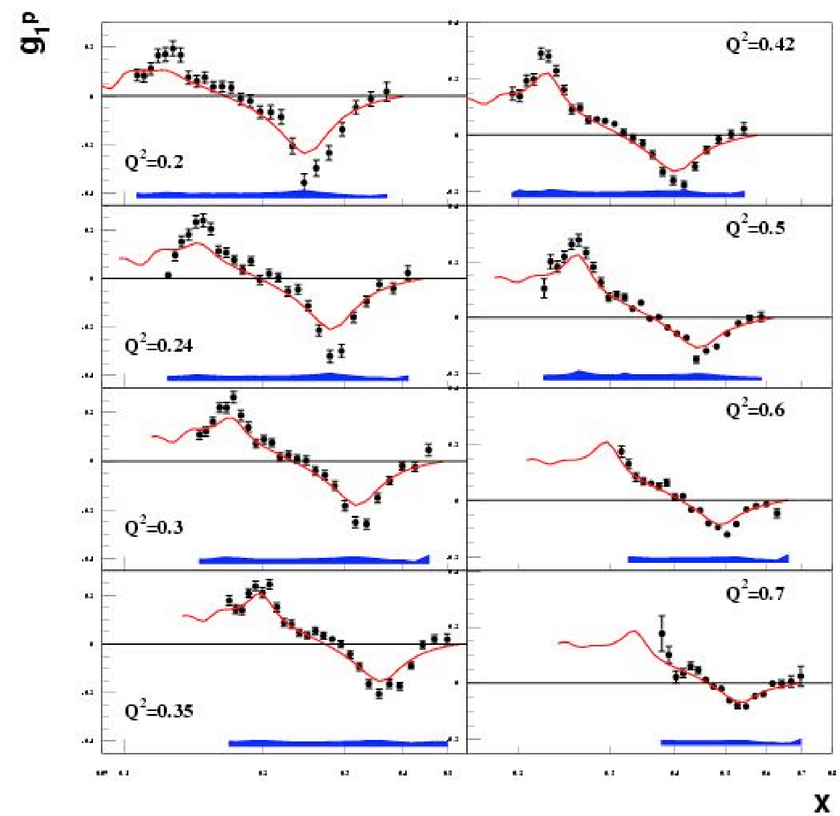
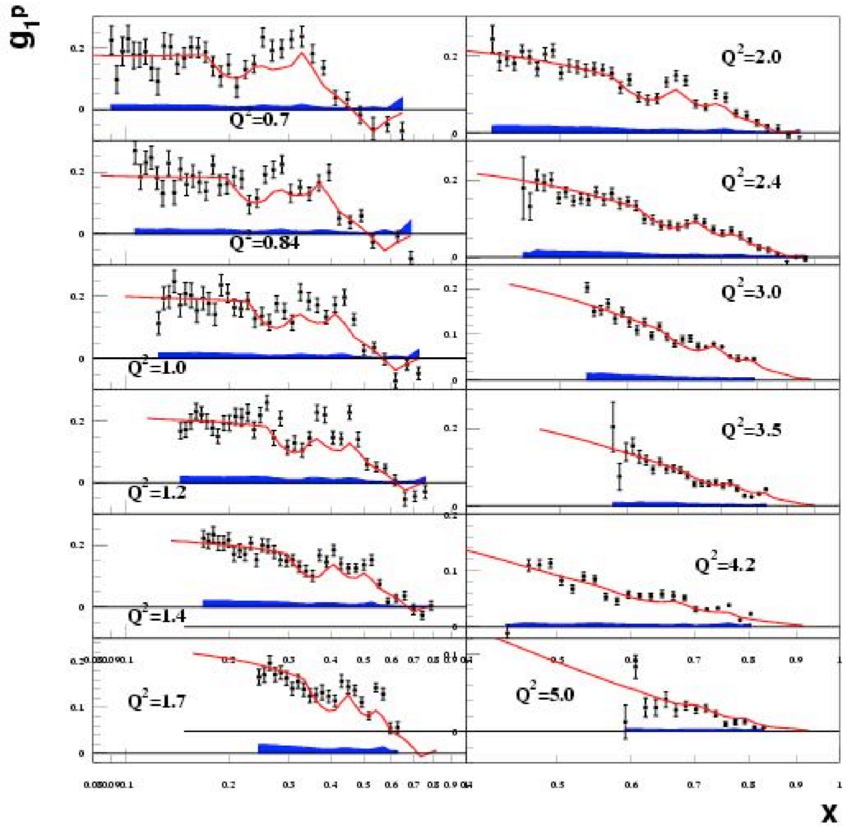
CLAS EG1 g_1^p $(Q^2<0.7)$

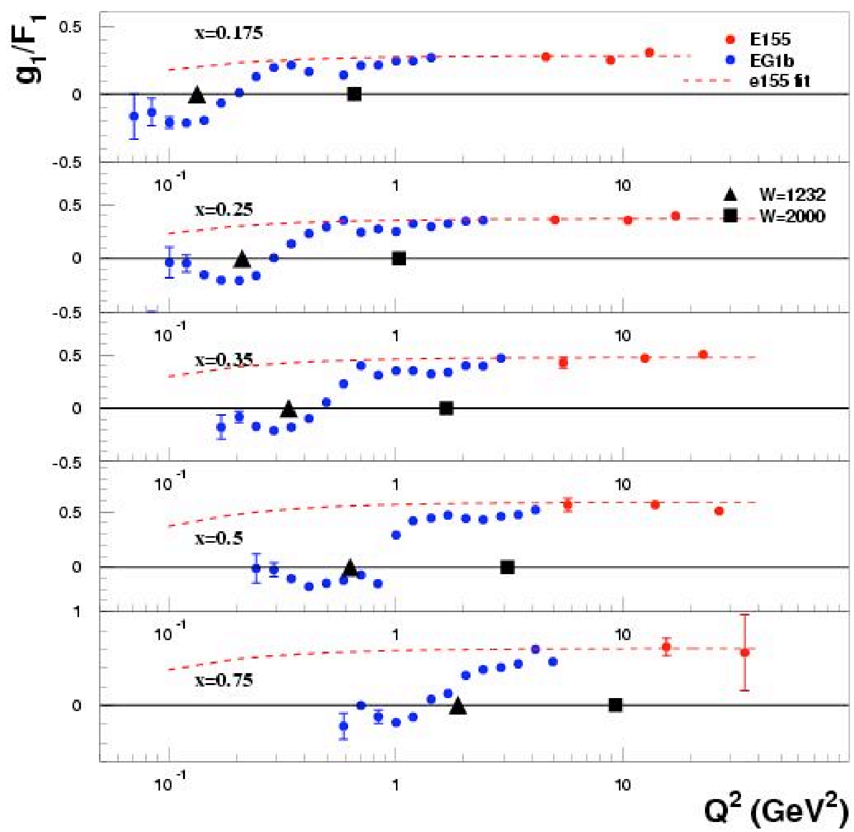




- At low Q^2 the Δ resonance drives g_1 negative
- Extensive x-range at fixed Q^2 allows integration over x
- Red curve is the EG1 model used for radiative corrections

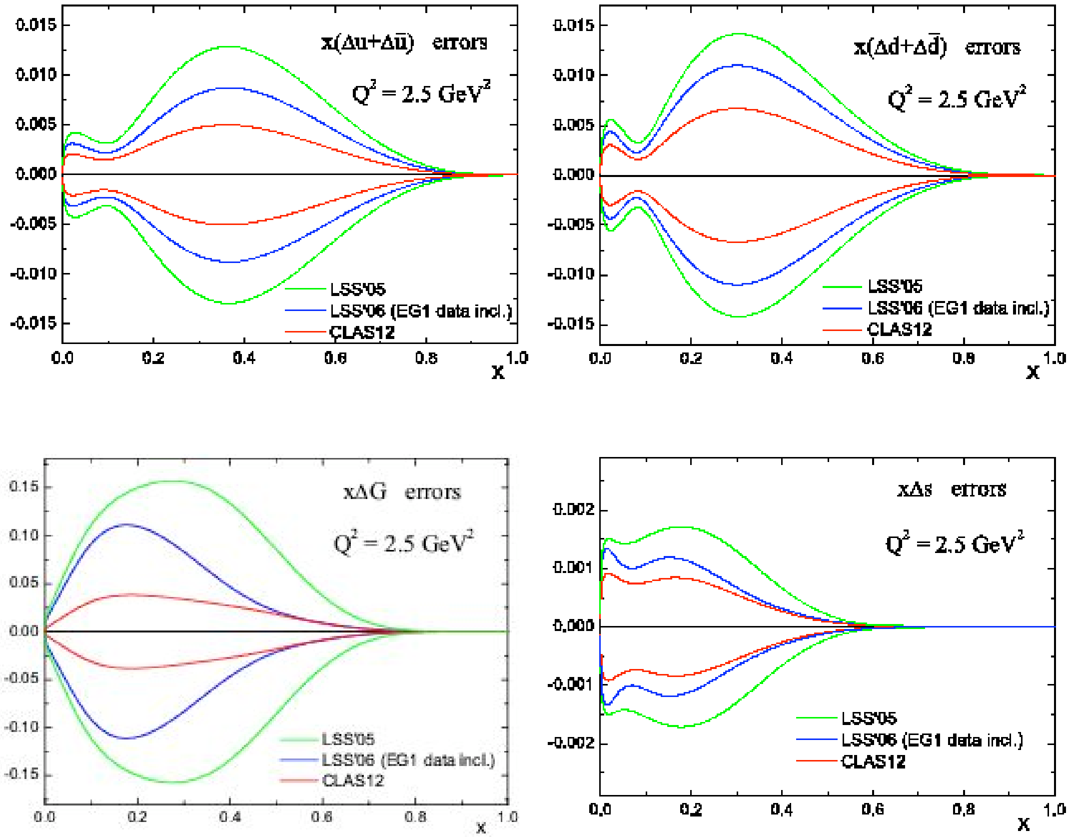
CLAS EG1 g_1^p (Q^2>0.7) & g_1/F_1





- At higher Q^2, g_1 becomes positive everywhere
- g_1/F_1 falls far below the DIS extrapolation at low Q^2
- Red curve is the EG1 model (dashed: DIS extrapolation)

PDFs and CLAS

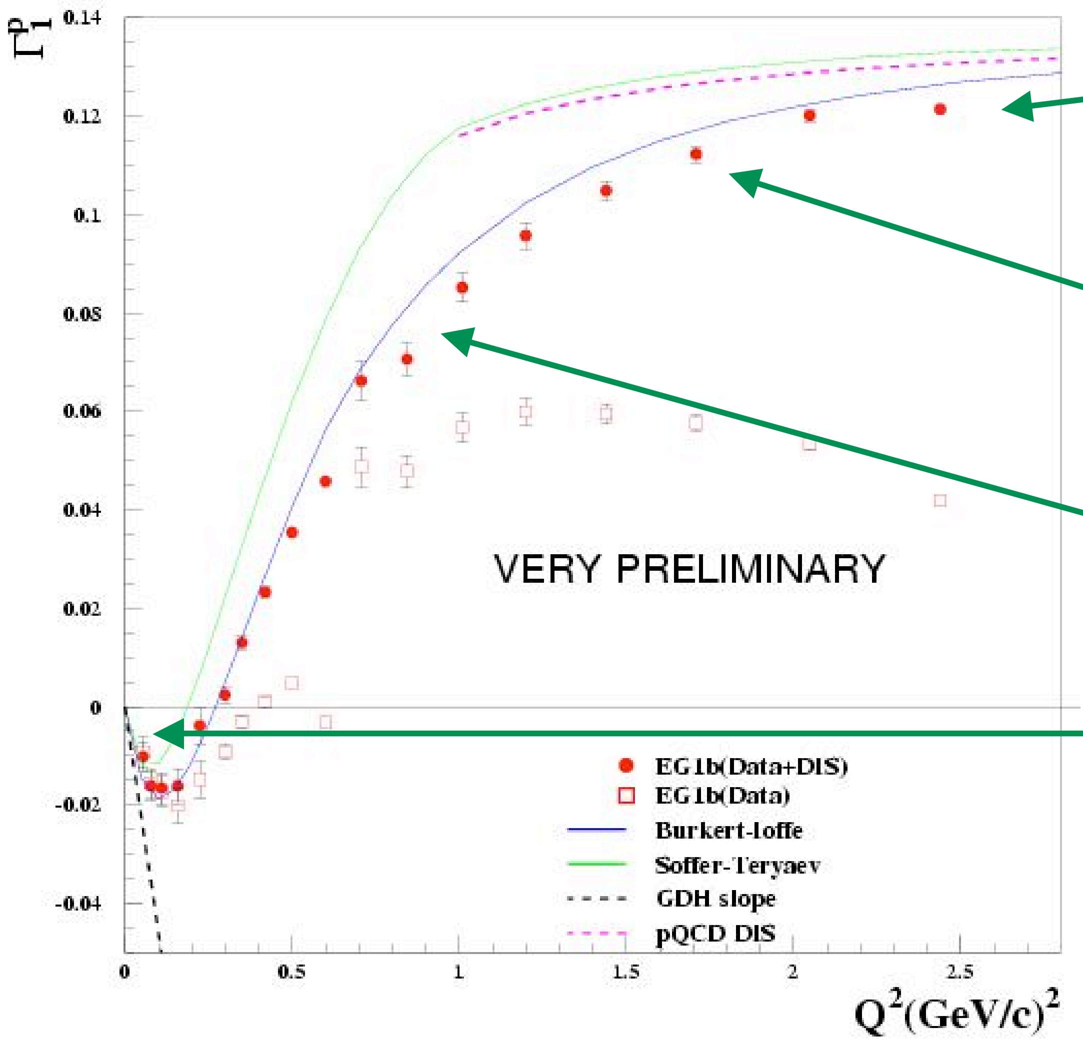


- Error envelopes for PDFs from LSS05 global analysis (green)
- CLAS EG1 data significantly improve errors on Δu, Δd, Δx and ΔG (blue)
- CLAS EG12 (12 GeV upgrade) will especially improve ΔG (red)

Regions of Q^2

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x, Q^2)dx$$

CLAS EG1 Data



scaling: $\ln Q^2$

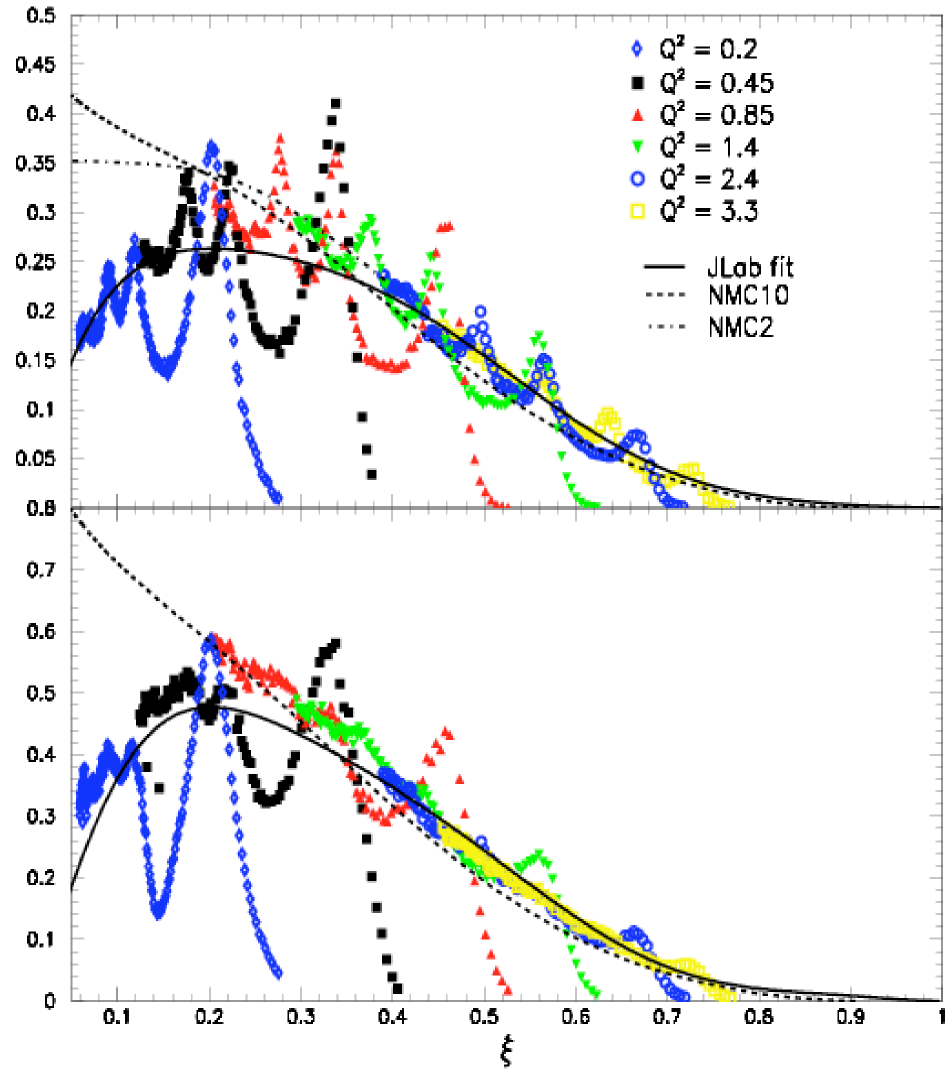
higher twist: $(1/Q^2)^n$

no nice expansion

χPT: $(Q^2)^n$

Duality

Hall C
PRL85(00)1182
Global duality to 10%
Local duality to 10%
W=1.232, 1.535, 1.680 GeV



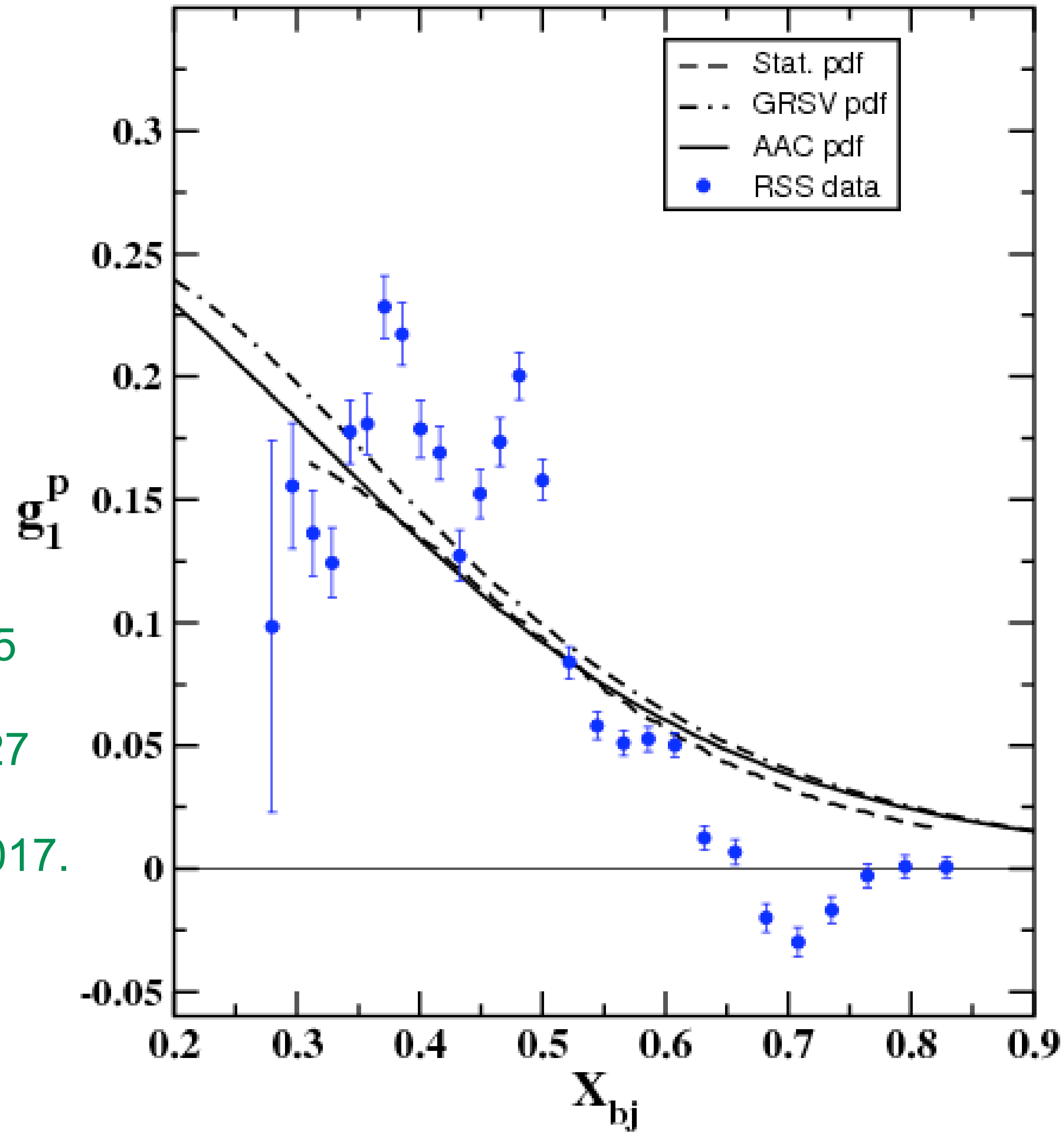
Duality - structure functions averaged over resonances behave according to DIS systematics

Global - all resonances

Local - one resonance

Polarized Duality

Hall C
RSS, Wesselmann, Slifer
Q^2=1.379 GeV^2
Target Mass Corrections
applied to PDFs
No duality for Δ
PRL98(07)132003



GRSV: Phys. Rev. D 53, (1996) 4775

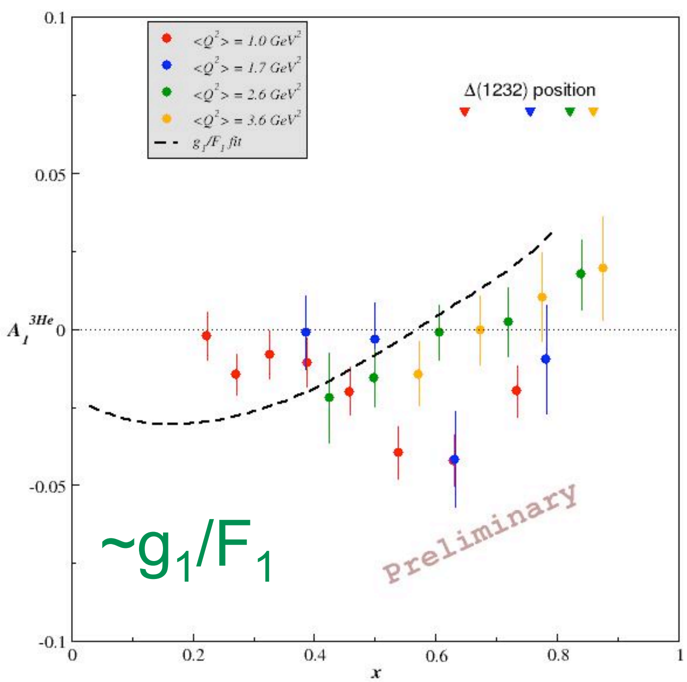
BSB : Eur. Phys. J. C 41, (2005) 327

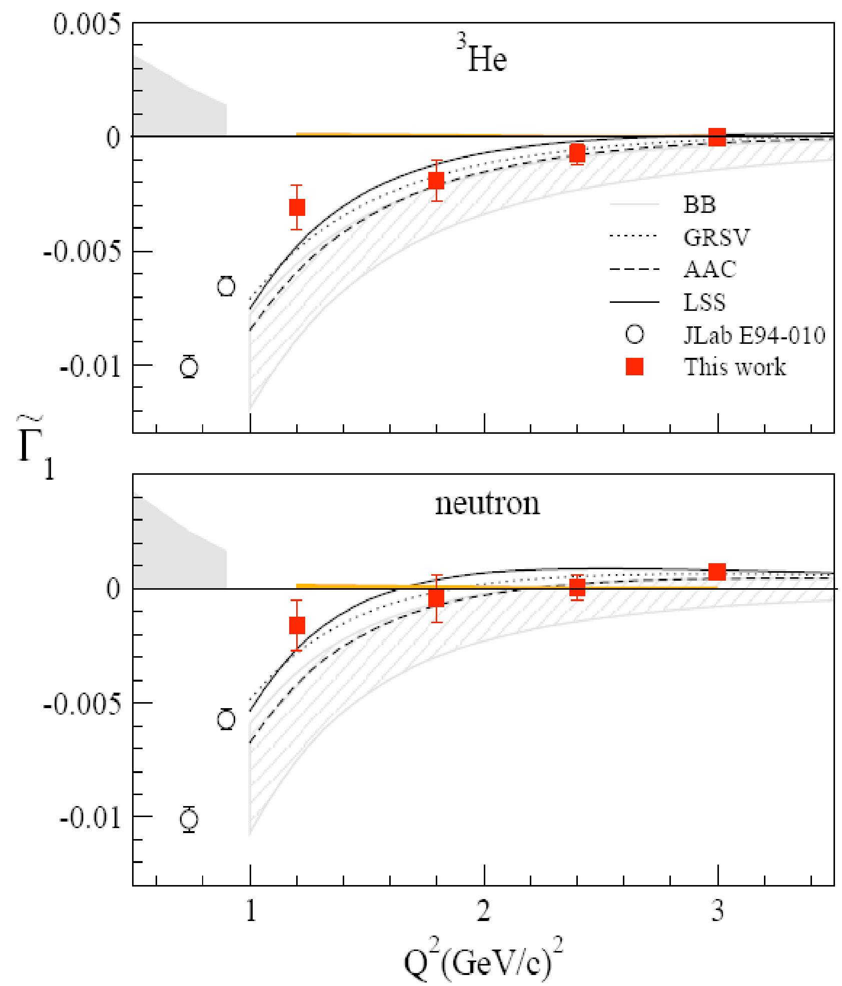
AAC : Phys. Rev. D 62, (2000) 034017.

Duality in ^{3}He

$$\tilde{\Gamma}_1^{res} = \int_{x_{\min}}^{x_{\max}} g_1^{res}(x,Q^2)dx \qquad \tilde{\Gamma}_1^{dis} = \int_{x_{\min}}^{x_{\max}} g_1^{dis}(x,Q^2)dx$$

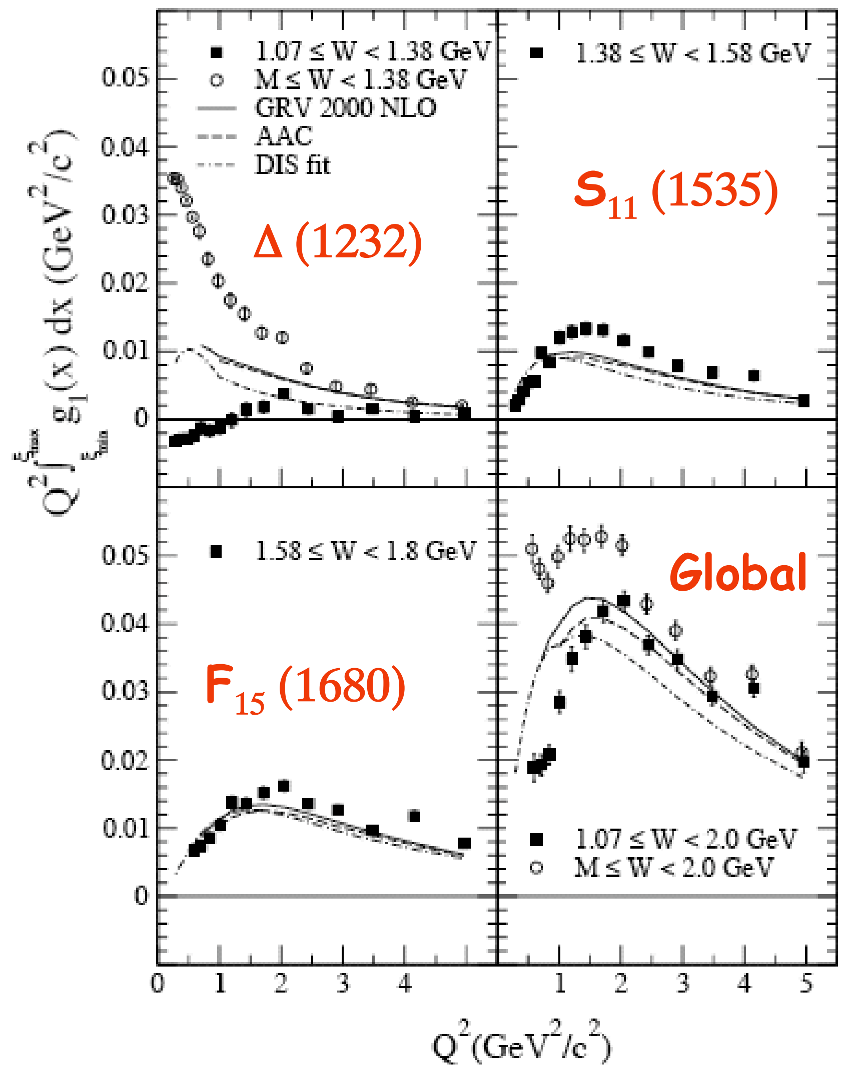
Hall A E01-012
Liyanage, Chen, Choi, Solvignon
Global duality for $Q^2>1.8$ GeV^2
Δ violates duality for $Q^2<1.7$ GeV^2



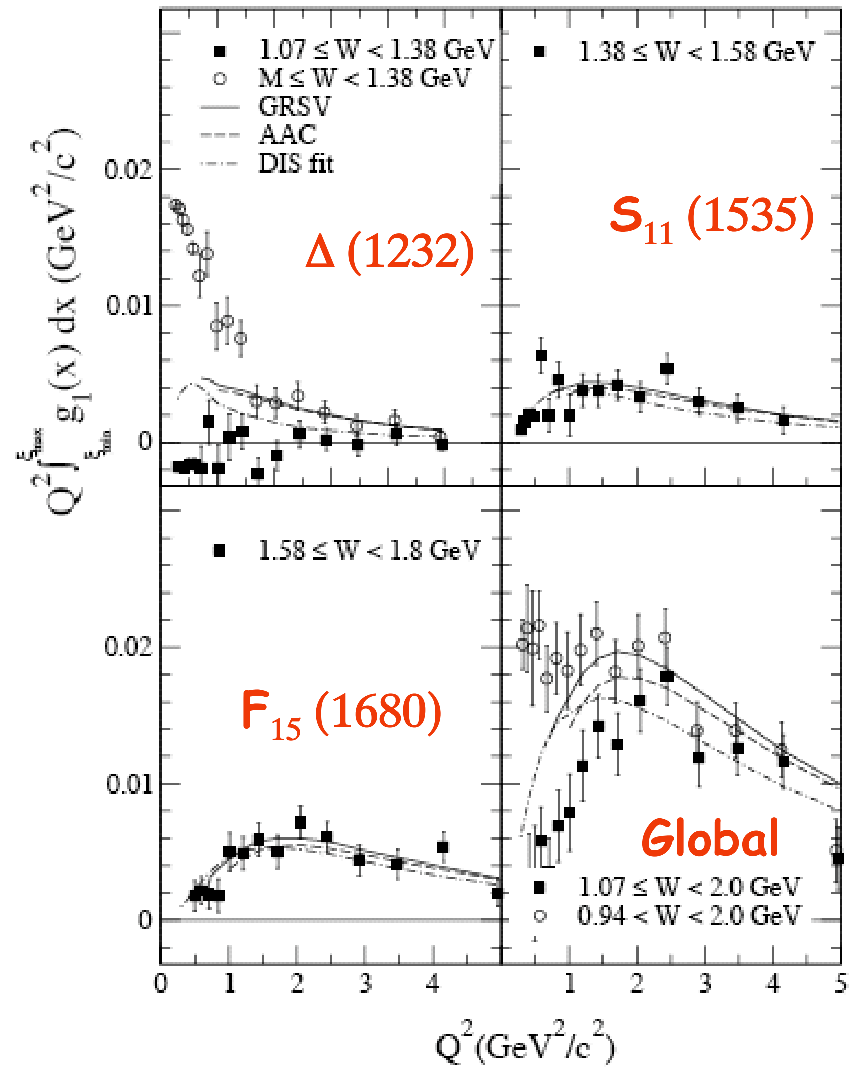


Duality at CLAS

Proton



Deuteron



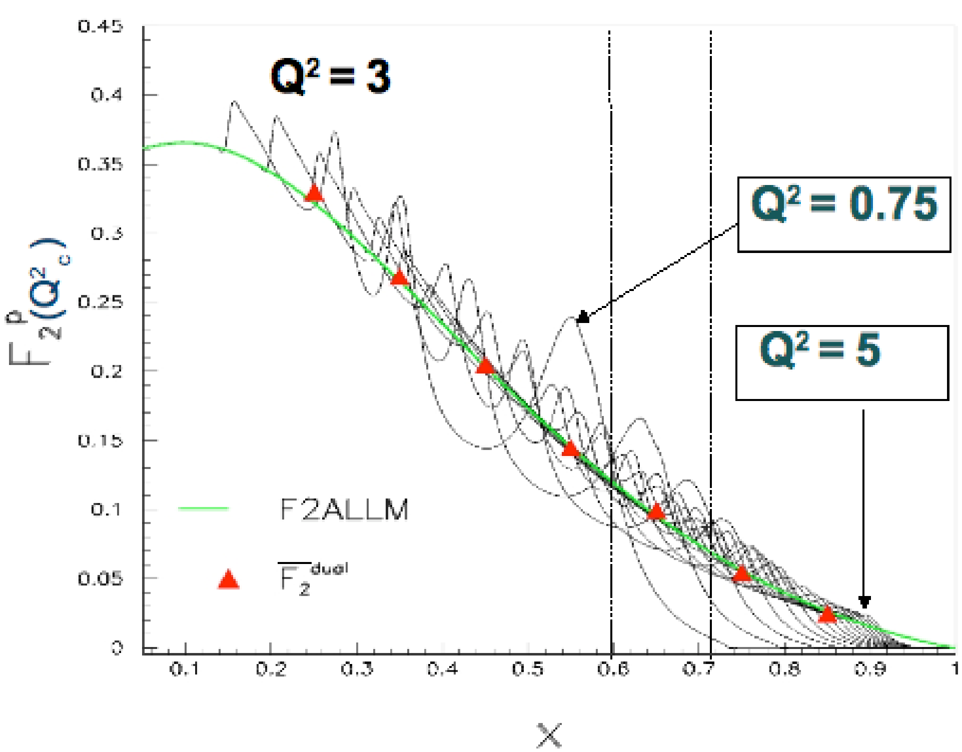
Duality-Averaging

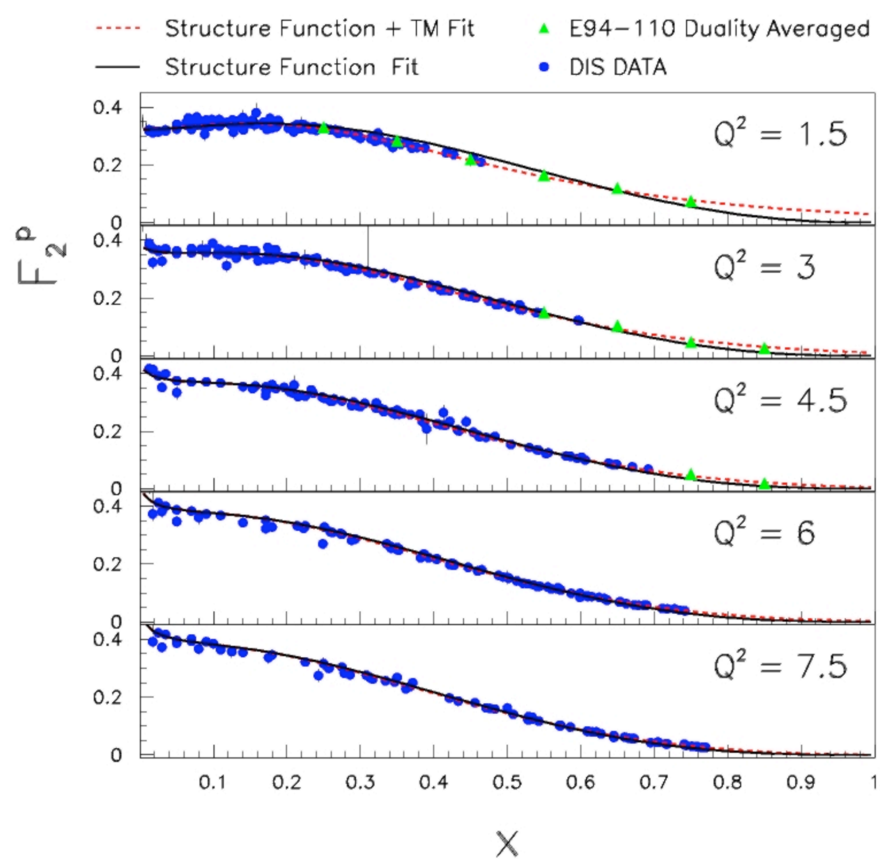
$$F_2^{\mathrm{TMC}}(x,Q^2) = \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi) + \frac{6M^2x^3}{Q^2r^4} h_2(\xi) + \frac{12M^4x^4}{Q^4r^5} g_2(\xi)$$

Hall C
Christy

$$h_2(\xi,Q^2) = \int_\xi^1 du \, \frac{F_2^{(0)}(u,Q^2)}{u^2} \qquad \xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}$$

$$g_2(\xi,Q^2) = \int_\xi^1 du \; h_2(u,Q^2)$$





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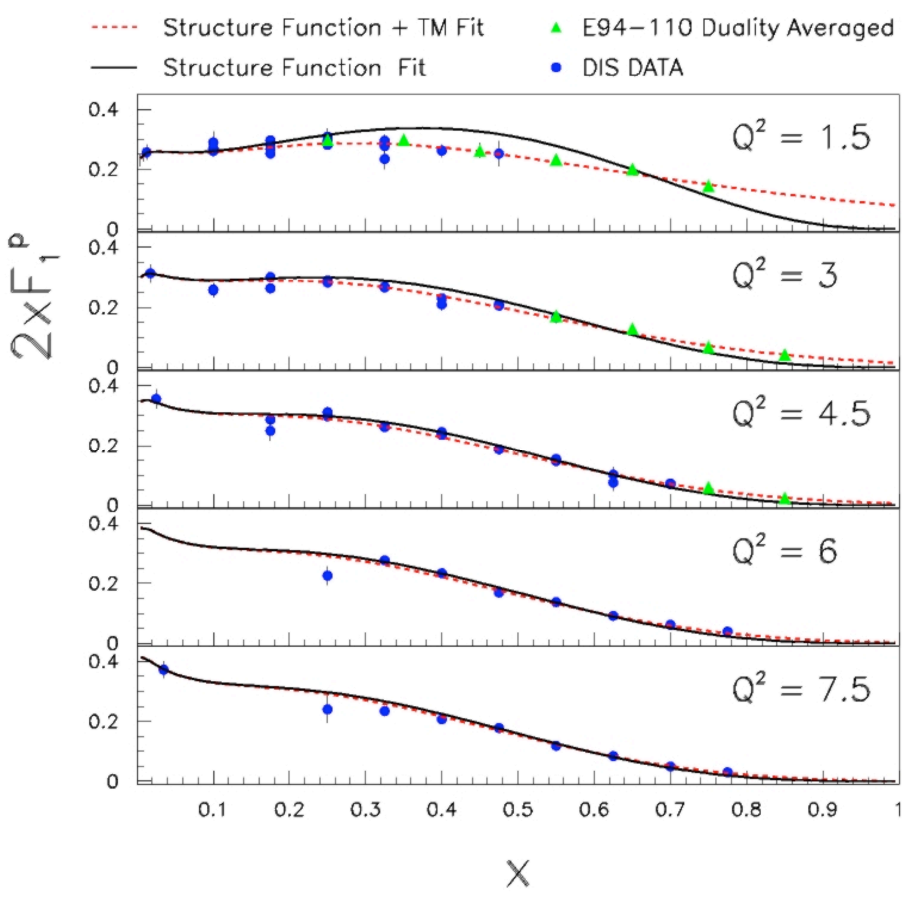
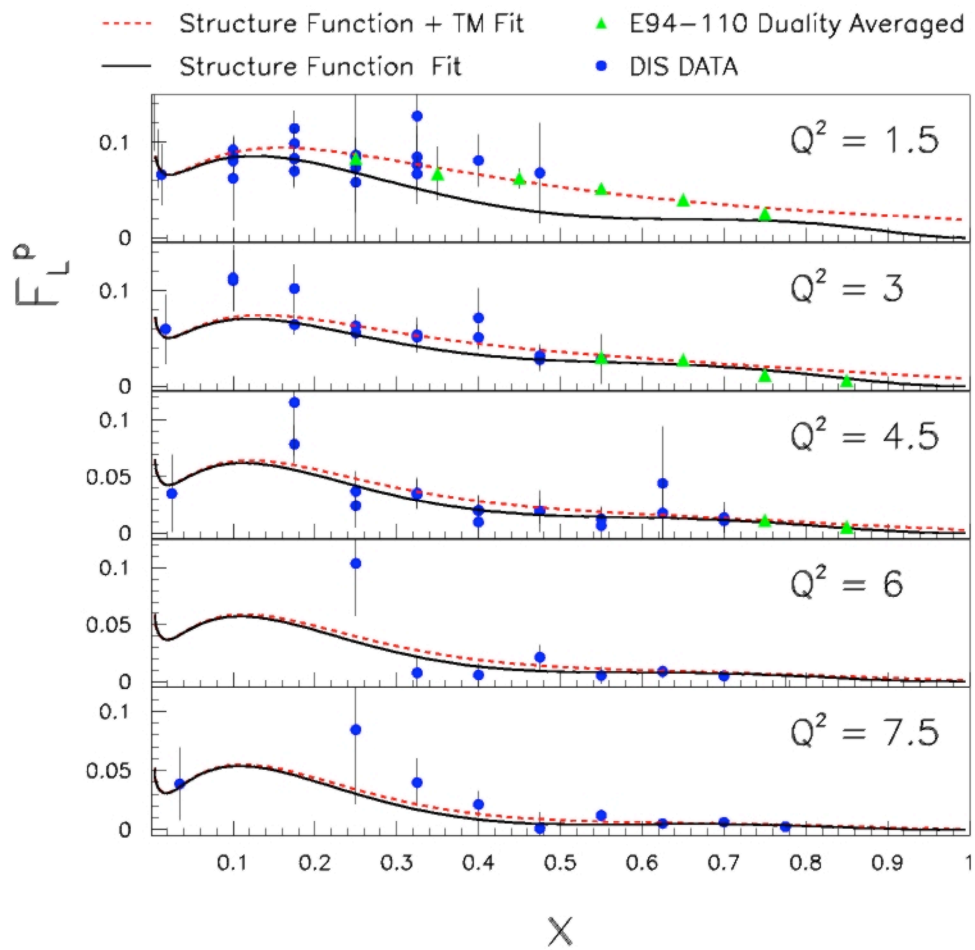
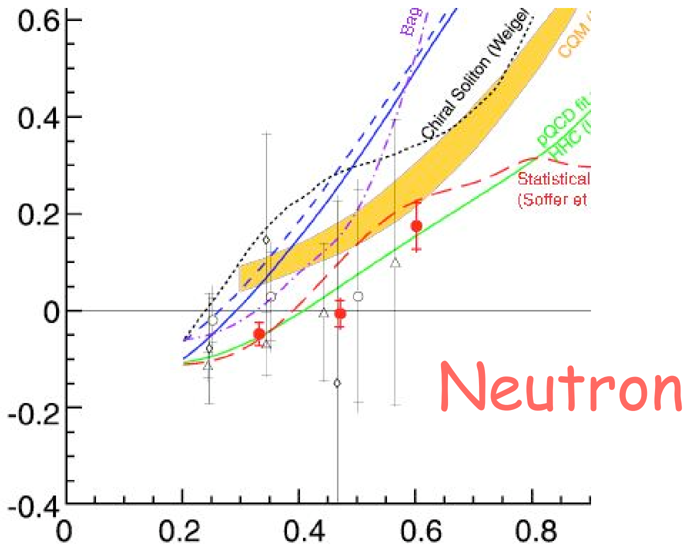
Duality Averaged F_1 and F_L

Hall C: Christy
hepph/0709.1775





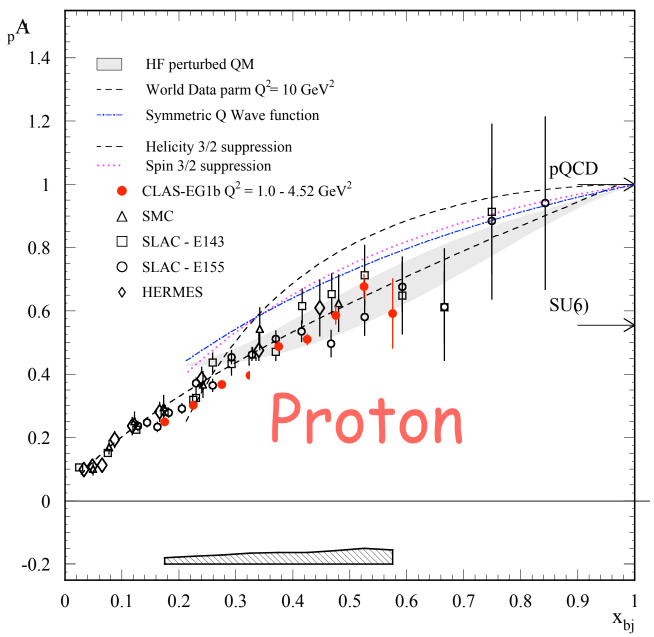
A_1 Data from EG1

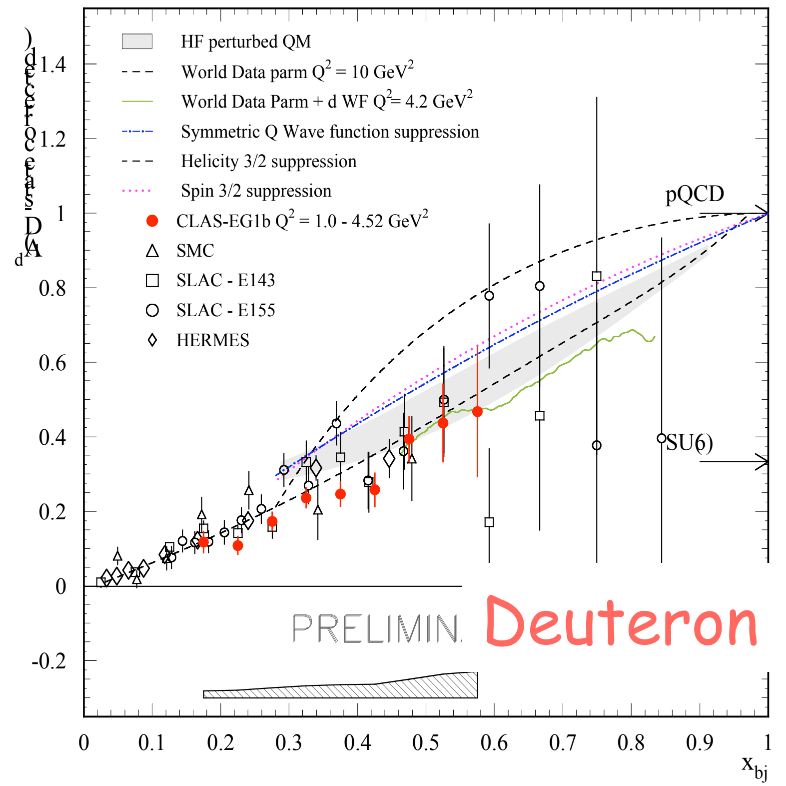


$\sim g_1/F_1$

Close and Melnitchouk, PRC **68**, 035210 (2003)

Isgur, PRD **59**, 034013 (2003)

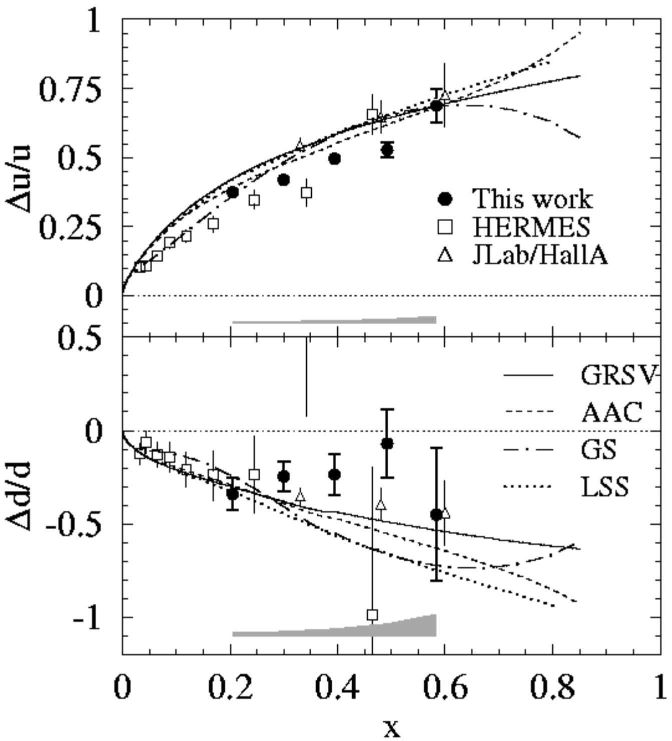




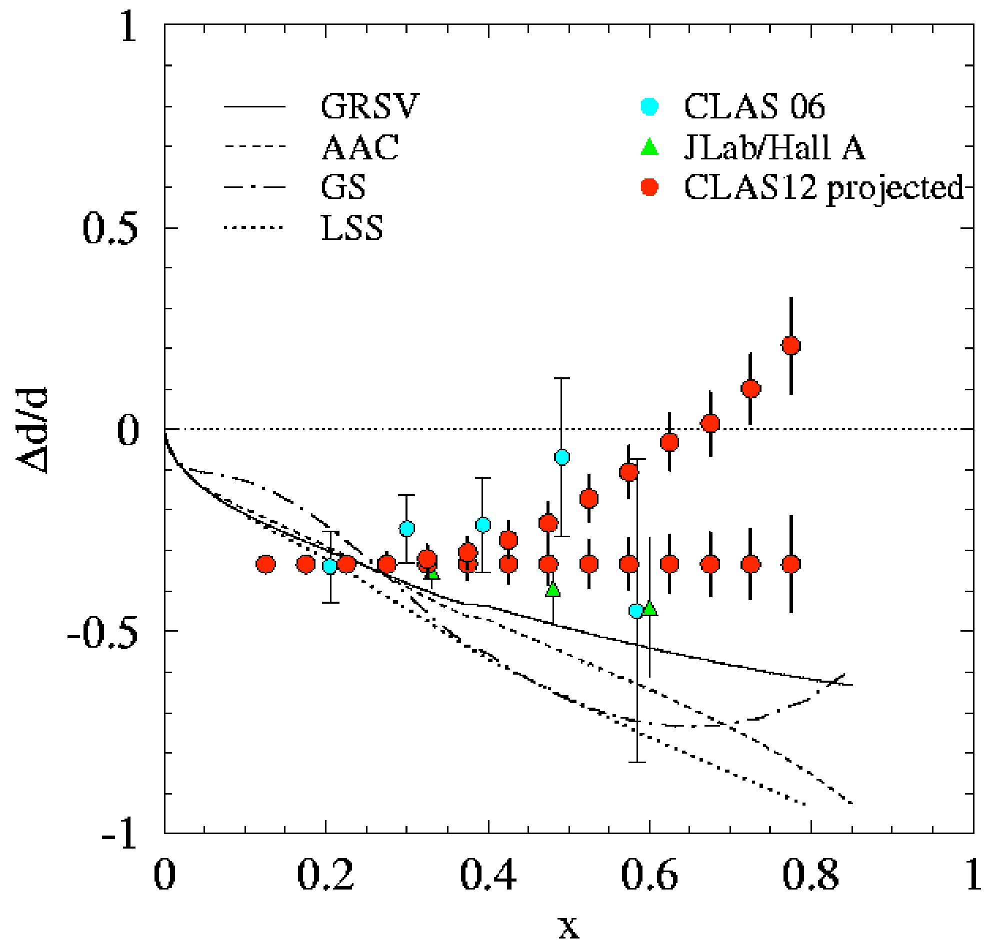
Quark polarization in the valence limit

$$A_1(x,Q^2) = \frac{\sum e_i^2 \Delta q_i(x,Q^2)}{\sum e_i^2 q_i(x,Q^2)}$$

CLAS EG1 Data



Simulated Data for EG12 Extracted from A_1^p, A_1^d and d/u



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RSS g_2^p

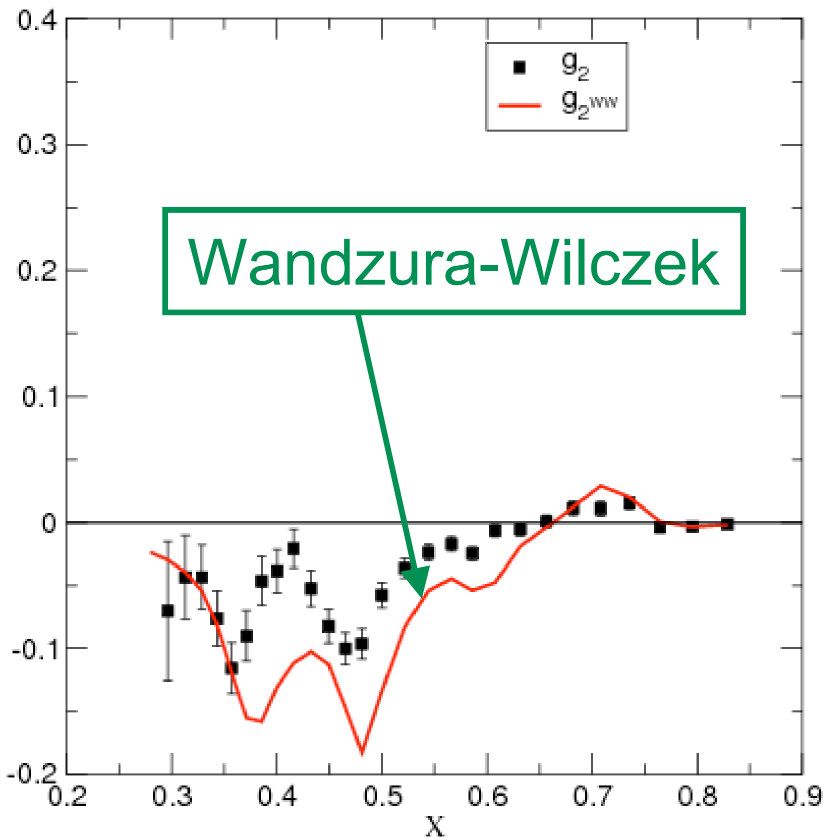
$$g_2^{WW} = -g_1 + \int_x^1 \frac{g_1}{y} dy$$

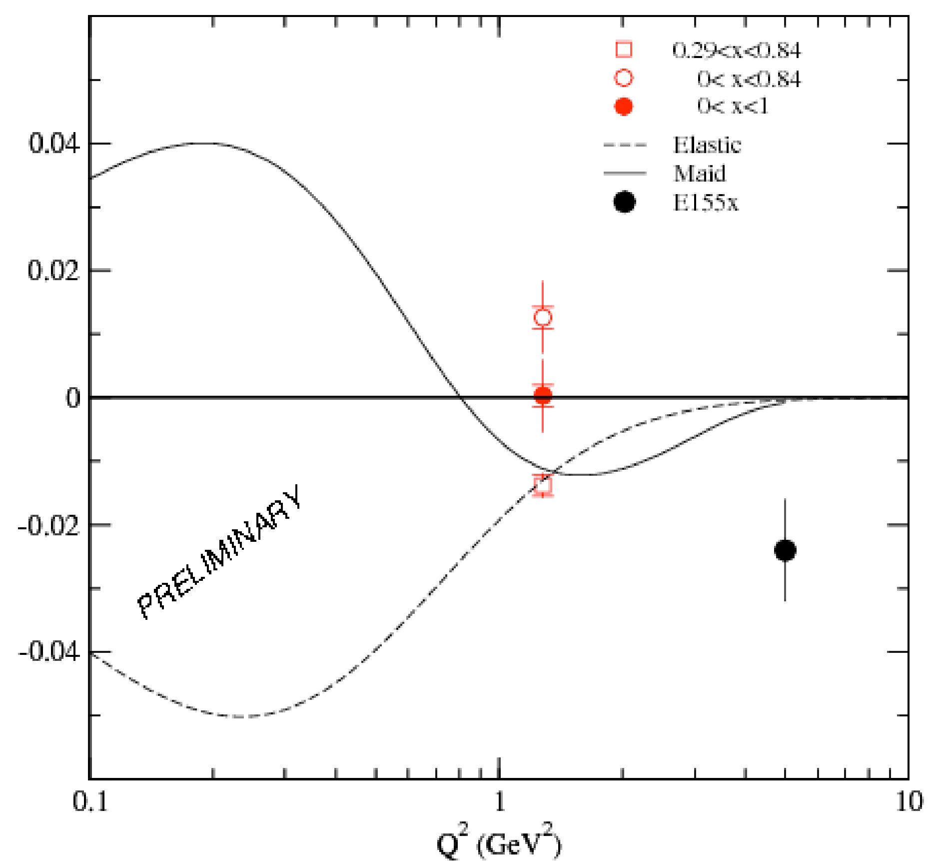
$$g_2 = g_2^{WW} + \overline{g}_2$$

Burkhardt-Cottingham Sum Rule

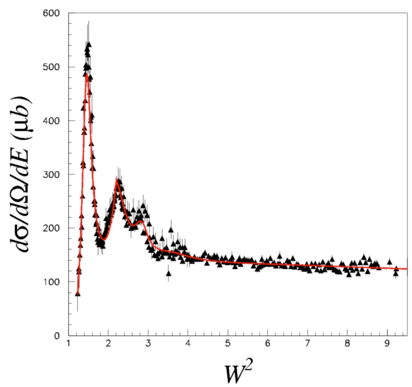
$$\Gamma_2 = \int_0^1 g_2(x, Q^2) dx = 0$$

Hall C: Wesselmann, Slifer



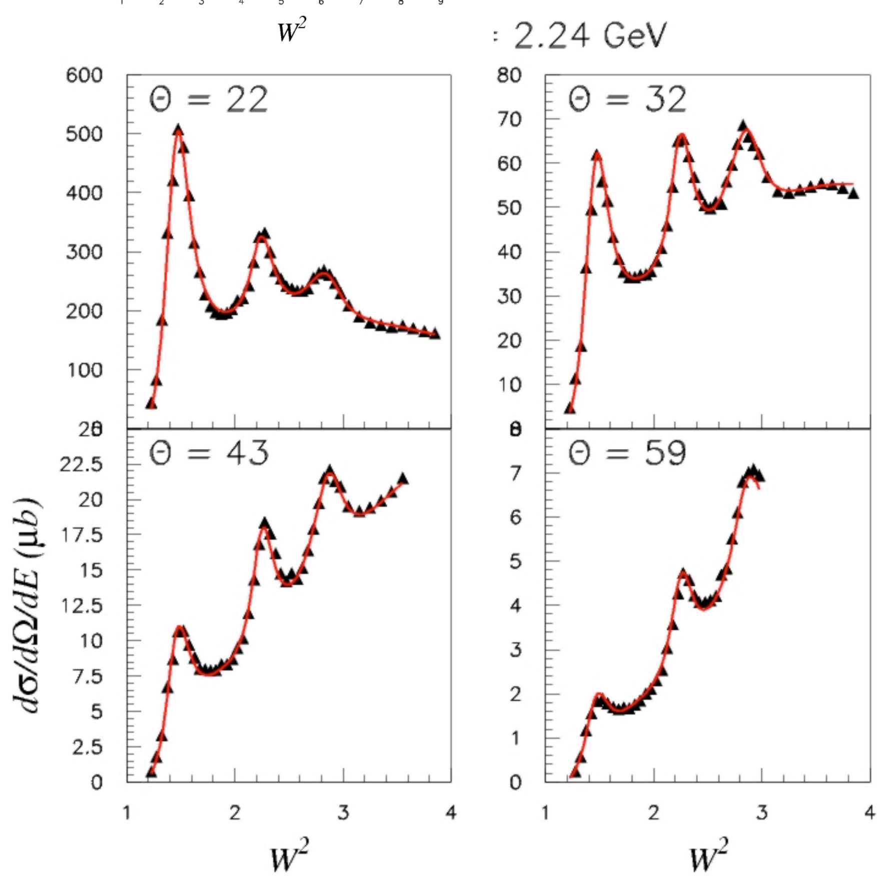


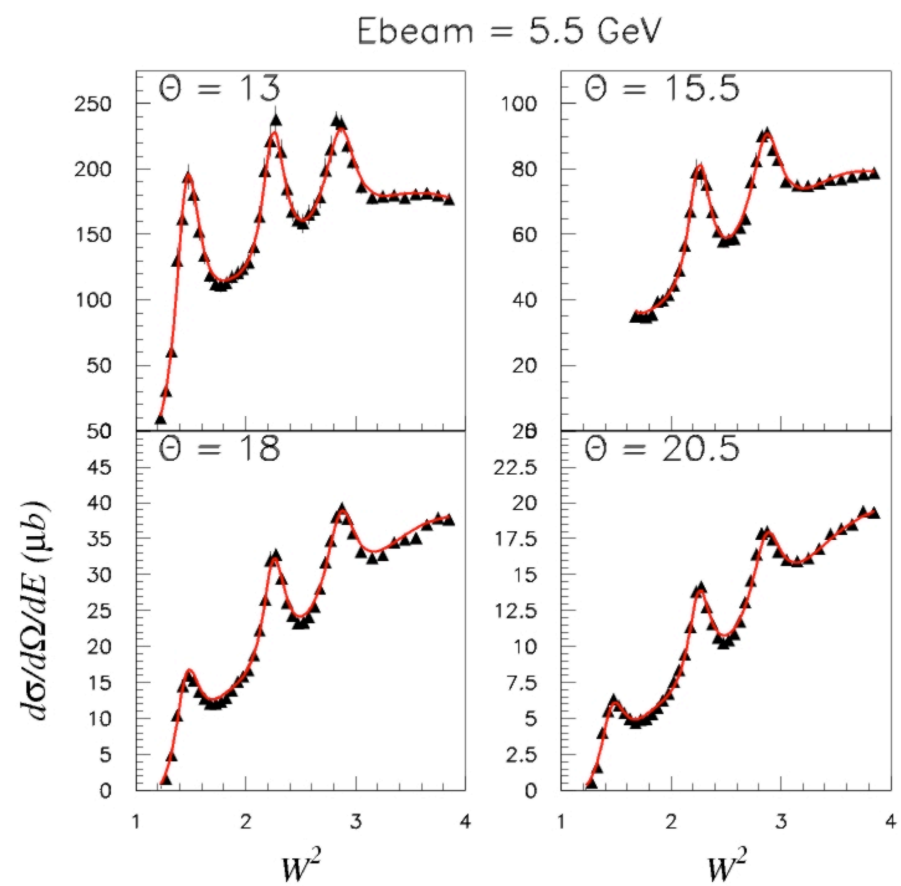
Proton Resonance Fits



$0<Q^2<8$ GeV^2
$1<W<3$ GeV
Reproduces σ to 3%

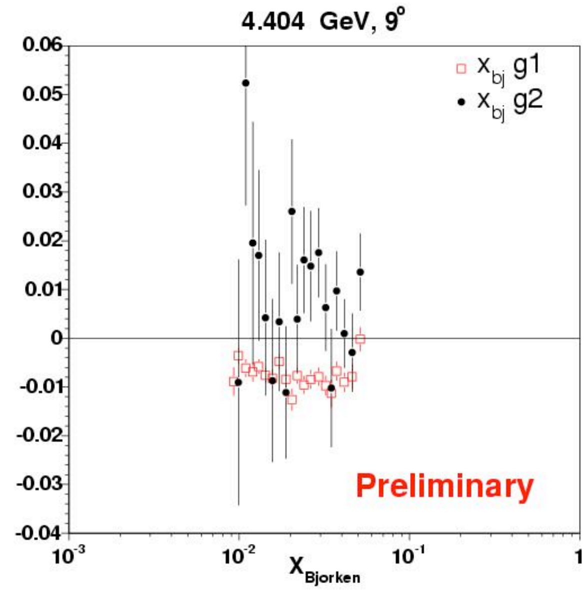
Hall C Data
M.E. Christy

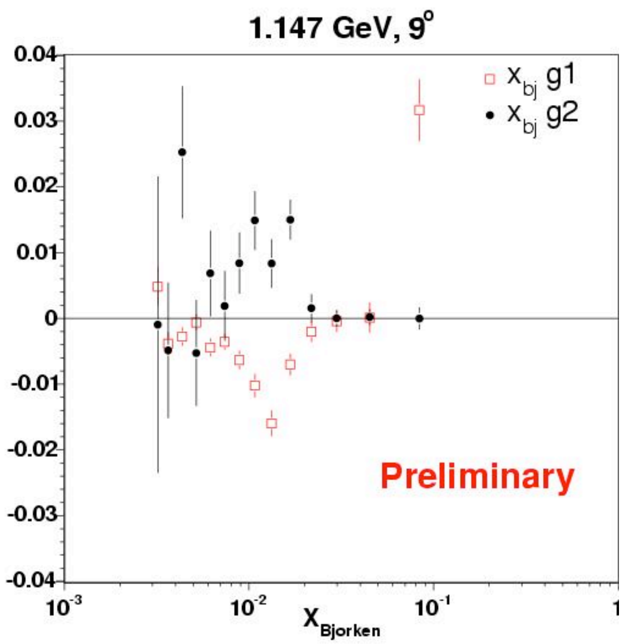


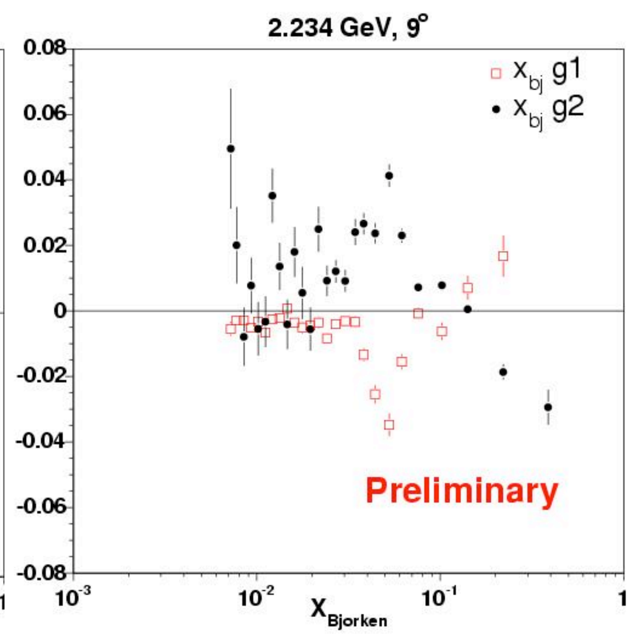


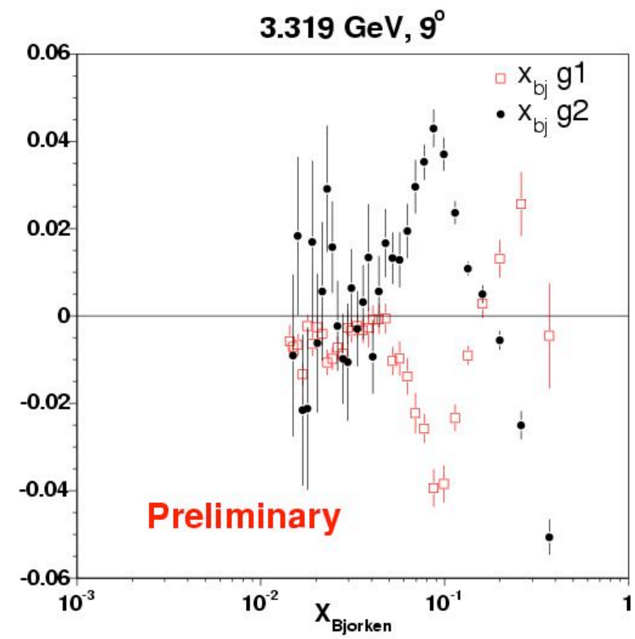
Small Angle GDH

Hall A
E97-110
Chen, Deur,Garibaldi, Sulkosky
$0.02<Q^2<0.3$ GeV^2
$g_2^n = - g_1^n$ at low Q^2









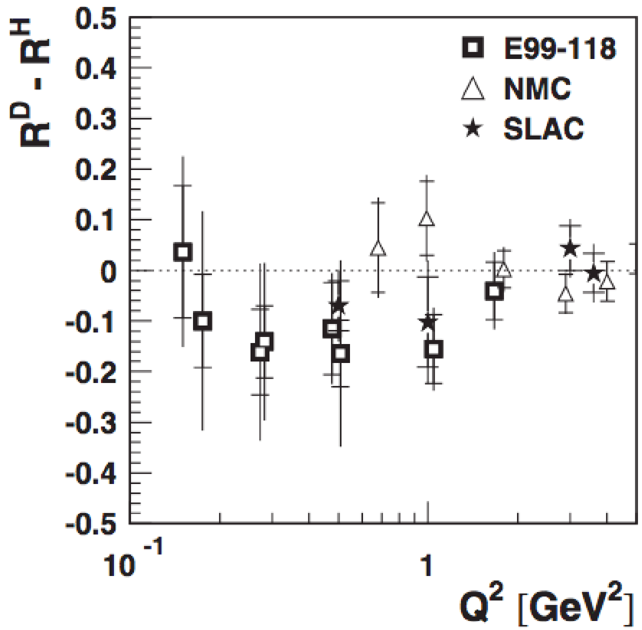
$R = \sigma_L/\sigma_T$ (DIS)

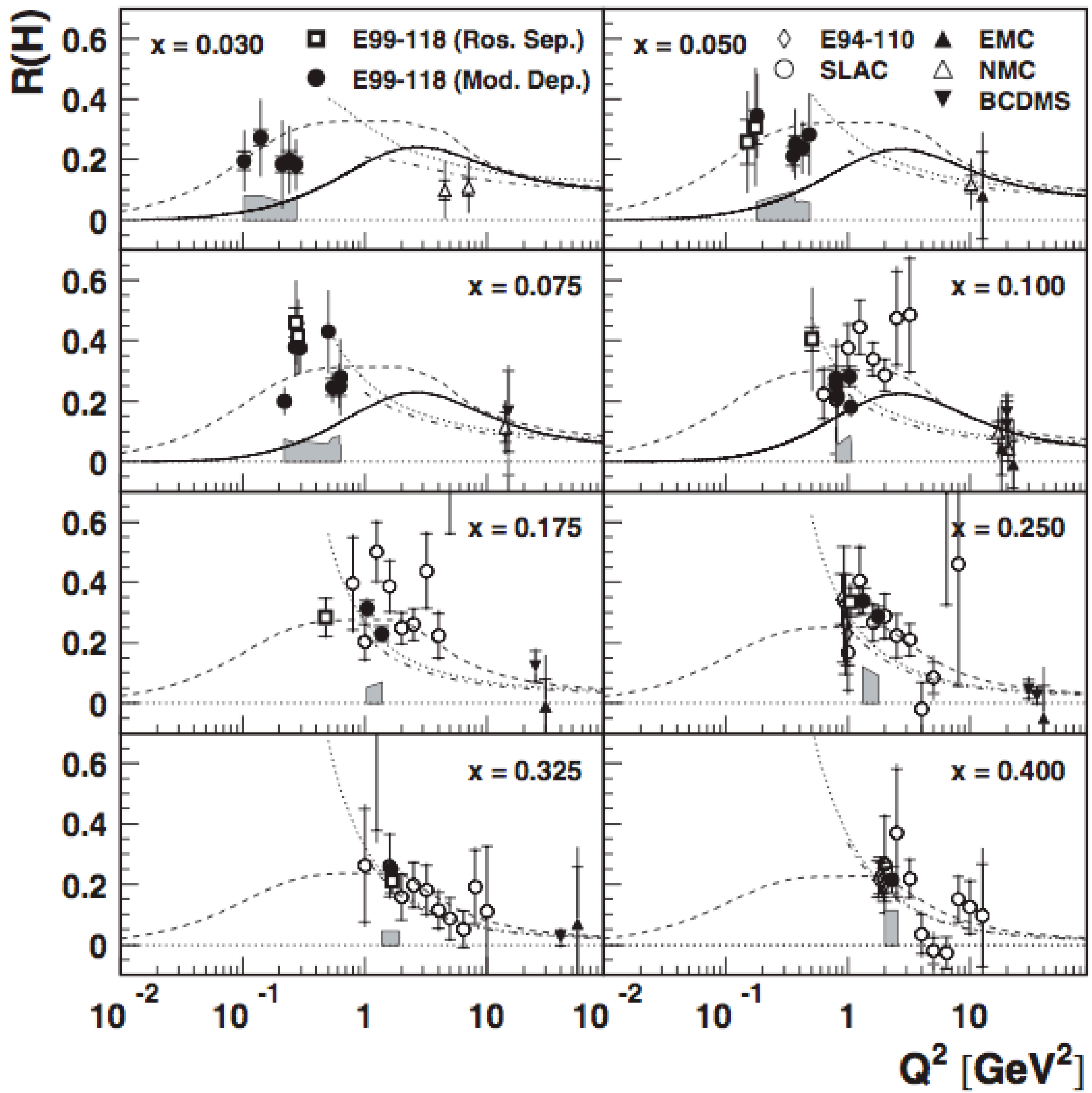
Hall C: E99-118
Tvaskis, PRL98(07)142301

$$\frac{1}{\Gamma}\frac{d^2\sigma}{d\Omega dE'} = \sigma_T + \varepsilon\sigma_L$$

$$R(x, Q^2) \equiv \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1}$$

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\rm Mott}\frac{2MxF_2}{Q^2\varepsilon}\left(\frac{1+\varepsilon R}{1+R}\right)$$





$R = \sigma_L/\sigma_T$ (Resonance)

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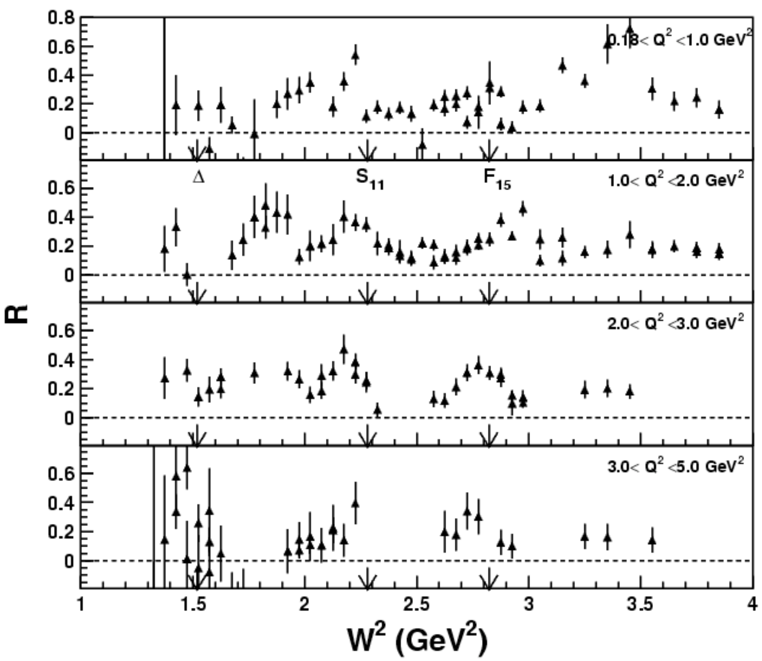
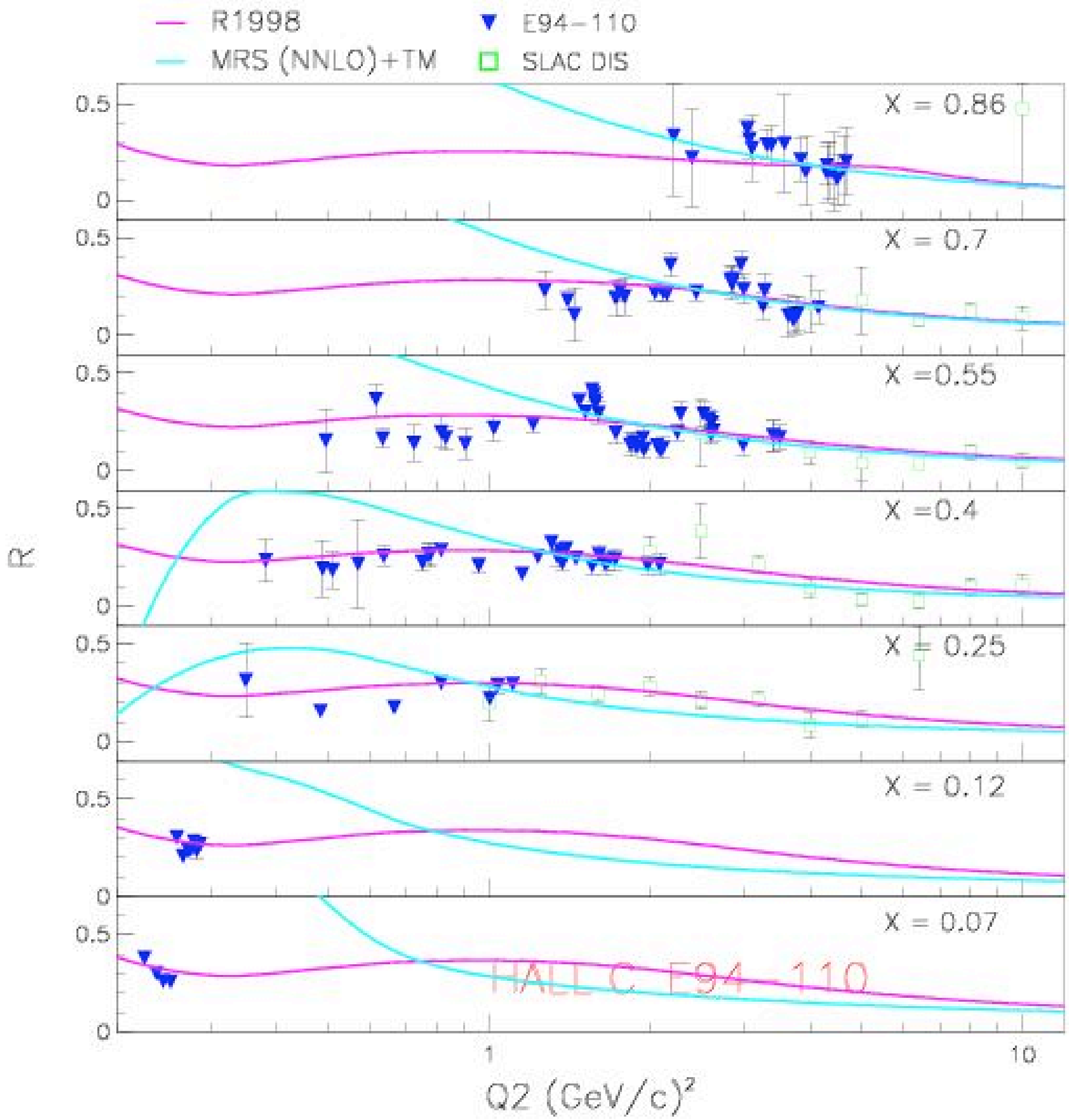
Hall C: E94-110
Liang, nucl-ex/0410027

$$\frac{1}{\Gamma}\frac{d^2\sigma}{d\Omega dE'} = \sigma_T + \varepsilon\sigma_L$$

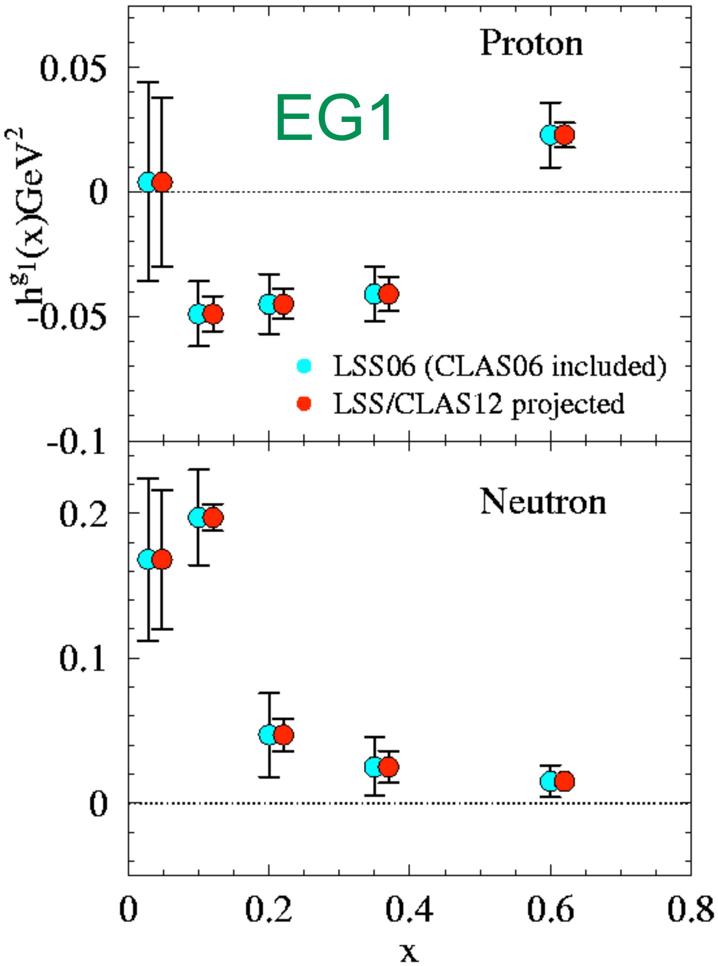
$$R(x, Q^2) \equiv \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1}$$





Higher Twist from g_1 in CLAS

$$\left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\rm exp} F_1(x,Q^2)_{\rm exp} = g_1(x,Q^2)_{\rm exp} = g_1(x,Q^2)_{LT} + h^{g_1}(x)/Q^2$$



$$1 < Q^2 < 5\ \mathrm{GeV}^2, \quad 2 < W < 3.5\ \mathrm{GeV}$$

$$\int_0^1 dx h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)$$

- F_1 from NMC fit to F_2 and 1998 SLAC fit to R
- g_1 (leading twist) from NLO fit at high Q^2
- h from fit to all data, especially CLAS in the pre-asymptotic region
- d_2: twist-3, f_2: twist-4

Bjorken Sum & Higher Twist

$$\Gamma_1^{(n)} = \int_0^1 x^n g_1(x,Q^2)dx = \frac{a_n}{2}, \quad n=0,2,4,\ldots,$$

$$\Gamma_2^{(n)} = \int_0^1 x^n g_2(x,Q^2)dx = \frac{1}{2}\frac{n}{n+1}(d_n - a_n), \quad n=2,4,\ldots,$$

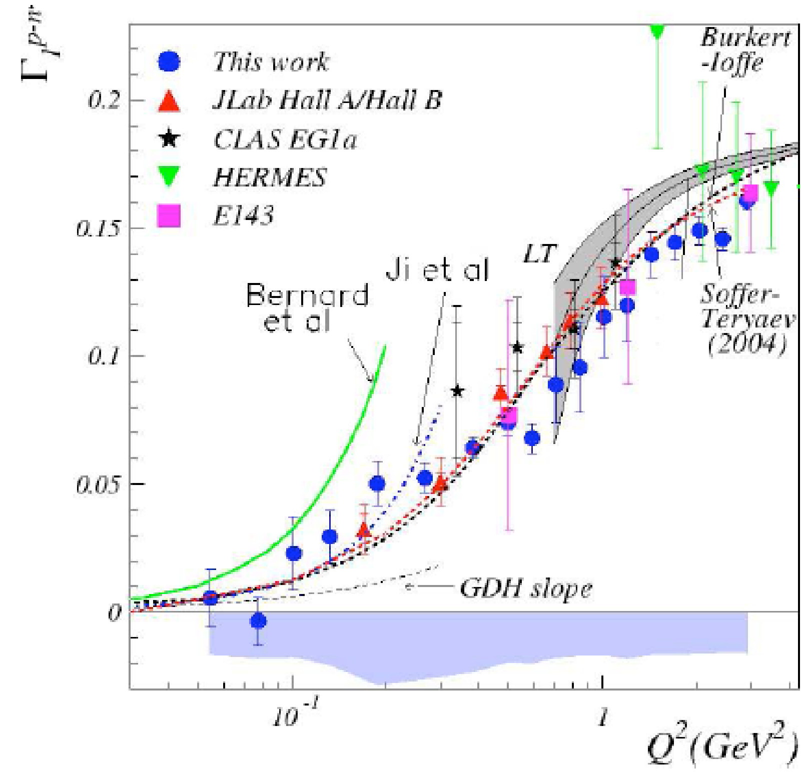
Bjorken Sum Rule:

CLAS: Deur

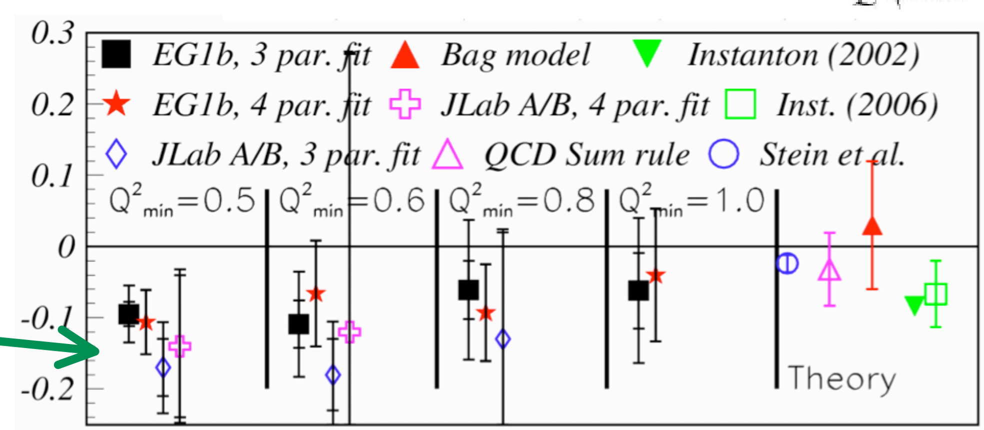
$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3\right] + \frac{\mu_4^{p-n}}{Q^2} + \ldots$$

$$\mu_4^{p-n} = \frac{M^2}{9}\left(a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n}\right)$$

$$d_2^{p-n} = \int_0^1 dx\, x^2 \left(2g_1^{p-n} + 3g_2^{p-n}\right)$$



Fit Γ_1^{p-n} to powers of $1/Q^2$ and extract f_2^{p-n}



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Nachtmann Moments

$$M_1(Q^2) = \int_0^1 dx \frac{\xi^2}{x^2} \left\{ g_1(x, Q^2) \left(\frac{x}{\xi} - \frac{1}{9} \frac{M^2 x \xi}{Q^2} \right) - g_2(x, Q^2) \frac{4}{3} \frac{M^2 x^2}{Q^2} \right\},$$

$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

$$M_1(Q^2) = \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + \cdots$$

$$\mu_4(Q^2) = 4 f_2(Q^2)/9M^2$$

$$f_2 = 0.039 \pm 0.022(\text{stat}) \pm^{0.000}_{0.018}(\text{sys}) \pm 0.030(\text{low } x) \pm^{0.007}_{0.011}(\alpha_s),$$

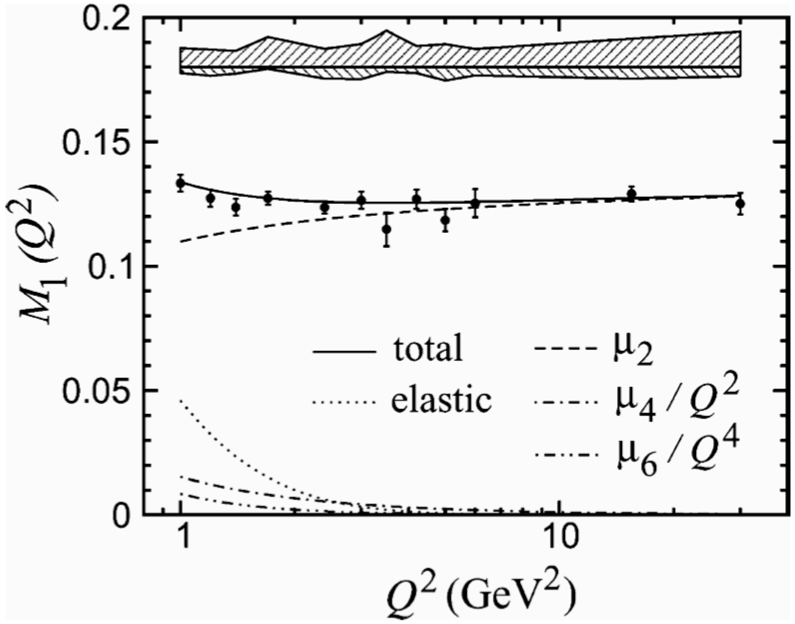
$$d_2(Q^2) = \int_0^1 dx\, x^2[2g_1(x, Q^2) + 3g_2(x, Q^2)]$$

$$\chi_E = \frac{2}{3}(2d_2 + f_2)$$

$$\chi_B = \frac{1}{3}(4d_2 - f_2)$$

CLAS, Osipenko

PLB609(05)259



$$\chi_E = 0.026 \pm 0.015(\text{stat}) \pm^{0.021}_{0.024}(\text{sys}),$$

$$\chi_B = -0.013 \mp 0.007(\text{stat}) \mp^{0.010}_{0.012}(\text{sys})$$

^{3}He Higher Twist

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- Higher twist terms from world neutron data

$\mu_4 = 0.019(24)M^2$

$\mu_6 = -0.019(17)M^2$

- Twist-4 term

$\mu_4 = M^2/9\ (a_2+4d_2+4f_2)$

SLAC E155x: $d_2=0.0079(48)$

E99-117: $d_2=0.0062(28)$

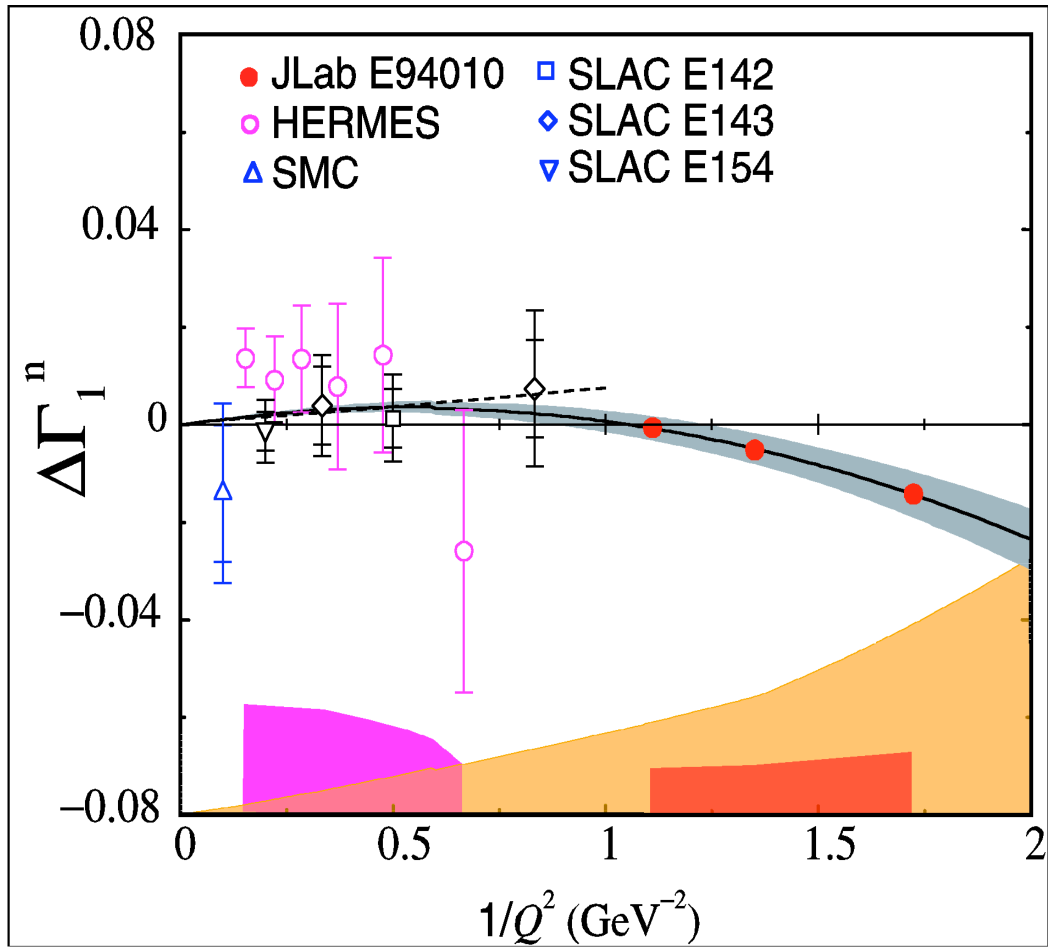
$f_2 = 0.034(43)$ (total)

- Color polarizabilities

$\chi_E = 0.033(29)$

$\chi_B = -0.001(16)$

Hall A
E94-010



Color Polarizability

- Osipenko, CLAS, proton, PLB609(05)249
  - $f_2 = 0.039(39)$ $\chi_E = 0.026(27)$ $\chi_B = -0.013(13)$
- E94-010, Hall A, neutron
  - $f_2 = 0.034(43)$ $\chi_E = 0.033(29)$ $\chi_B = -0.001(16)$
- Deur, CLAS, Bjorken (p-n)
  - $f_2 = -0.101(74)$ $\chi_E = -0.077(50)$ $\chi_B = 0.024(28)$
- More accurate determinations are needed.

Baldin Sum Rule

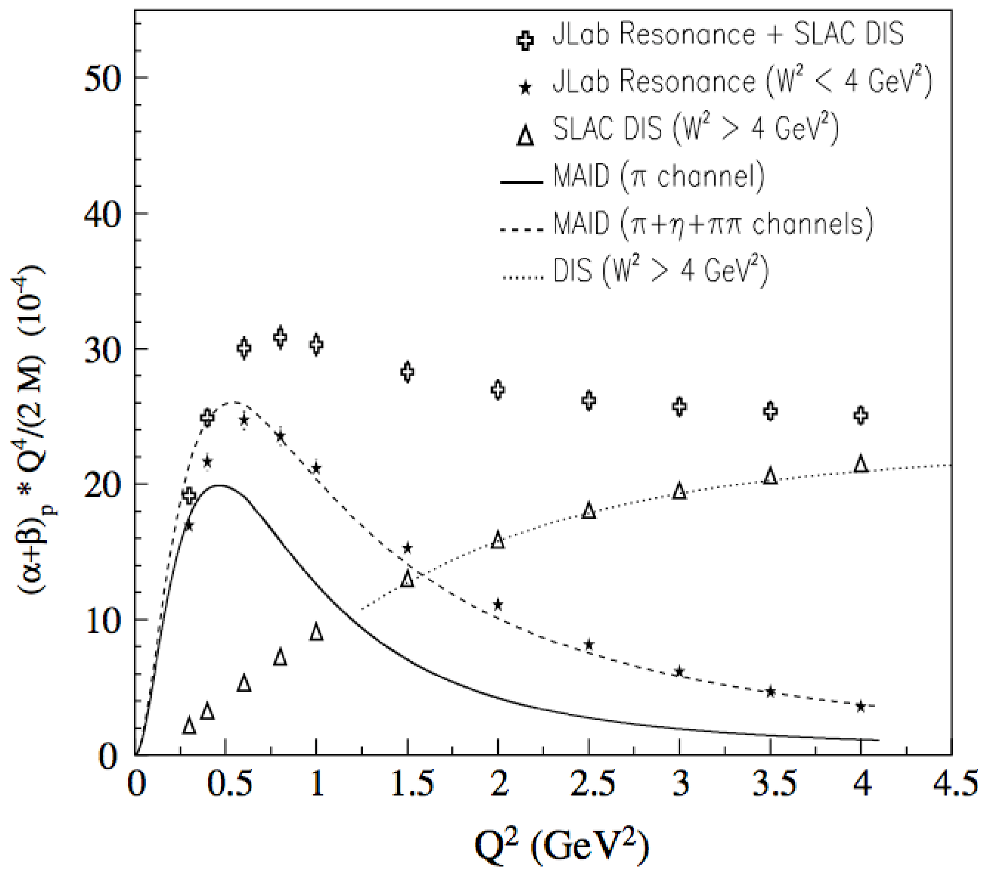
$$\alpha + \beta = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} \frac{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}}{\nu^2} d\nu$$

$$\alpha(Q^2) + \beta(Q^2) = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} \frac{K}{\nu}\frac{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}}{\nu^2} d\nu = \frac{e^2 M}{\pi Q^4}\int_0^{x_0} 2x F_1(x, Q^2) dx$$

α and β are E&M polarizabilities

Unpolarized analog to GDH

Hall C
Liang PRC73(06)065201



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CLAS Moments $\Gamma_1^{p,d}$

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x,Q^2)dx \qquad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

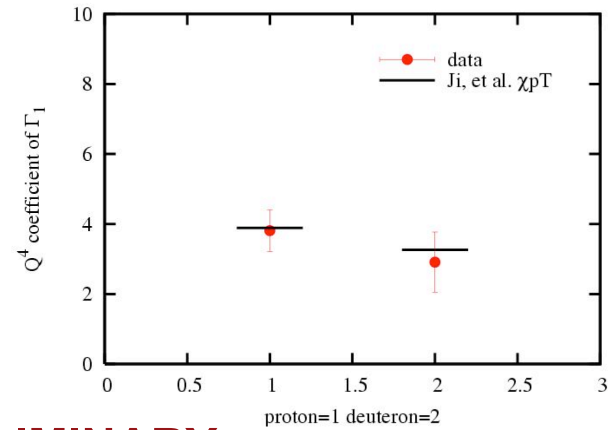
low Q^2 fit

$$\Gamma_1^d(Q^2) = (1-1.5\omega_D)\left\{\Gamma_1^p(Q^2) + \Gamma_1^n(Q^2)\right\}$$

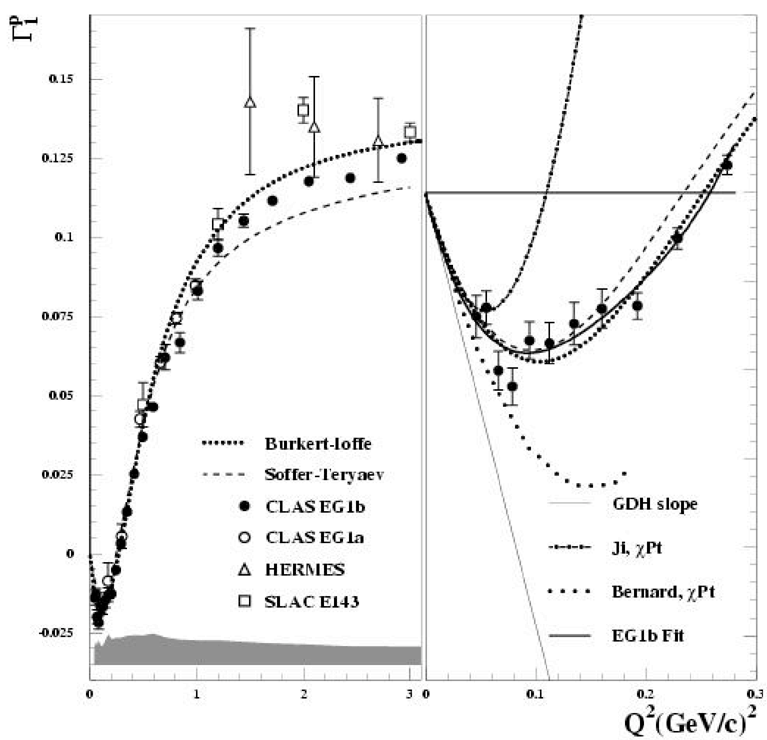
$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^2}{8M^2}Q^2 + 3.89Q^4 + \ldots$$

$$\Gamma_1^n(Q^2) = -\frac{\kappa_n^2}{8M^2}Q^2 + 3.15Q^4 + \ldots.$$

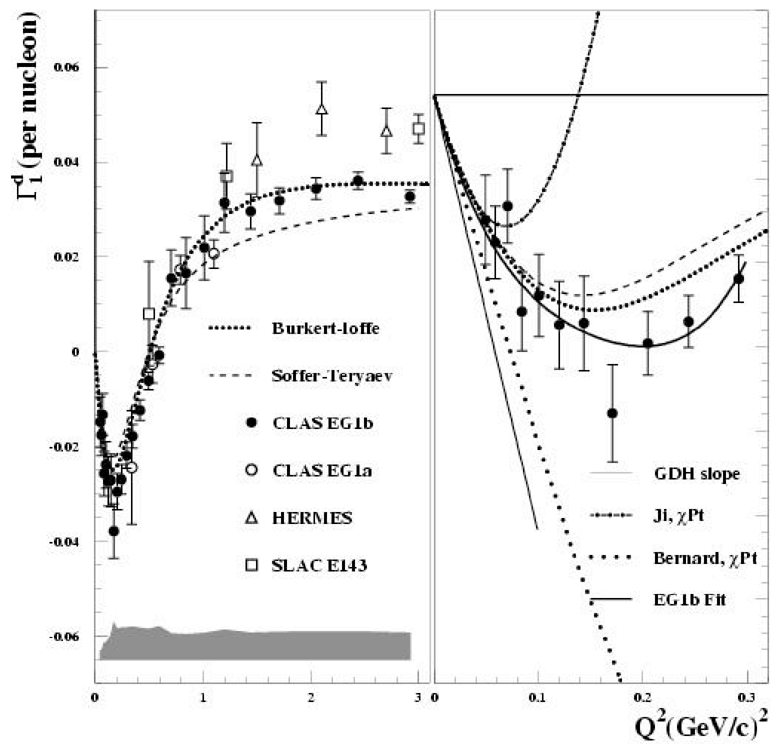
GDH + χpT



PRELIMINARY



PRELIMINARY



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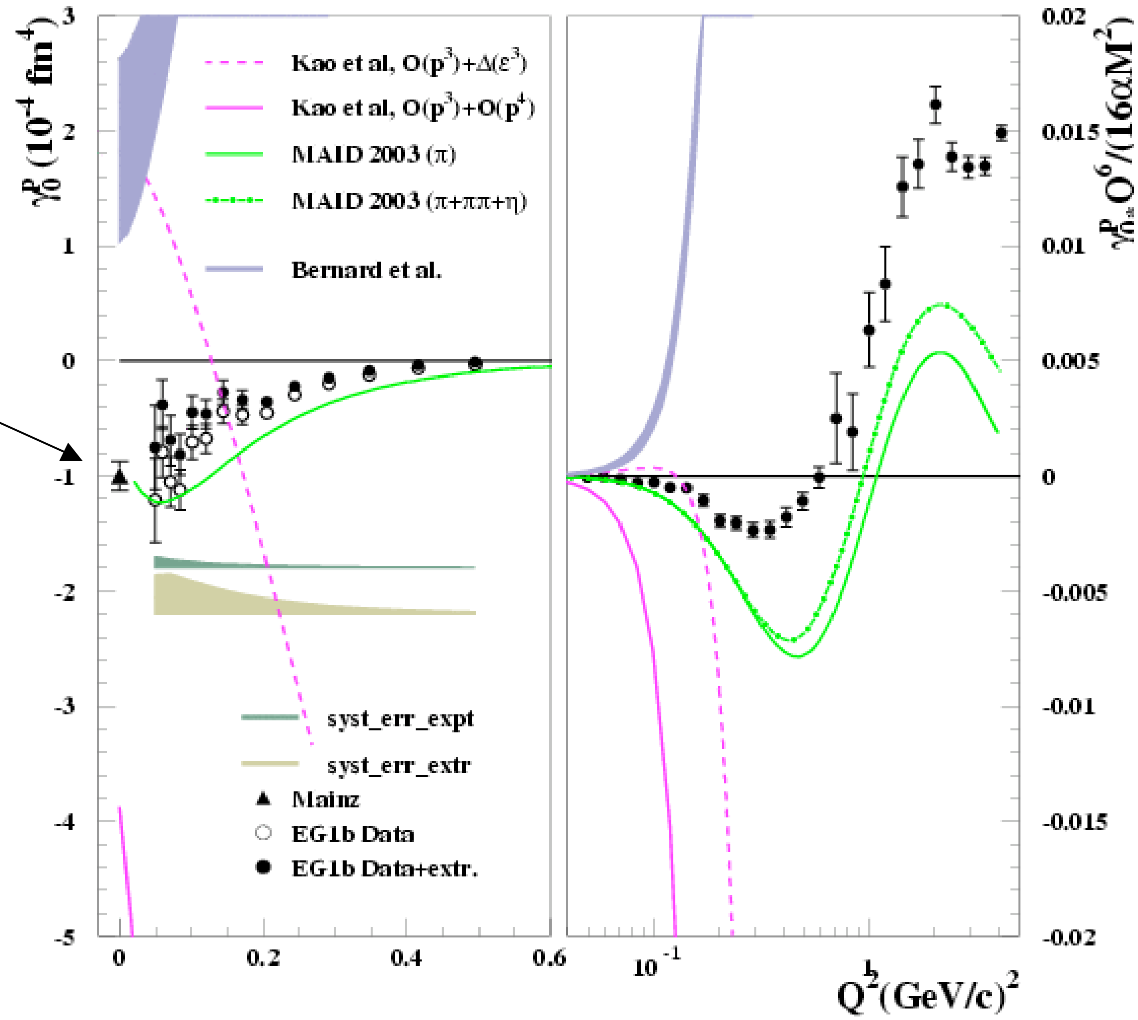
Forward Spin Polarizability

$$\gamma_0(Q^2) = \frac{4e^2M^2}{\pi Q^6}\int_0^{x_0} dx\, x^2 \left\{ g_1(x,Q^2) - \gamma^2 g_2(x,Q^2) \right\}$$

PRELIMINARY

real photon point

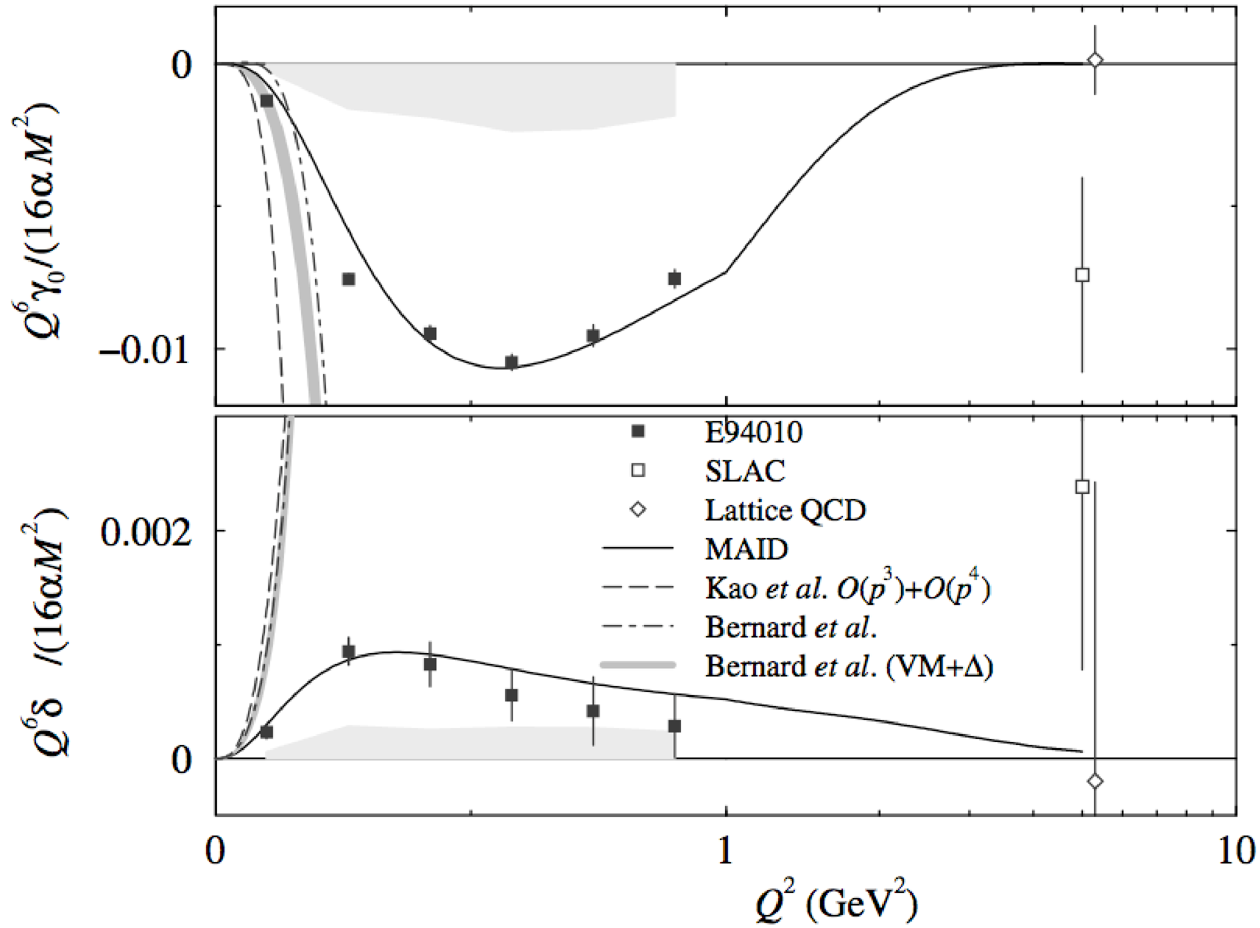
CLAS EG1 Data



^{3}He Polarizabilities

$$\delta_{LT}(Q^2) = \left(\frac{1}{2\pi^2}\right)\int_{\nu_0}^{\infty}\frac{K(\nu,Q^2)}{\nu}\frac{\sigma_{LT}(\nu,Q^2)}{Q\nu^2}d\nu$$
$$= \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2[g_1(x,Q^2)+g_2(x,Q^2)]dx$$

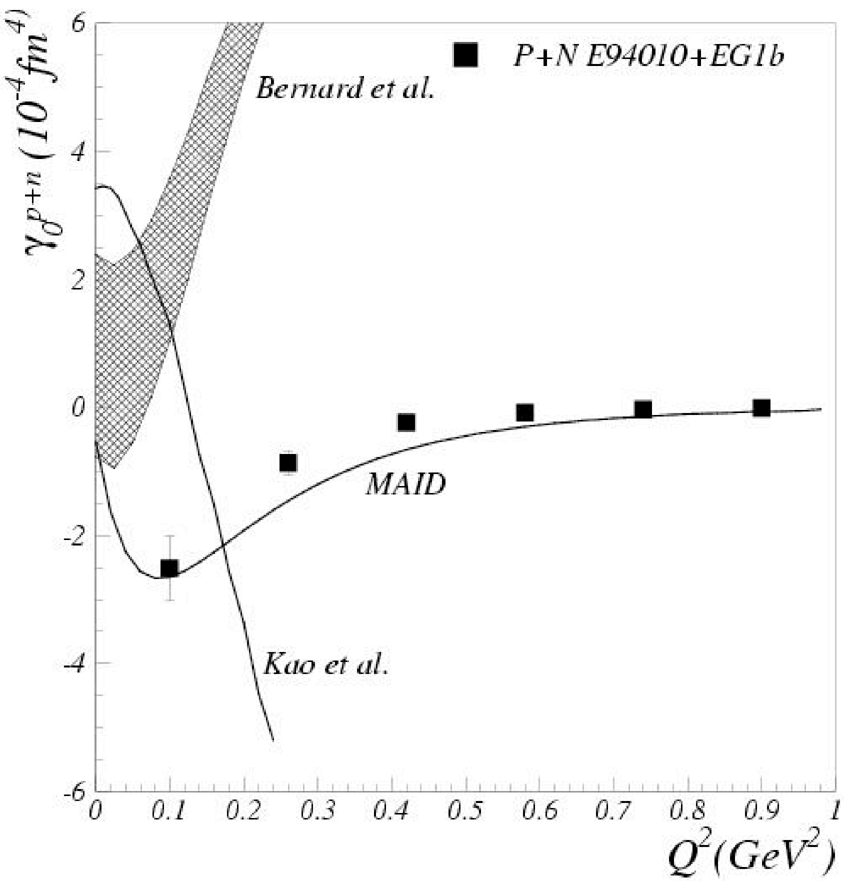
Hall A
Amarian,
PRL93(04)1152301

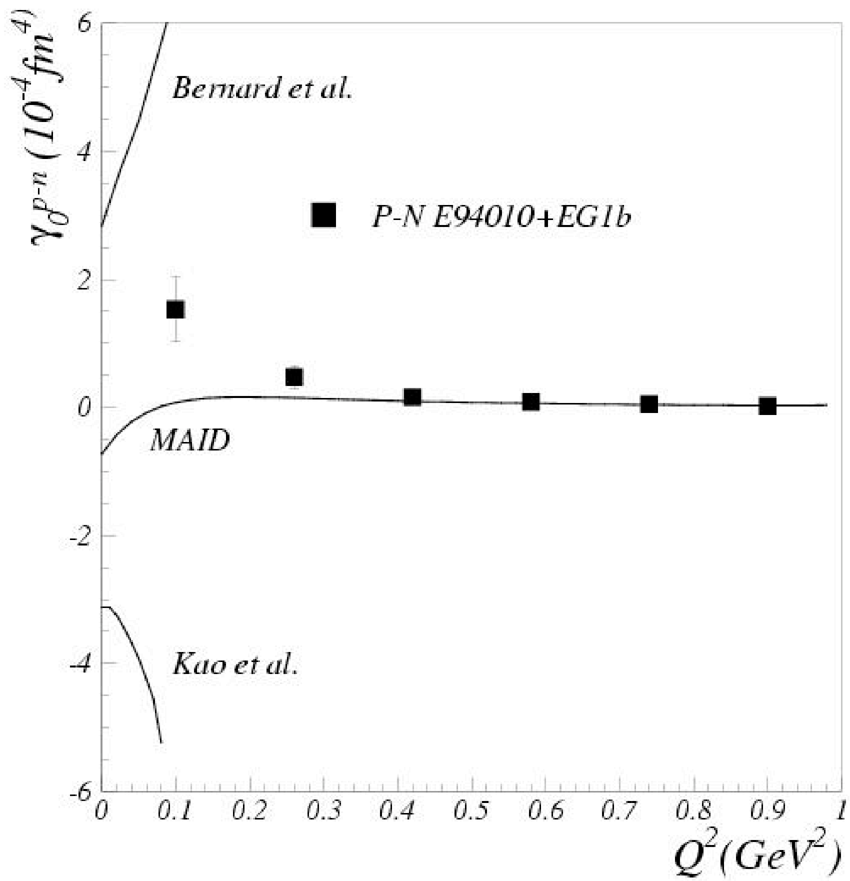


γ_0^{p-n} and γ_0^{p+n}

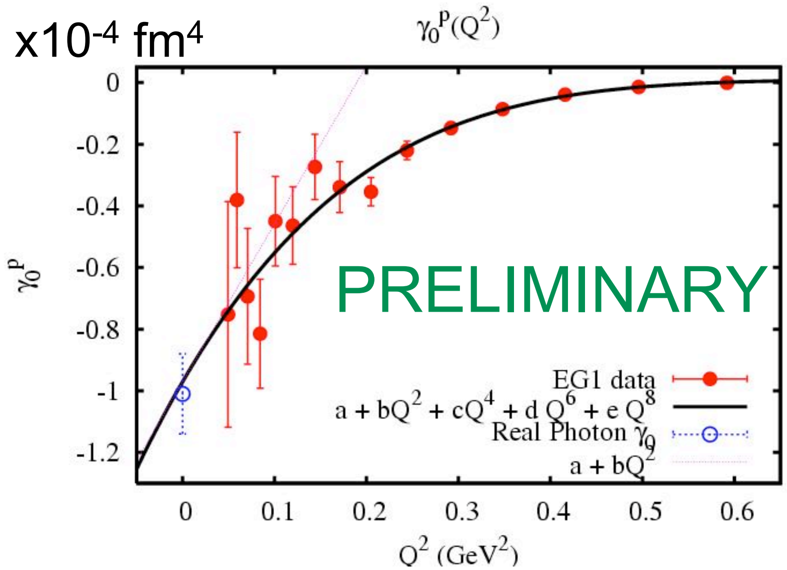
A Deur
CLAS + Hall A

For isovector (p-n) case
Δ contribution cancels



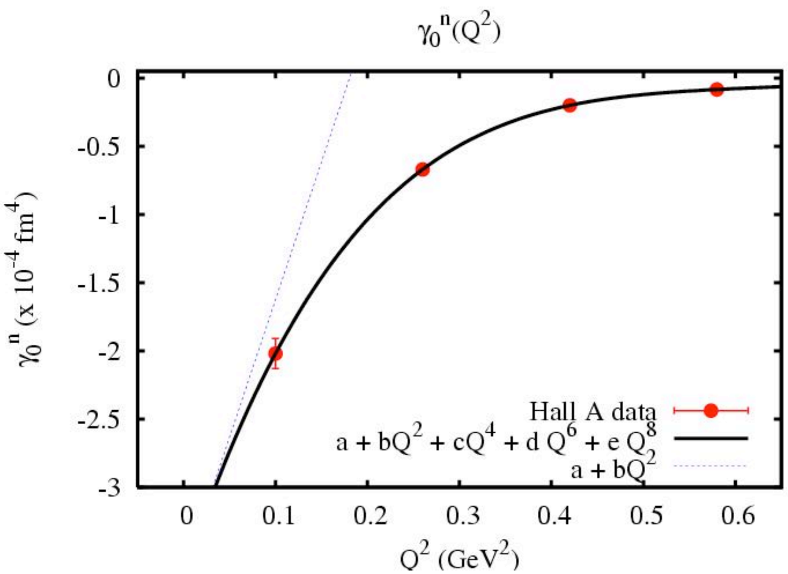


γ_0 Fits at Low Q^2



$a = -0.97(11)$
$b = 5.13(94)$
[targets for χPT]

Prok et al., CLAS EG1



$a = -3.643(1)$
$b = 20.180(8)$
[targets for χPT]

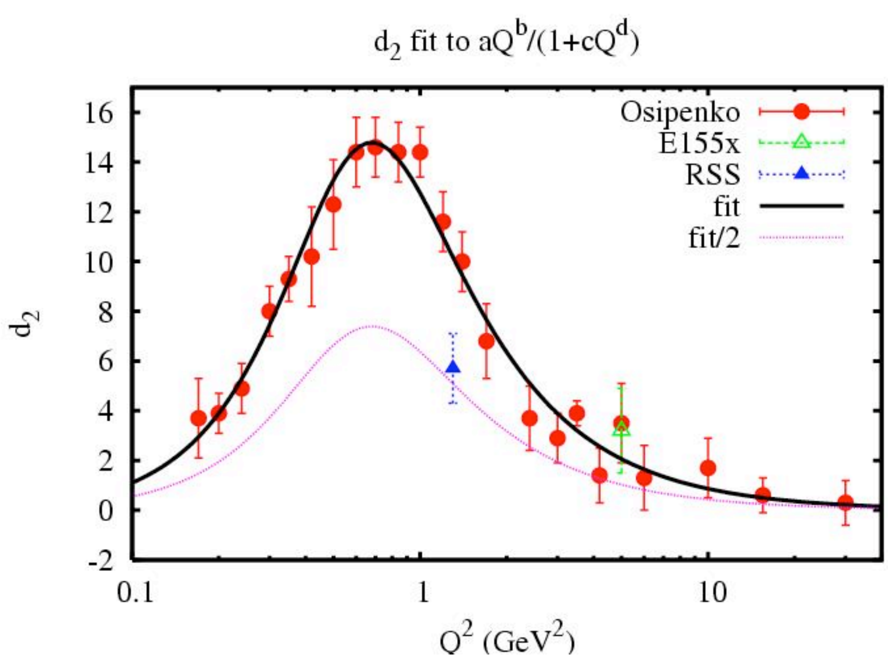
Amarian et al., PRL93(04)152301

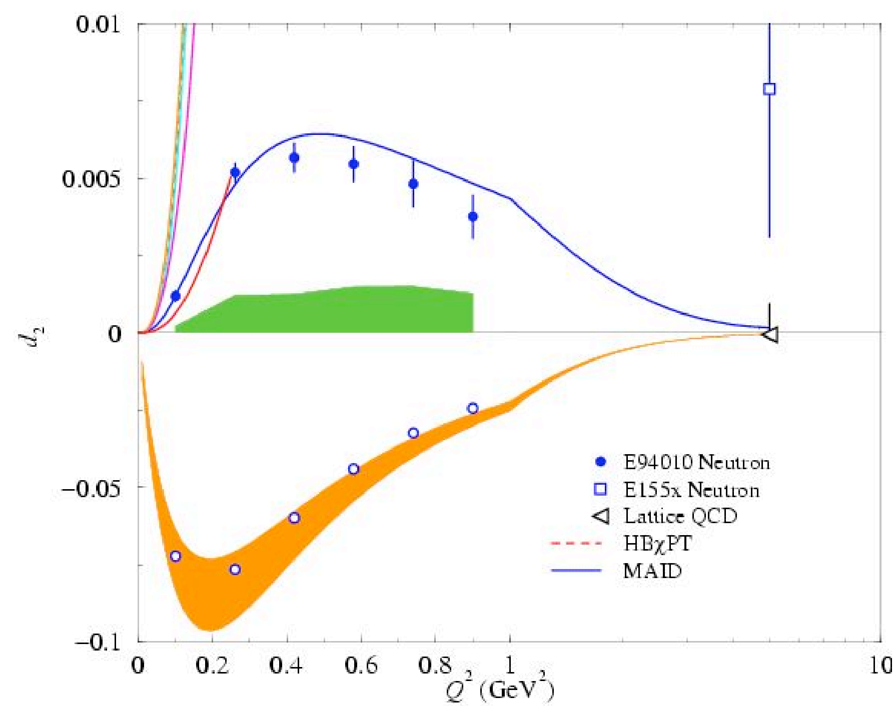
Higher Twist d_2

$$d_2(Q^2) = \int_0^1 dx\, x^2 [2g_1(x,Q^2) + 3g_2(x,Q^2)]$$

CLAS EG1 (proton)
Osipenko, PRD71(05)054007
Model-dependent determination

Hall A (neutron)
E94-010
Amarian, PRL92(04)022301





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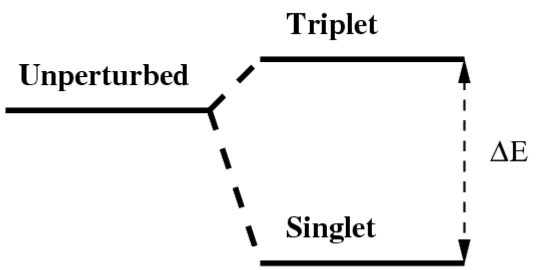
Hydrogen Hyperfine Splitting

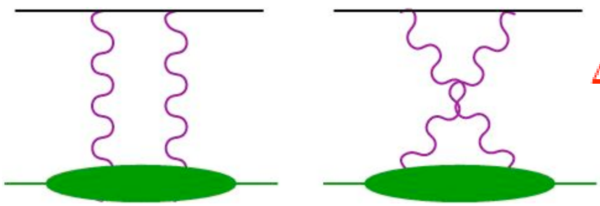
$$E_{\mathrm{HFS}}(e^- p) = 1.4204057517667(9)\mathrm{GHz} = (1+\Delta_{QED}+\Delta_R^p+\Delta_S)E_F^p$$



$$\Delta_S = \Delta_Z + \Delta_{\mathrm{pol}} \qquad \delta_Z^{\mathrm{rad}} = \frac{\alpha}{3\pi}\left[2\ln\frac{\Lambda^2}{m^2} - \frac{4111}{420}\right]$$

Zemach: $\Delta_Z = -2\alpha m_e \langle r\rangle_Z (1+\delta_Z^{\mathrm{rad}})$

$$\langle r\rangle_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa} - 1\right]$$



$\Delta_S = -38.62(16)$ ppm $\quad \Delta_Z = -41.0(5)$ ppm

$\Delta_{\mathrm{pol}} = 2.38(58)$ ppm

$$\Delta_{\mathrm{pol}} = \frac{\alpha m_e}{2\pi(1+\kappa)M}(\Delta_1+\Delta_2) = (0.2264798\ \mathrm{ppm})(\Delta_1+\Delta_2)$$

$$\tau = \nu^2/Q^2$$

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\} = 6.48(89)$$

$$\Delta_2 = -24m_p^2\int_0^\infty \frac{dQ^2}{Q^4}B_2(Q^2). = -0.57(57)$$

$$B_1 = \int_0^{x_{\mathrm{th}}} dx\,\beta(\tau)g_1(x,Q^2)\,,$$

$$B_2 = \int_0^{x_{\mathrm{th}}} dx\,\beta_2(\tau)g_2(x,Q^2)\,,$$

$$\beta(\tau) = \frac{4}{9}\left(-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}\right)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}\,,$$

Δ_{pol} = 1.34(24) ppm from CLAS

Lexicon of Sums

- $\Gamma_1^{(n)}$ moments of g_1
- $\Gamma_2^{(n)}$ moments of g_2
- γ_0 forward spin polarizability; moment of g_1 & g_2
- δ_{LT} forward spin polarizability; moment of g_1 & g_2
- a_n OPE twist-2 coefficients
- d_2 OPE twist-3 coefficient
- f_2 OPE twist-4 coefficient
- χ_E color electric polarizability
- χ_B color magnetic polarizability
- $\alpha + \beta$ electric and magnetic polarizability
- $\Delta_1 + \Delta_2$ hyperfine splitting polarizability

Conclusions

- A wealth of data exists for g_1, g_2, F_1 and F_2
- What's still missing:
  - high x: $g_1^{p,d,n}$ and F_2^n (JLab12 and BoNuS)
  - g_2^p on the proton (transverse target)
  - precision and full kinematic coverage for $1 < Q^2 < 10$ (JLab12)
  - low Q^2 evolution of $g_1^{p,d}$ (EG4)
- What's gained:
  - understanding three regions
    - Q^2 near 0 (χPT)
    - Q^2 from 0.1-10 GeV^2 (TMC, higher twists, resonances, the transition)
    - Q^2 near infinity (pQCD)