How Does the Proton Spin?

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Colloquium

Calvin College

23 November 2010

Scattering

Most of what we know about the world comes from scattering experiments.

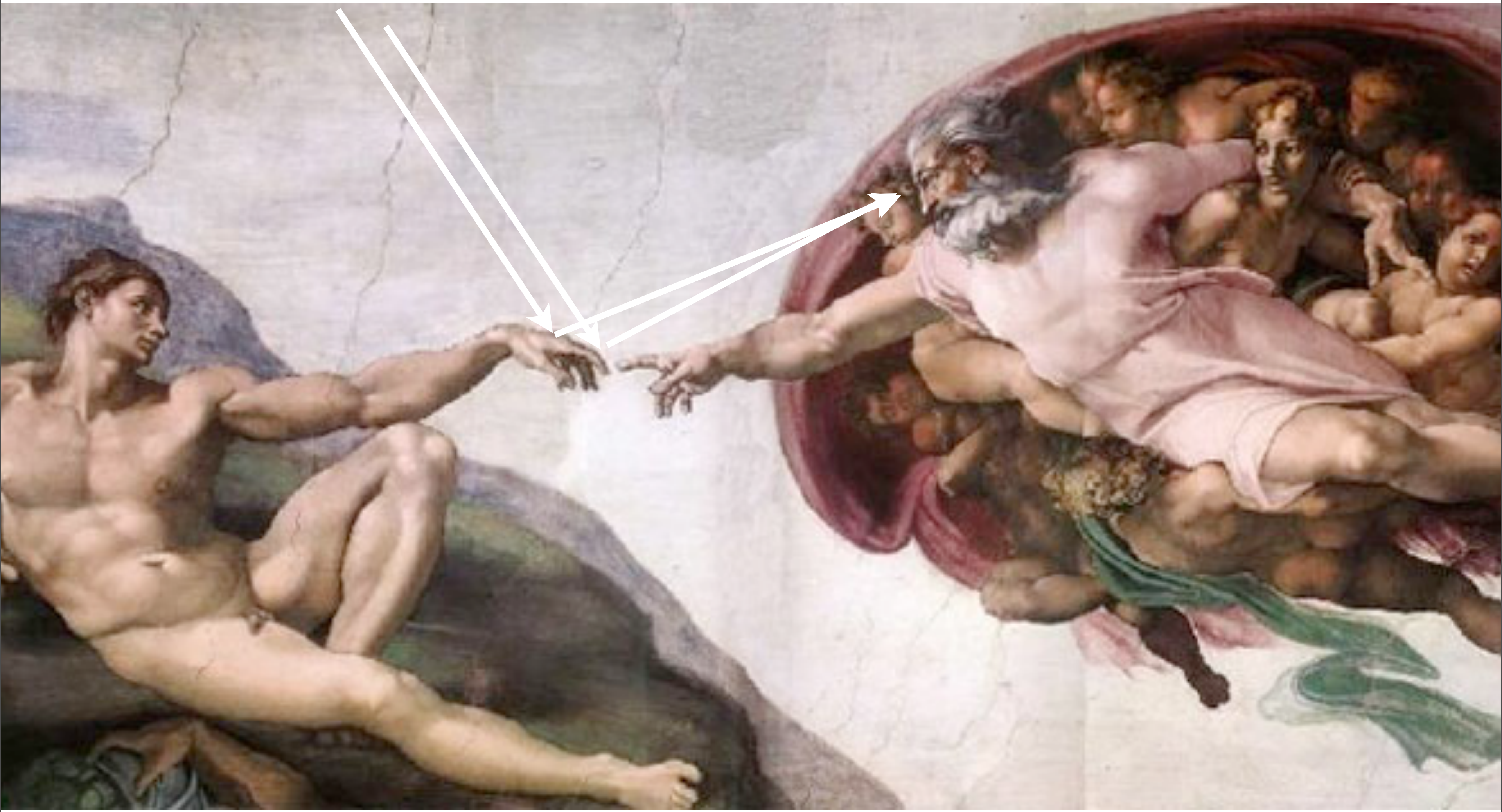
Rouen Cathedral, Claude Monet, ~1893

Scattering

Scattered light forms the image of a hand

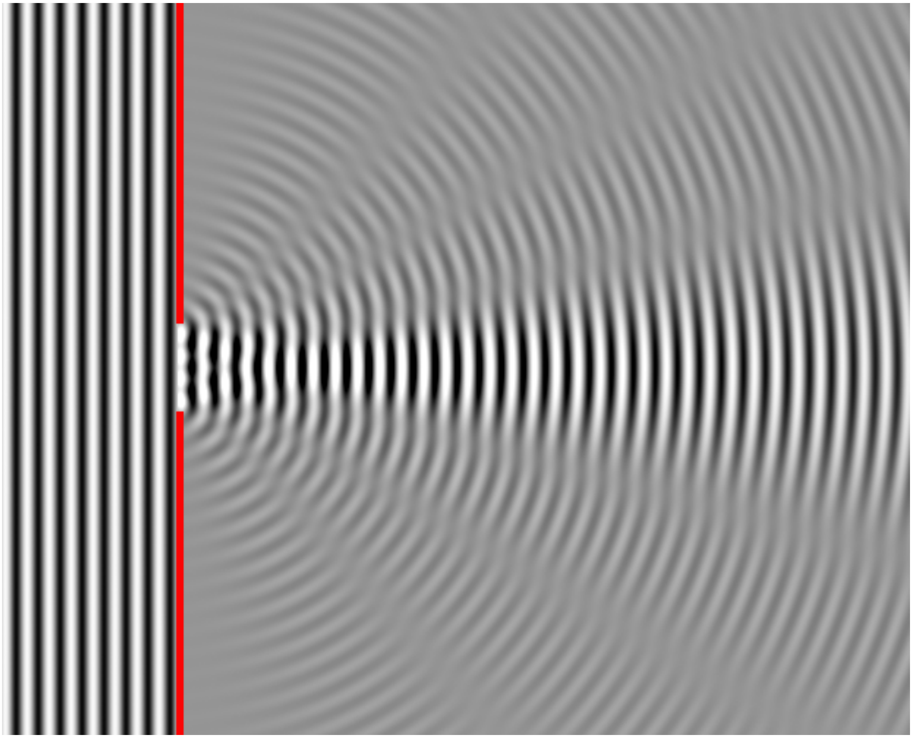
Scattering

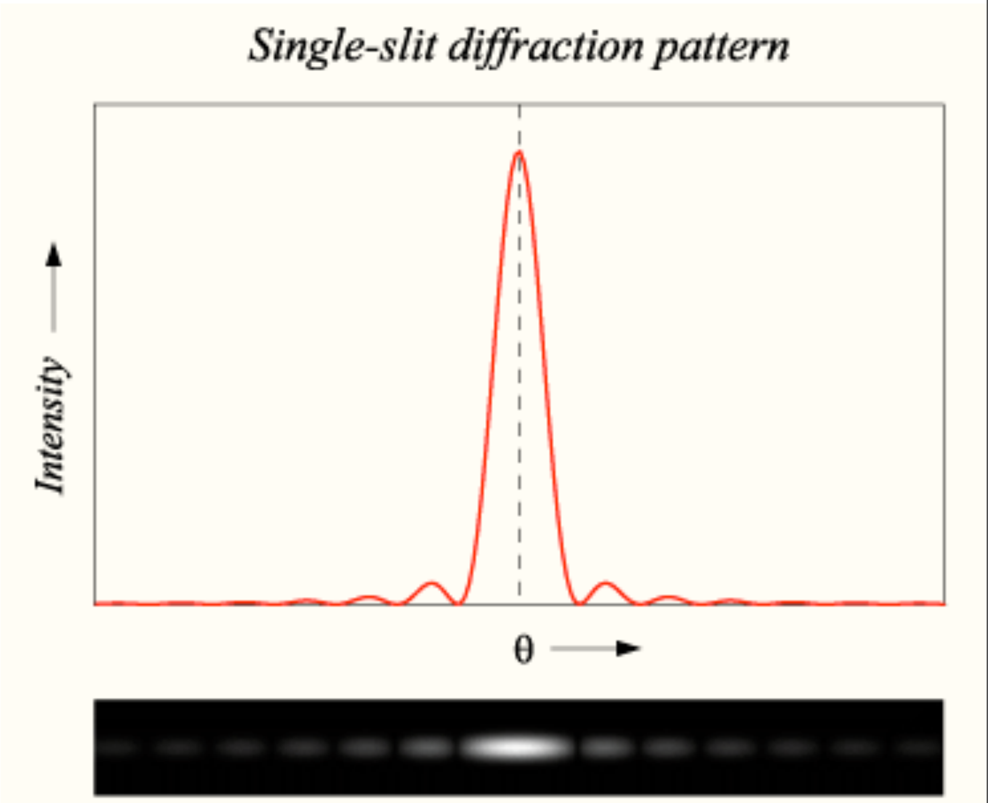
Scattered light forms the image of a hand on a detector



Fraunhofer Diffraction

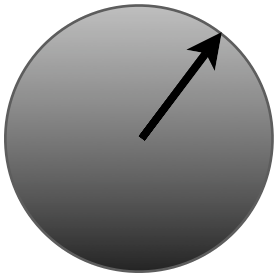
- Incident wave is monochromatic and parallel
- Image is far away
- Single slit: $a \sin \theta_n = n \lambda$ gives interference minima
- The size a can be determined if $\lambda < a$





Size of the Proton

$r \sim 1$ fm $= 10^{-15}$ m

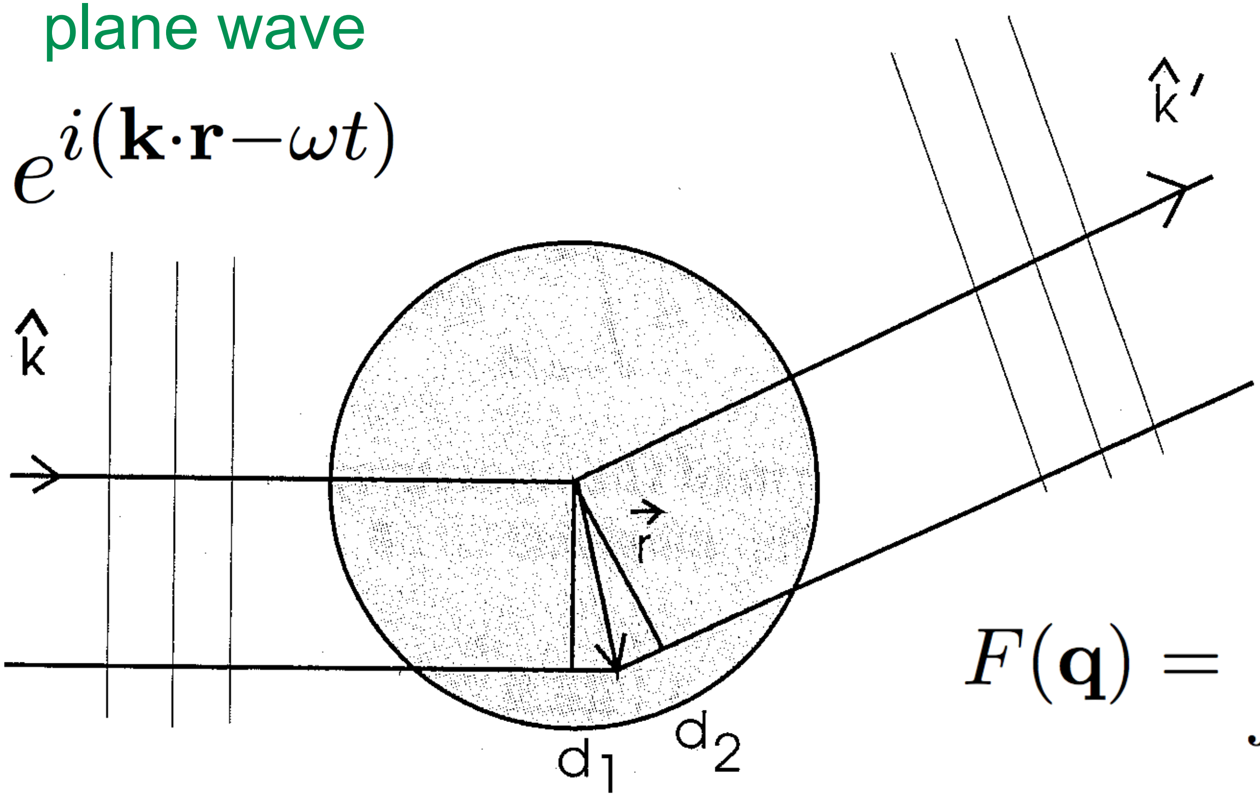


- We need a wave with < 1 fm wavelength
- Why not use electrons?
- DeBroglie wavelength is $\lambda = h/p$
- $pc = hc/\lambda$ = (1.24 GeV-fm)/(1 fm) = 1.24 GeV
- Electron mass m_e = 0.511 MeV is negligible
- Electron accelerators are required
  - Jefferson Lab: E_{beam} = 6 GeV
  - DESY: E_{beam} = 27 GeV
  - SLAC: E_{beam} = 50 GeV
  - CERN: E_{beam} = 200 GeV for muons
  - Fermilab: E_{beam} = 500 GeV for muons

Scattering from a Sphere

plane wave

$$e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$



Interference pattern is the Fourier transform of the density

form factor

$$F(\mathbf{q}) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r$$

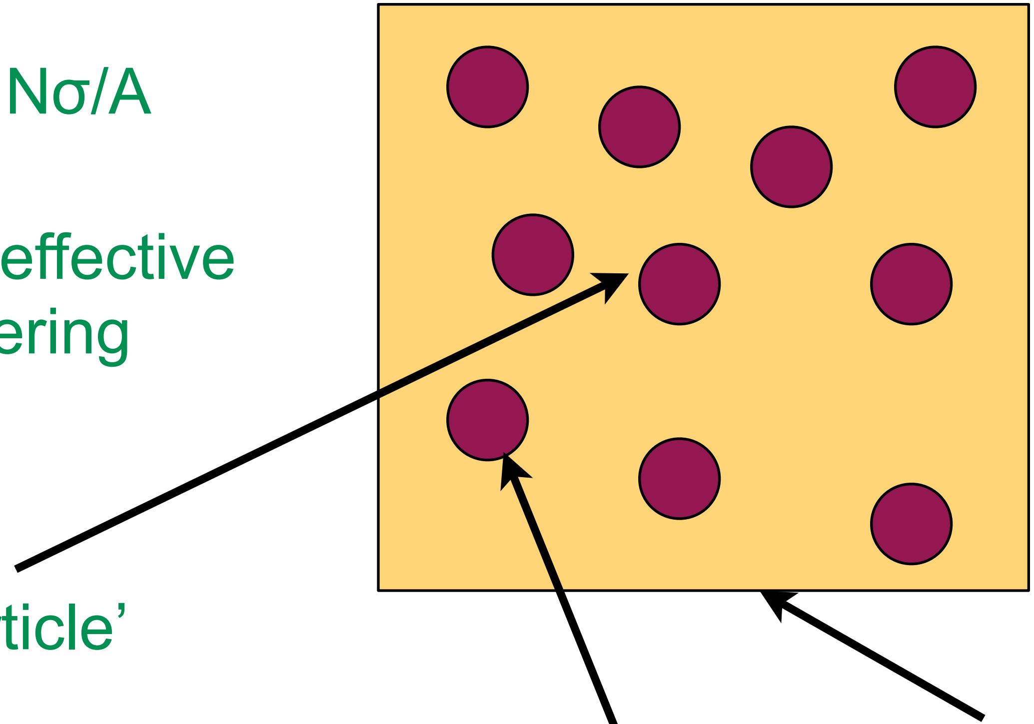
$$\Delta = \frac{2\pi}{\lambda}(d_1 + d_2) = \frac{2\pi}{\lambda}(\hat{\mathbf{k}}\cdot\mathbf{r} - \hat{\mathbf{k}}'\cdot\mathbf{r}) = \mathbf{q}\cdot\mathbf{r}$$

additional path length

Cross Section

Probability of scattering = $N\sigma/A$

We generalize σ to be an effective area that for a given scattering probability.



'Tiny probe particle'

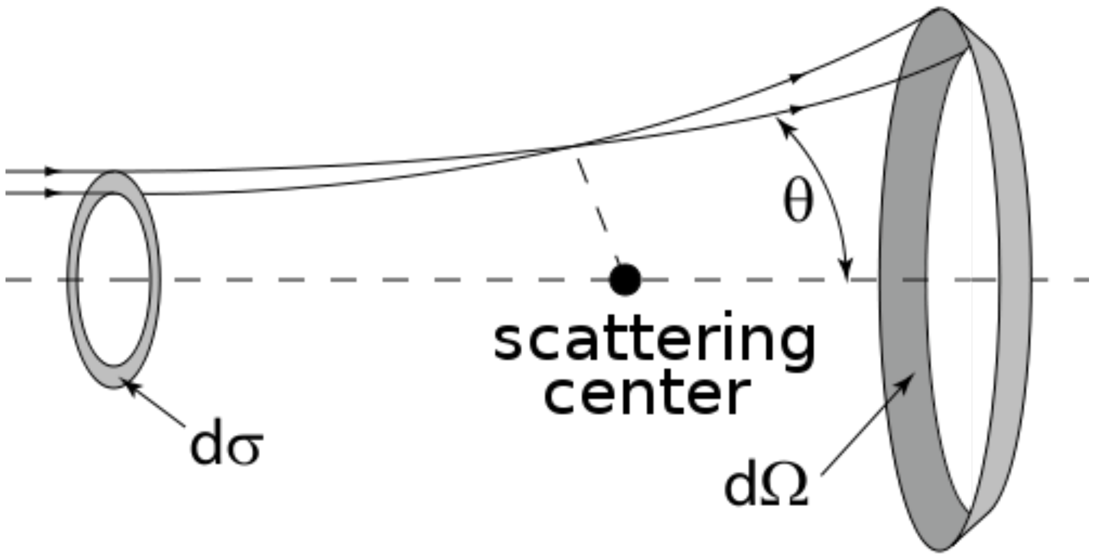
Probability of scattering = N_{scat} / N_{inc}

Area σ Area A

$N = N_A \rho t A / M$ $N_A = 6.02 \times 10^{23}$ ρ = target density

t = target thickness M = molar mass

Point Cross Sections



α ~ 1/137 is the fine structure constant

E is the incident and E' the scattered energy

$$\left(\frac{d\sigma}{d\Omega}\right)^{e}_{\text{Rutherford}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}$$

for an electron on a charge-1 target

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \cos^2 \frac{\theta}{2}$$

recoil

for a relativistic electron on a charge-1 target

Form Factors

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2$$

Differential cross section for an extended object

Point cross section

Form factor (internal structure)

$$F(\mathbf{q}) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r = \int \left(1 + i\mathbf{q}\cdot\mathbf{r} - \frac{1}{2}(\mathbf{q}\cdot\mathbf{r})^2 + \ldots\right)\rho(\mathbf{r})d^3r = 1 - \frac{1}{6}q^2\langle r^2\rangle + \ldots$$

Fourier expansion of the form factor

RMS particle radius

$$\int \rho(\mathbf{r})d^3r = 1$$

Density normalization

Form factor for $\rho(r)=e^{-\Lambda q}$

$$F(q^2) = \left(1 - \frac{q^2}{\Lambda^2}\right)^{-2}$$

Elastic ep Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)^{ep}_{\text{Elastic}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)}{\epsilon(1+\tau)}$$

Point cross section

Electric form factor

Magnetic form factor

$$\epsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1} \qquad \tau = Q^2/4M^2$$

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71}\right)^{-2}$$

Dipole form factor

Q^2 is momentum transfer squared minus energy transfer squared

M is the mass of the proton

Proton Form Factors

Arrington, Melnitchouk, Tjon, PRC**76**(07)035205

Nobel Prize

Current world data

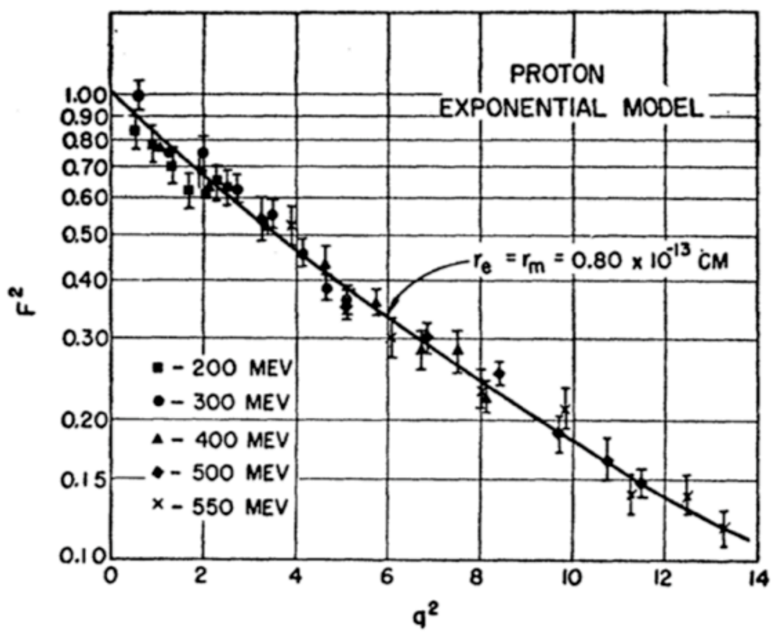
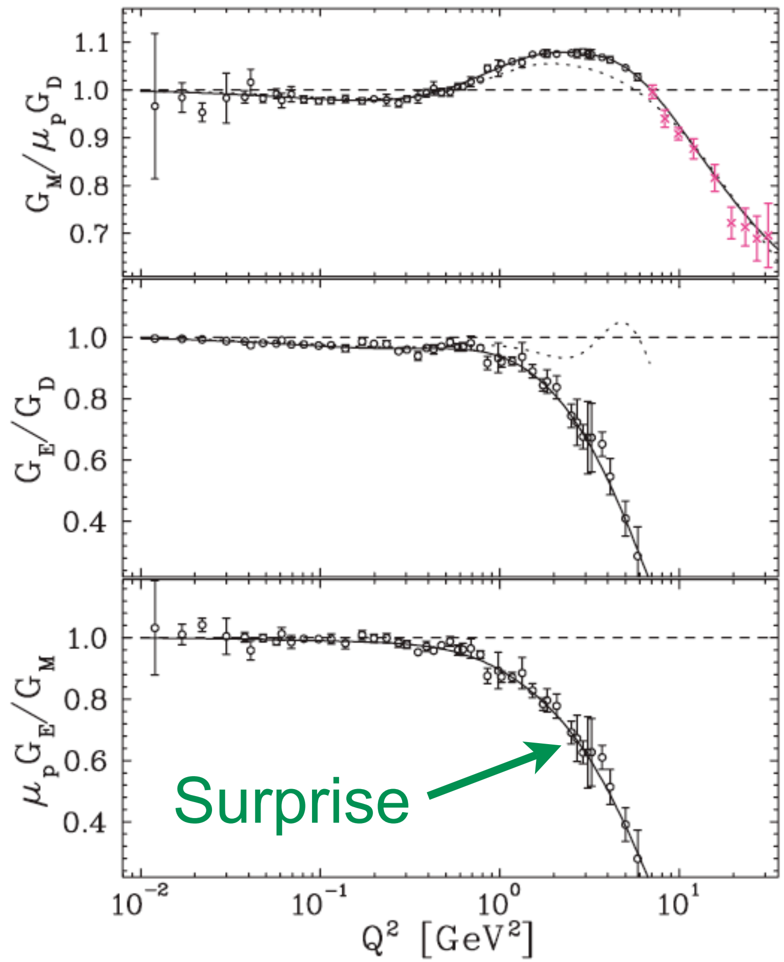
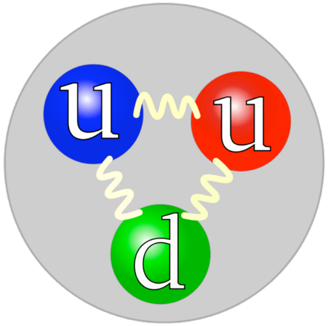


FIG. 11. Summary of the comparison between the exponential model with equal radii (0.8×10^{-13} cm) and the experimental points. The square of the form factor is plotted against q^2, where q is given by Eq. (3). q^2 is given in units of 10^{-26} cm^2.



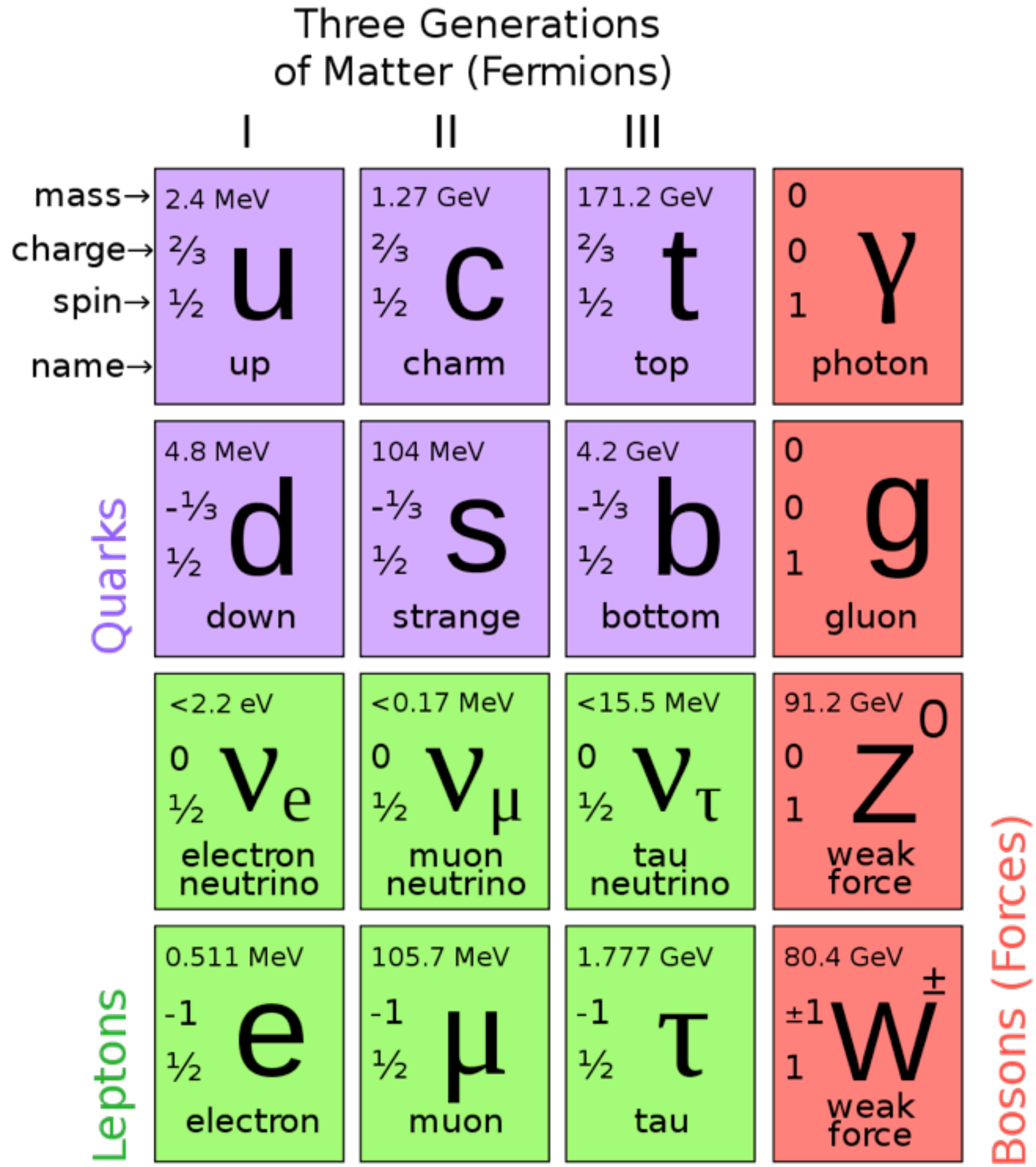
Chambers & Hofstadter, PR**103**(56)1454

Quarks



Because the proton has a form factor, we know it is made of smaller building blocks.

p: uud
n: ddu
π$^+$: u$\bar{d}$
Λ: uds



Forces

\+ photon –

$$F \sim \frac{\alpha}{r^2}$$

QED

Coulomb force
1 charge & 1 anticharge
1 photon with no charge

ΔEΔt ~ ħ
F ~ Δp/Δt
r ~ c Δt
ΔE = Δp
F ~ 1/r^2

Forces are carried by spontaneously created messenger particles

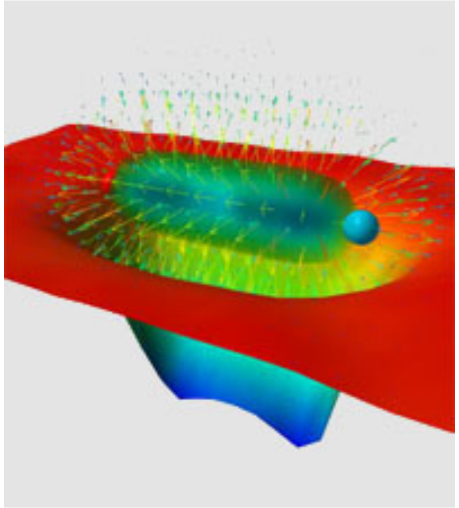
gluon

$$F \sim \alpha_s r$$

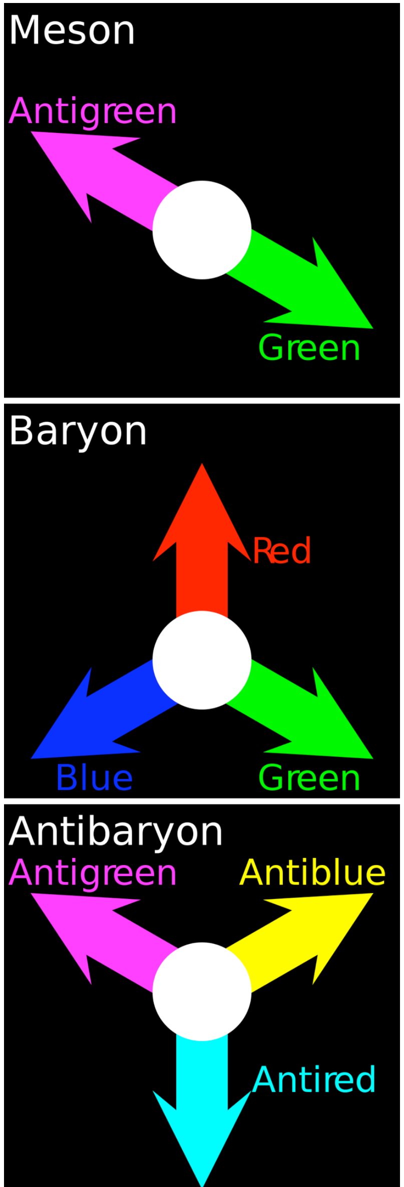
Strong (nuclear) force
3 charges & 3 anticharges
8 gluons with mixed charge and anticharge

QCD

Leinweber, Lattice QCD simulation



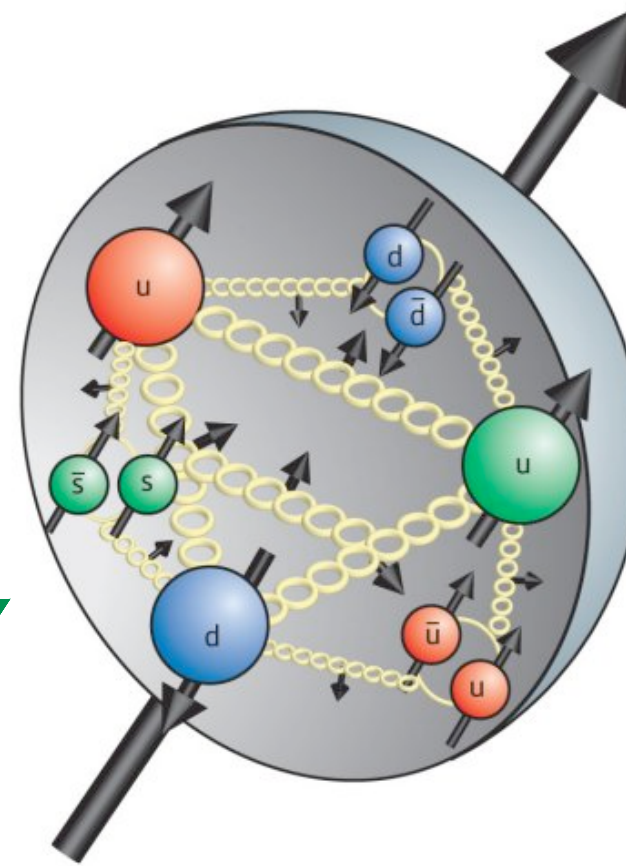
Hadrons



hadron: anything with a quark in it
meson: quark+antiquark
baryon: 3 quarks

Atoms are neutral wrt electric charge and hadrons are neutral wrt color charge

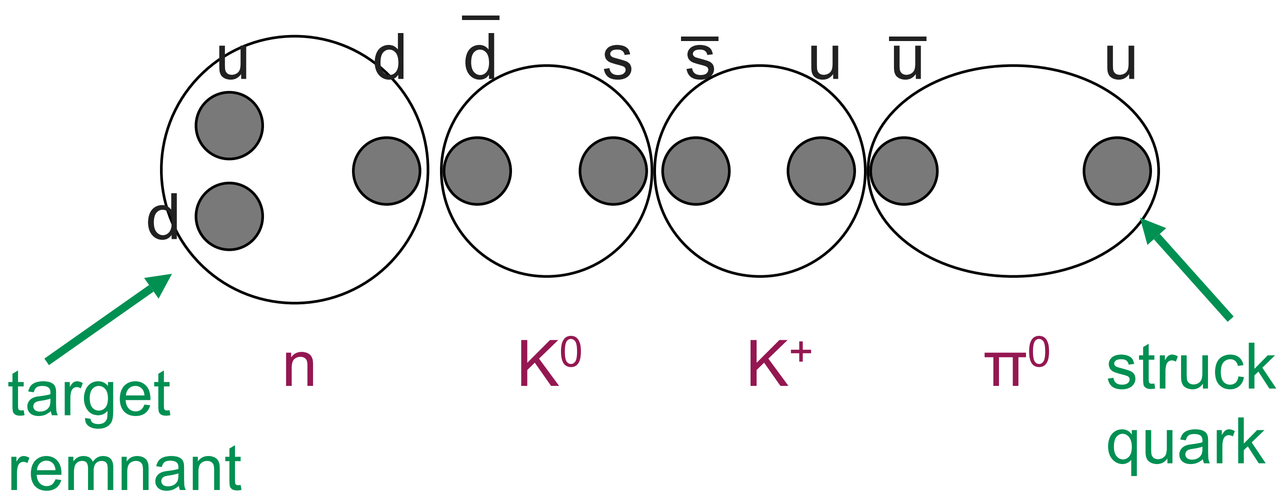
Updated cartoon of the inside of a proton



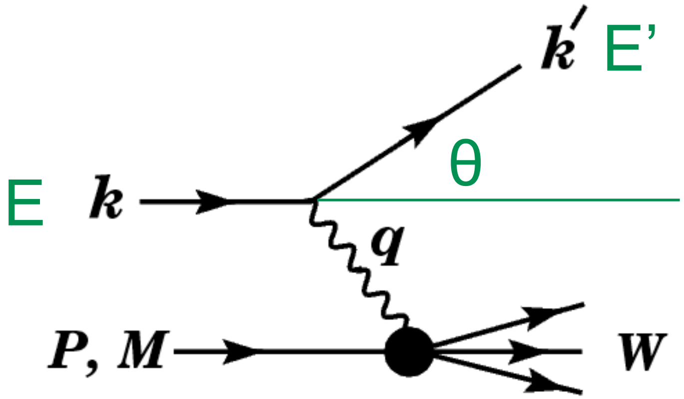
Quantum Weirdness

- When combining quantum mechanics with relativity, we give up the number 1.
- ‘Empty’ space teems with spontaneously created virtual particles.
- All real particles are clothed by a multitude of virtual ones
- Exchanges of virtual particles is how a force is felt between two real particles
- Although this solves the problem of action at a distance, we are left with unavoidable infinities.

Fragmentation
Hadronization



Relativistic Kinematics



$$E = \sqrt{(mc^2)^2 + (pc)^2} = \sqrt{m^2 + p^2}$$

$$(E, \vec{p}) \cdot (E, \vec{p}) \equiv E^2 - p^2 = m^2$$

$$W^2 = (M + \nu, \mathrm{q})^2 = M^2 + 2M\nu - Q^2$$

$$x = \frac{Q^2}{2M\nu}$$ =1 for elastic scattering

4-vectors:

k = (E,0,0,E)

k’ = (E’,E’sinθ,0,E’cosθ)

p = (M,0,0,0)

q = (ν, **q**) = k - k’

Lorentz invariants:

q•q = $-Q^2$

p•q/m = ν

$(p+q)^2 = W^2$

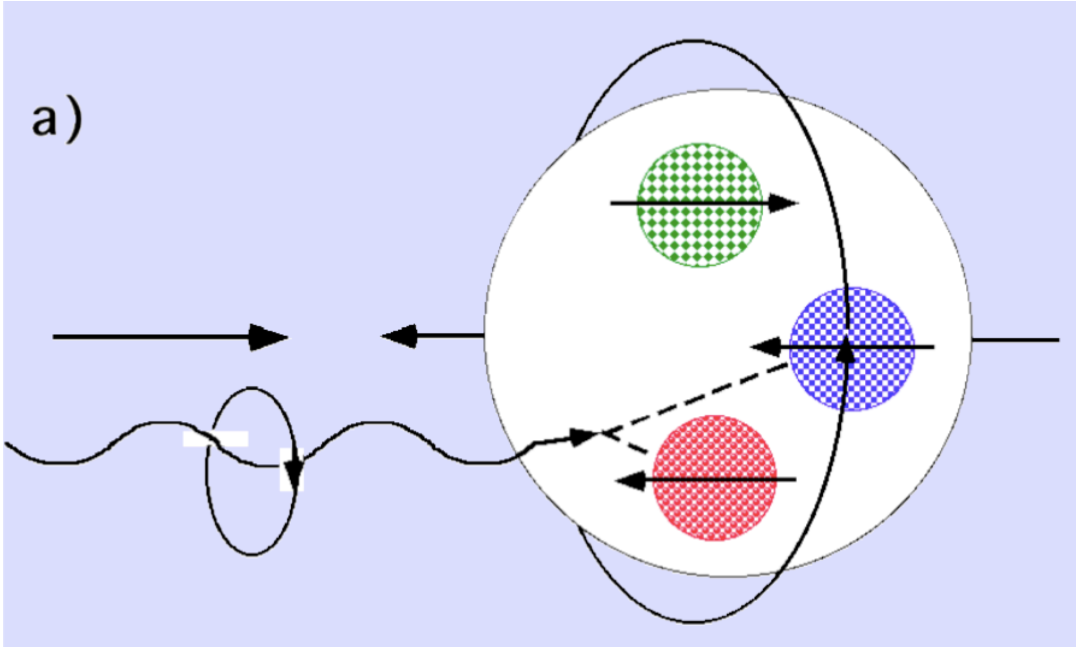
$(k+p)^2 = s$

-q•q/(2p•q) = x

p•q/p•k = y

How to Measure Quark Spin

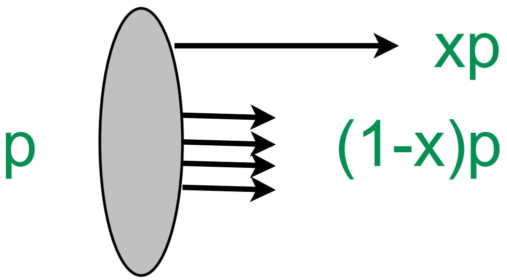
At high energies only one dimension is important



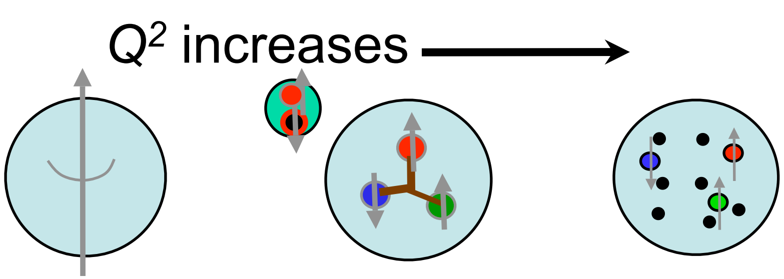
Conserve angular momentum:
$S_e = +1/2$
$S_e = -1/2$; $S_\gamma = 1$
$S_q = -1/2$; $S_\gamma = 1$
$S_q = +1/2$
$S_q \neq +3/2$ ever

Electrons can only scatter from quarks with opposite spin. The difference between electron scattering for spins opposite and along the proton's spin counts the quarks with spin along and opposite to the proton's spin.

Deep Inelastic Scattering



'Elastic' e-quark scattering:
$-q \bullet q/(2xp \bullet q) = 1$, ∴
$x = Q^2/2M\nu$ is the fraction of the proton momentum carried by the struck quark.



Unpolarized Cross Section:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{8\pi\alpha^2 y}{Q^4}\left[\frac{y}{2}F_1 + \frac{\xi}{2xy}F_2\right]$$

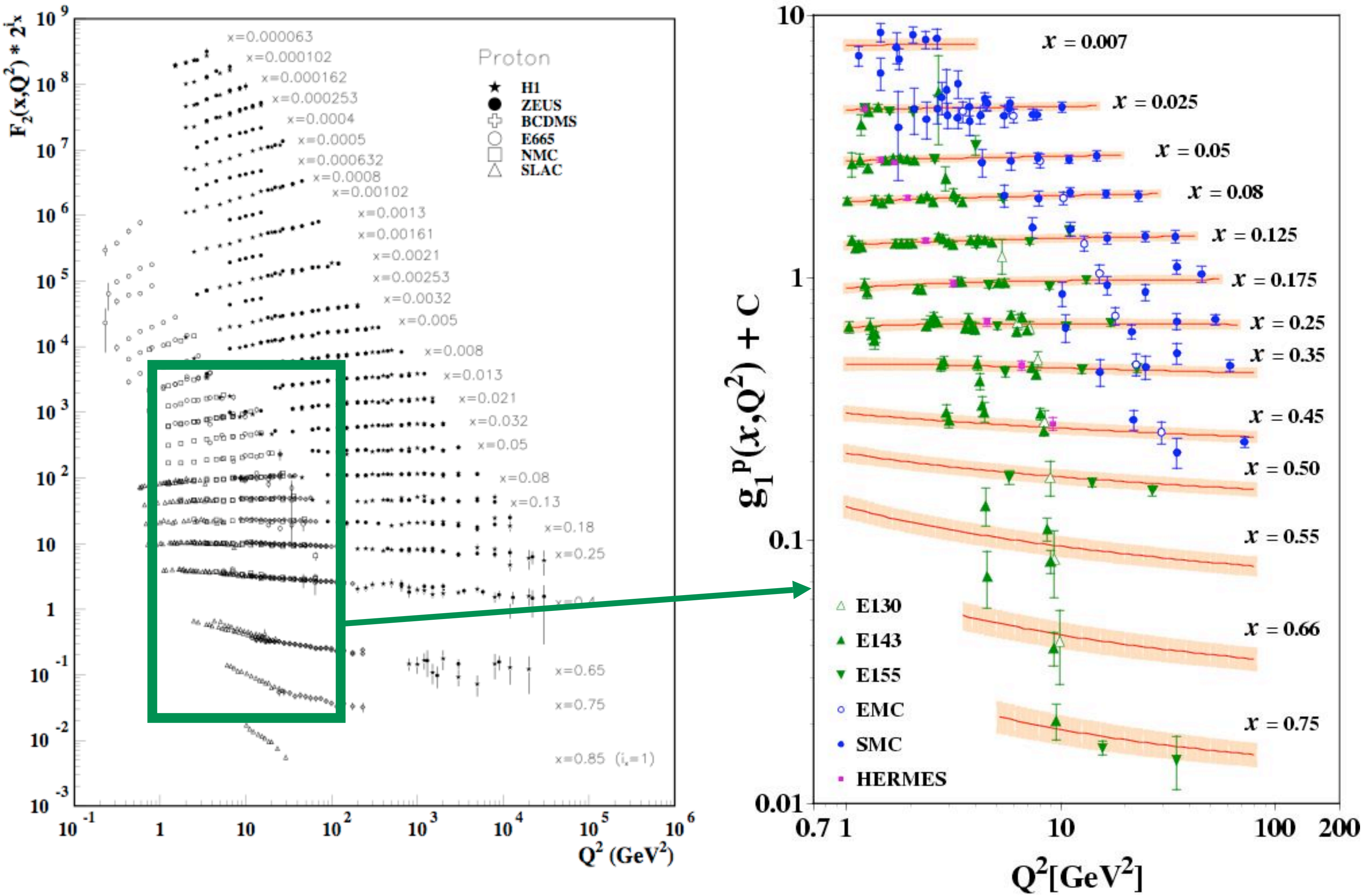
Polarized Cross Section:

$$\frac{d^2\Delta\sigma}{dx\,dQ^2} = \frac{8\pi\alpha^2 y}{Q^4}[\cos\alpha\{(\xi + \frac{y}{2})g_1 - \frac{\gamma y}{2}g_2\} - \sin\alpha\cos\phi\{\frac{y}{2}g_1 + g_2\}]$$

$\alpha =$ polar angle of target spin wrt the beam axis
$\phi =$ azimuthal spin angle wrt the scattering plane
$\alpha = 0°$ (longitudinal); $\alpha = 90°, \phi = 0°$ (transverse).
$\gamma^2 = 4M^2x^2/Q^2 = Q^2/\nu^2$
$\xi = 1 - y - \gamma y^2/4$

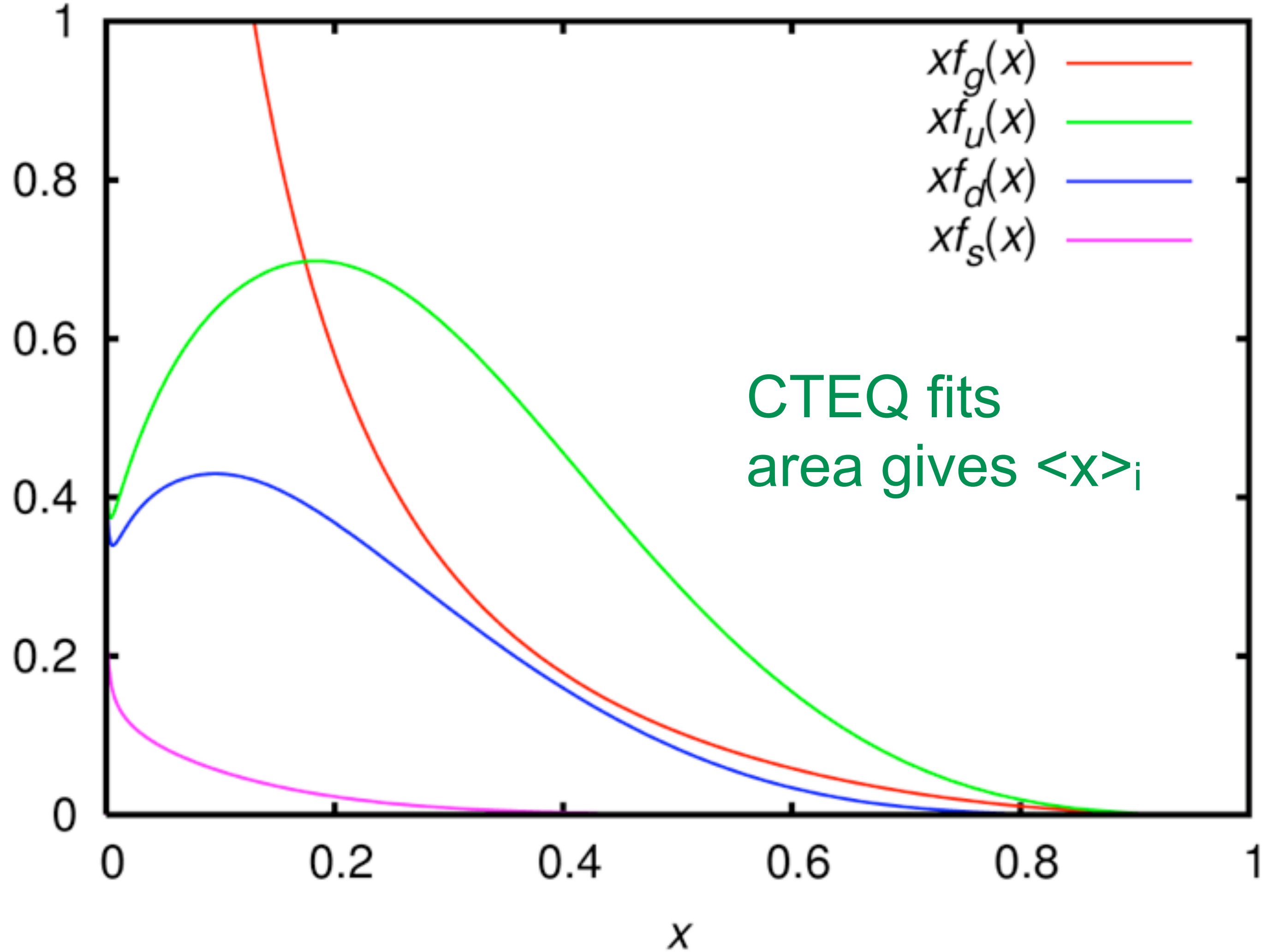
Parton Model:
$F_1(x, Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) + q^\downarrow(x) + \bar{q}^\uparrow(x) + \bar{q}^\downarrow(x))$
$F_2(x, Q^2) = 2xF_1(x, Q^2)$
$g_1(x, Q^2) = \frac{1}{2}\Sigma_i e_i^2(q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x))$

$F_2^p(x,Q^2)$ and $g_1^p(x,Q^2)$

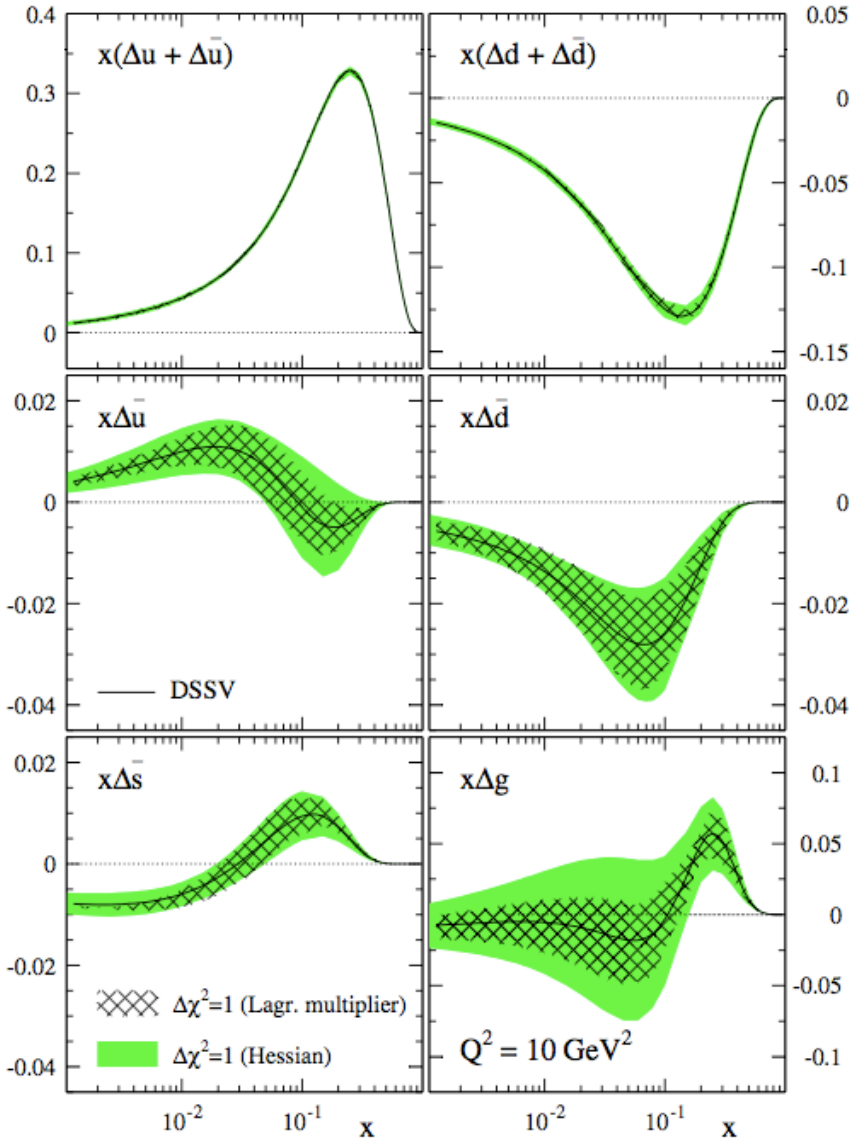


Parton Distribution Functions

Fits to all the world's data yield the probability distributions for finding a quark or a gluon with momentum fraction x.



Polarized PDFs



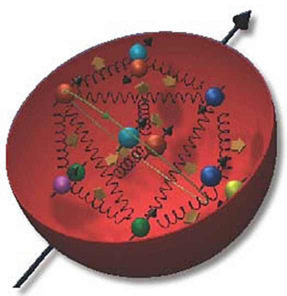
DSSV fits;
area gives $<\Delta x>_i$

Q^2 evolution is used to determine Δg

Large uncertainties remain

DSSV PDFs

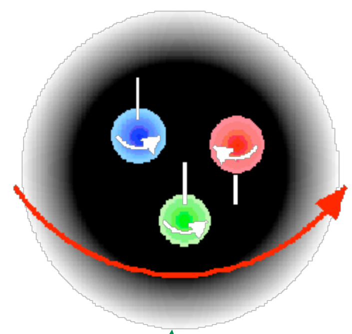
x range in Eq. (35)	Q^2 [GeV^2]	$\Delta u + \Delta\bar{u}$	$\Delta d + \Delta\bar{d}$	$\Delta\bar{u}$	$\Delta\bar{d}$	$\Delta\bar{s}$	Δg	$\Delta\Sigma$
0.001–1.0	1	0.809	−0.417	0.034	−0.089	−0.006	−0.118	0.381
	4	0.798	−0.417	0.030	−0.090	−0.006	−0.035	0.369
	10	0.793	−0.416	0.028	−0.089	−0.006	0.013	0.366
	100	0.785	−0.412	0.026	−0.088	−0.005	0.117	0.363
0.0–1.0	1	0.817	−0.453	0.037	−0.112	−0.055	−0.118	0.255
	4	0.814	−0.456	0.036	−0.114	−0.056	−0.096	0.245
	10	0.813	−0.458	0.036	−0.115	−0.057	−0.084	0.242
	100	0.812	−0.459	0.036	−0.116	−0.058	−0.058	0.238



More realistic

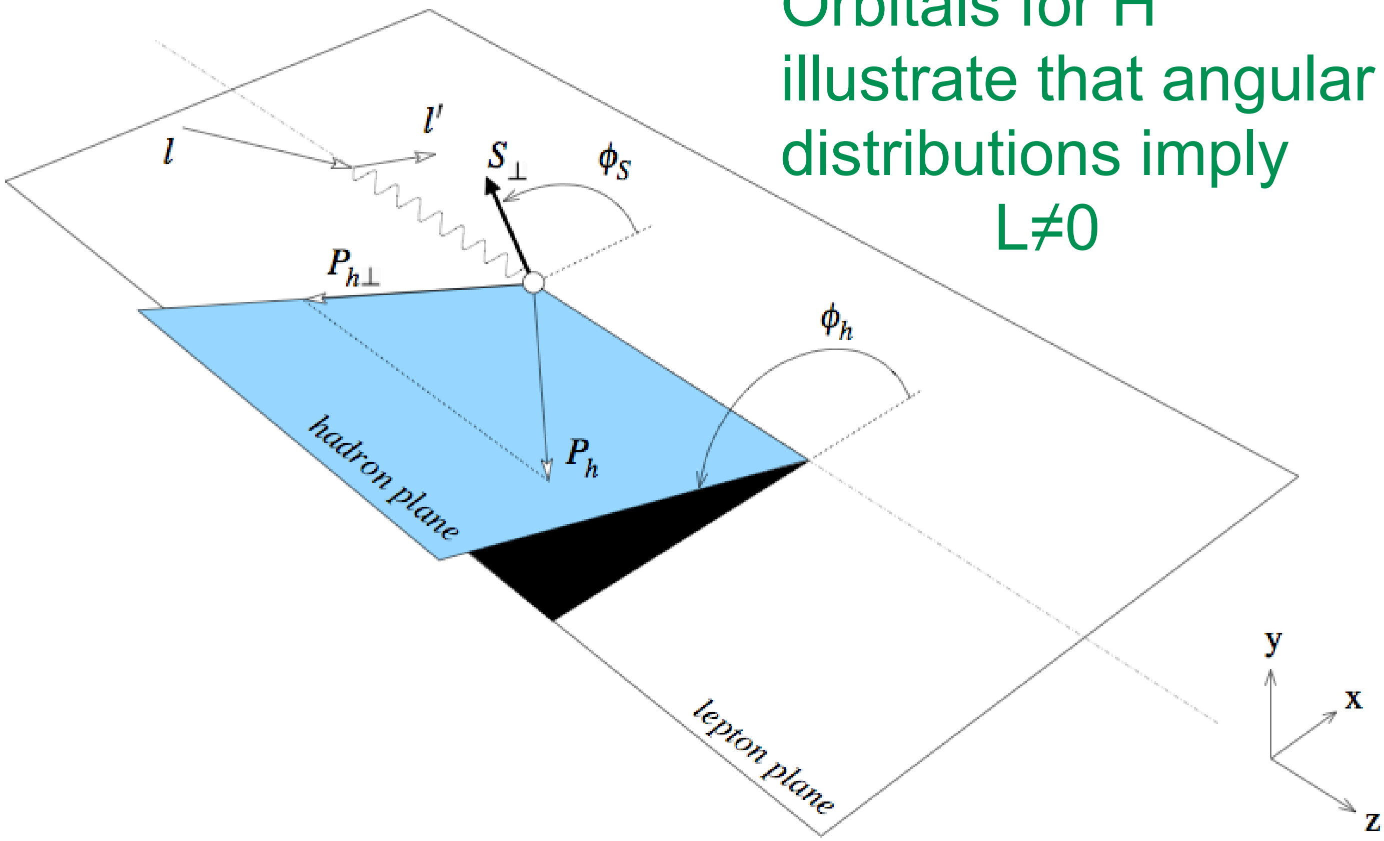
$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

- Significant contributions from x<0.001
- ΔG vanishes with increasing Q^2
- At Q^2=4 GeV^2, L_z = 0.474 (large)
- Errors on ΔG are still very large

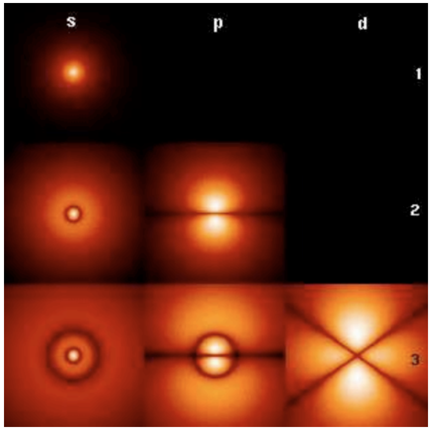


Way too naive

Semi-Inclusive DIS



Orbitals for H illustrate that angular distributions imply L≠0

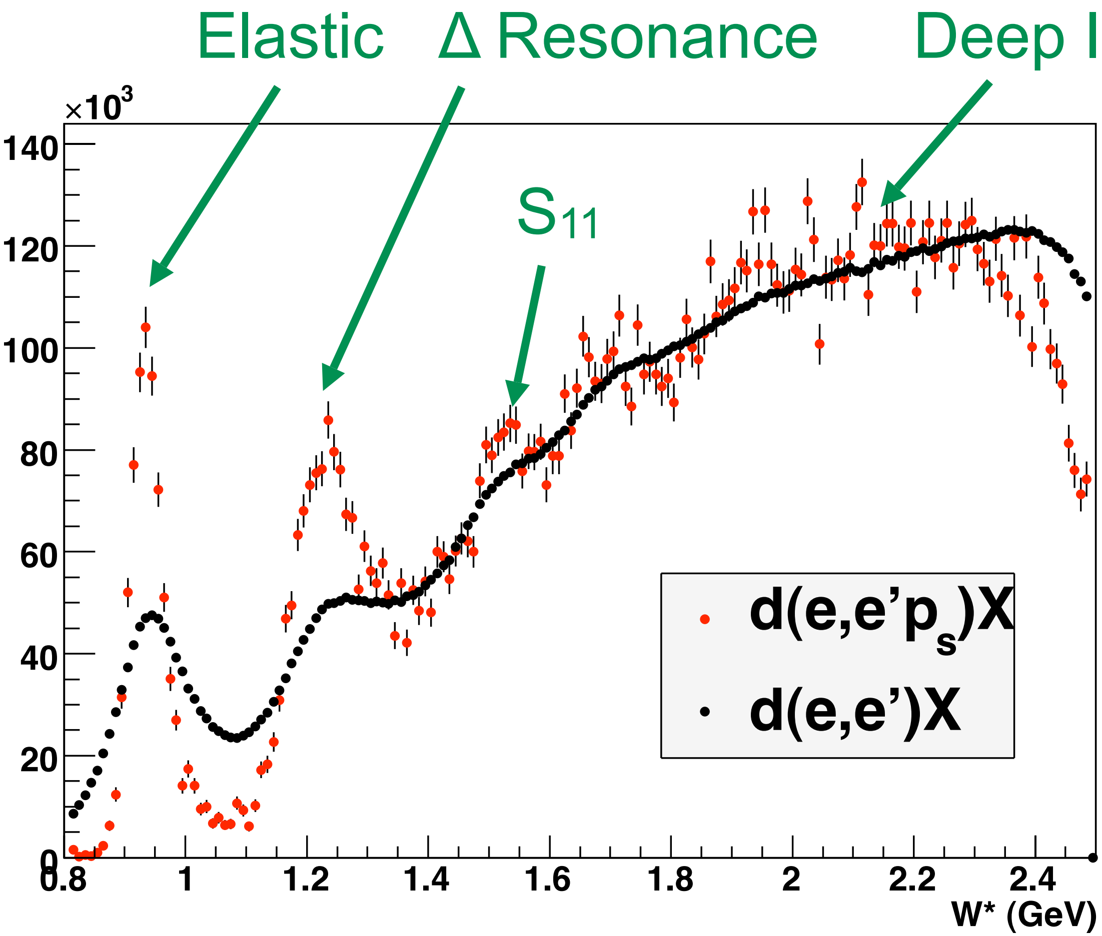


L=1 L=2

Azimuthal distributions of hadrons that contain the struck quark are sensitive to orbital angular momentum. These measurements are in progress.

What Can JLab Offer?

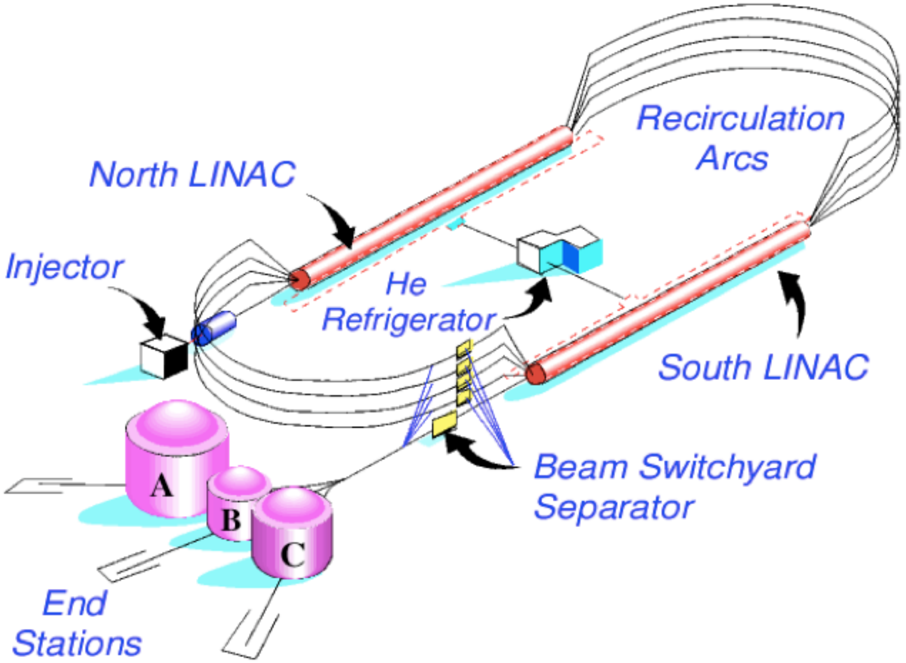
Jefferson Lab in Newport News, Virginia is a precise electron microscope for viewing the guts of protons and neutrons



Jefferson Lab:

- Continuous beam
- High Luminosity
- High Polarization
- 6 GeV beams
- 12 GeV beams in '14
- 3 experimental halls
- $0.01<Q^2<6$ GeV^2
- $0.8<W<3$ GeV

Jefferson Lab Accelerator

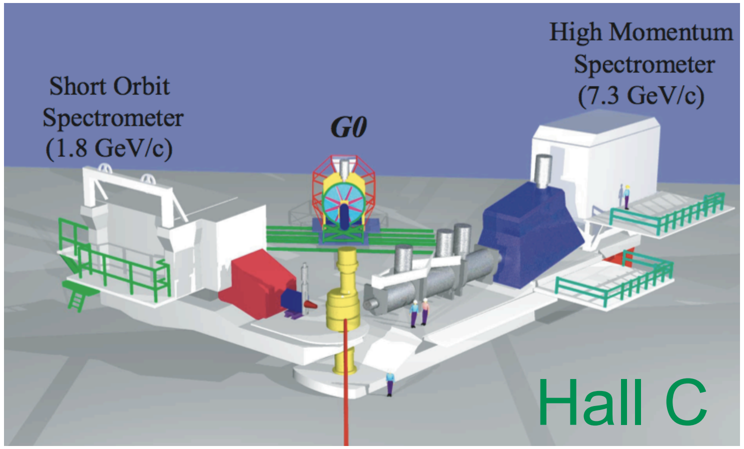


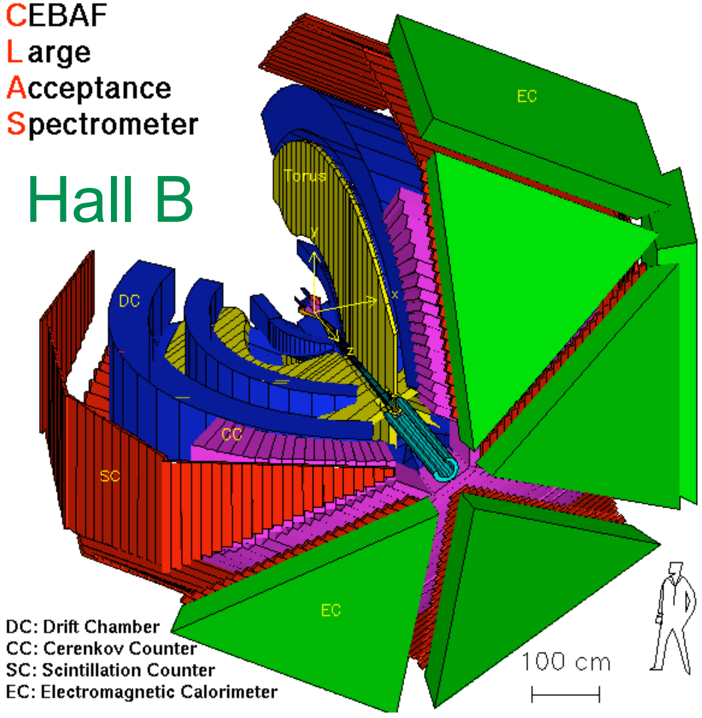
- Electron beams up to 6 GeV with >80% longitudinal polarization
- Beam currents of 1-50 nA in Hall B

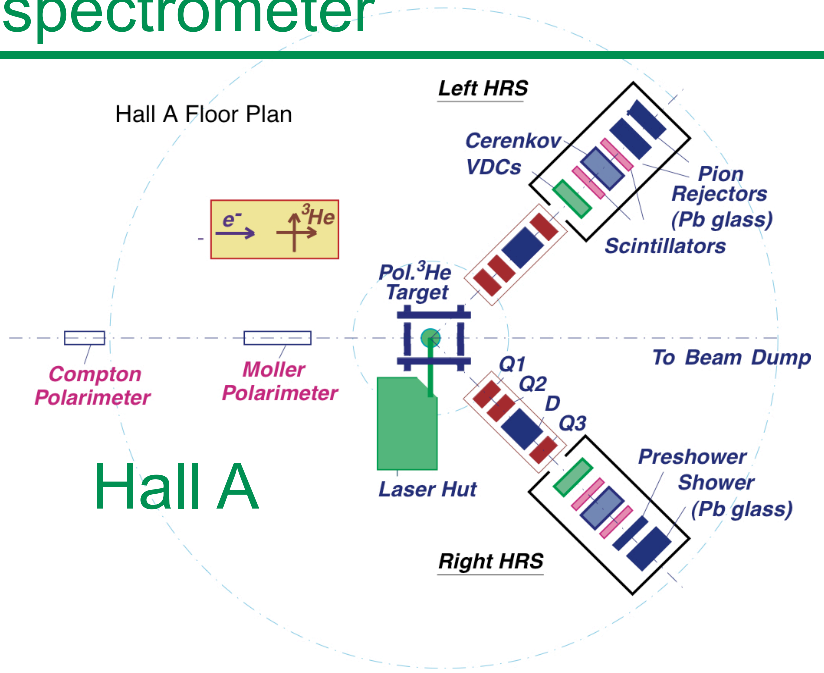
JLab Experimental Halls

The College of WILLIAM & MARY

- 3 experimental halls
- Hall A: 2 high-resolution spectrometers
- Hall B: 1 large-acceptance spectrometer
- Hall C: 1 electron & 1 proton spectrometer

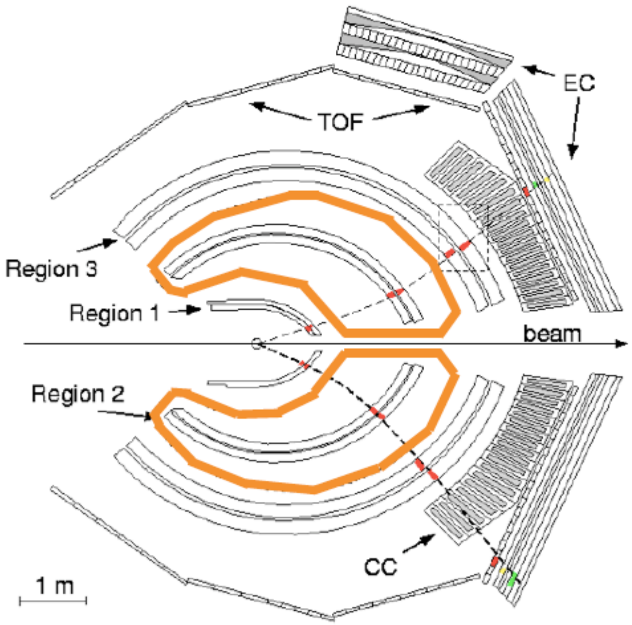


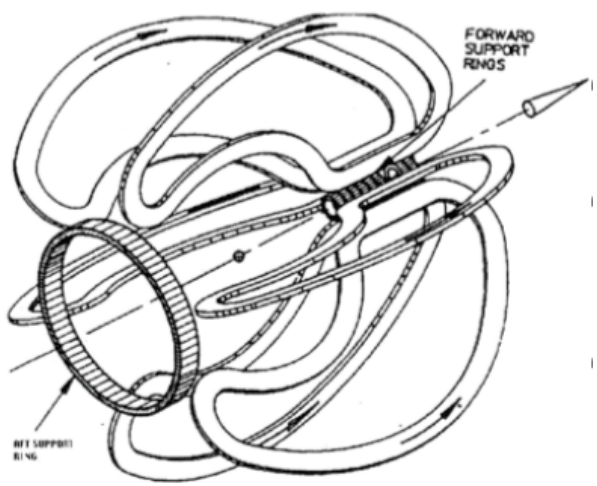




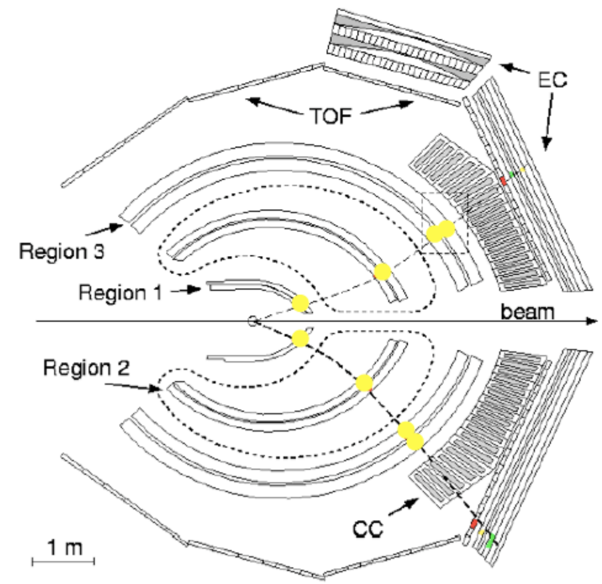
CLAS Detectors

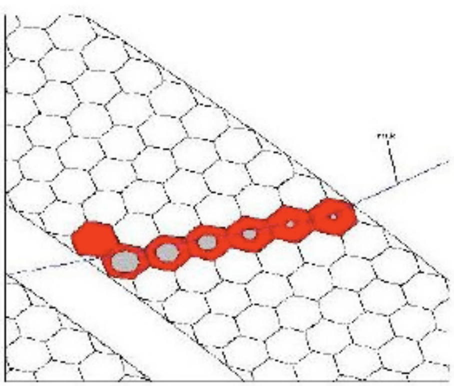
Torus Magnet





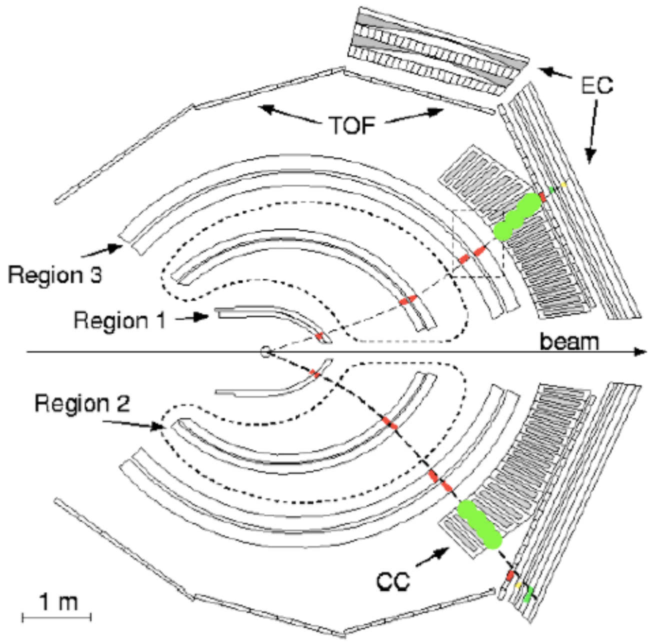
Drift Chambers (DC)

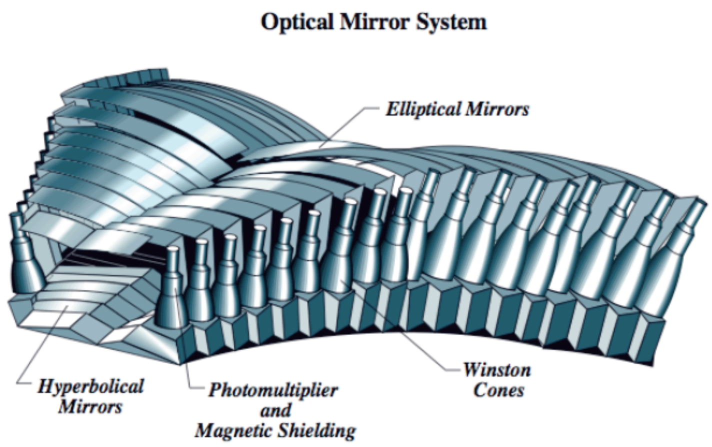




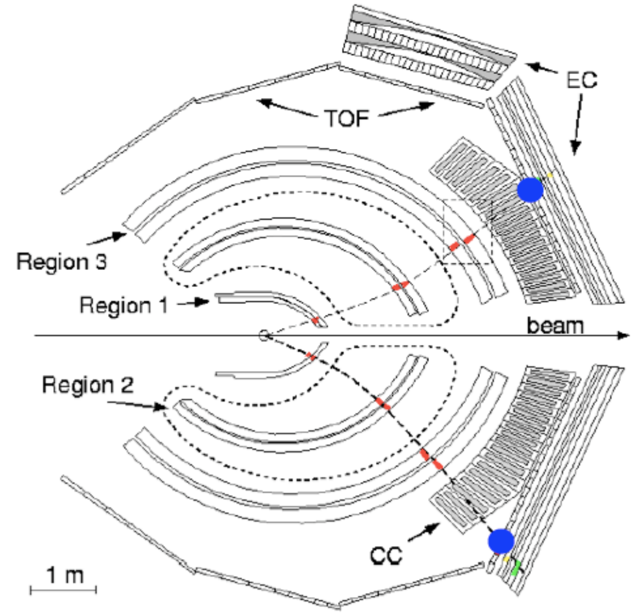
CLAS Detectors

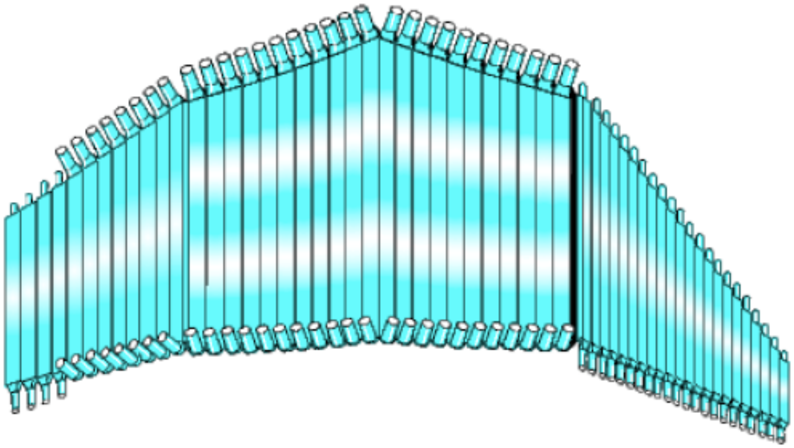
Cherenkov Counters (CC)





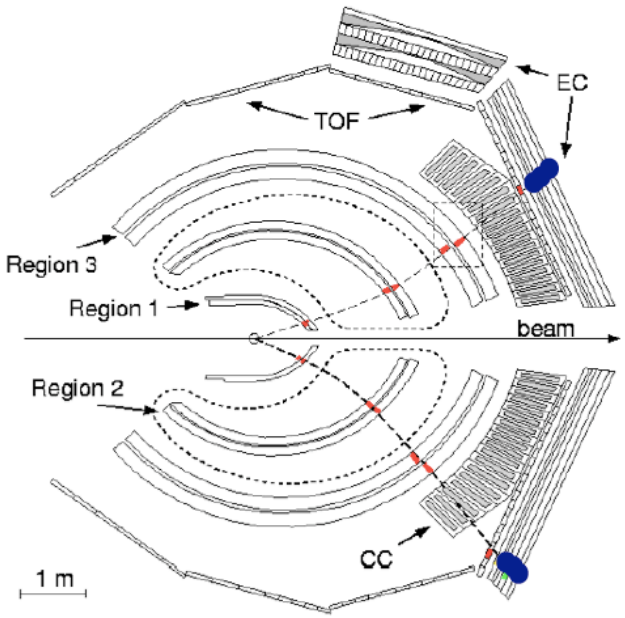
Scintillation Counters (SC)

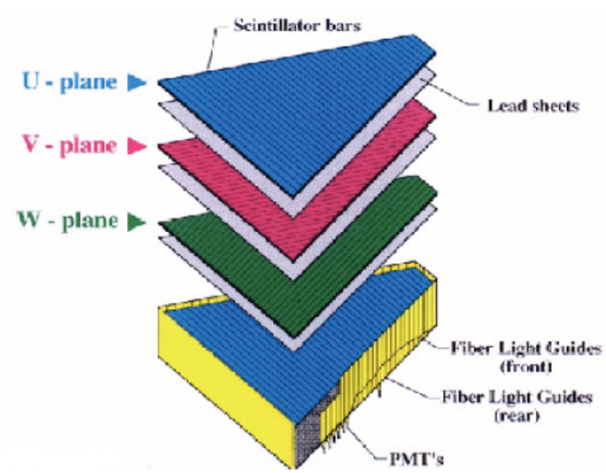


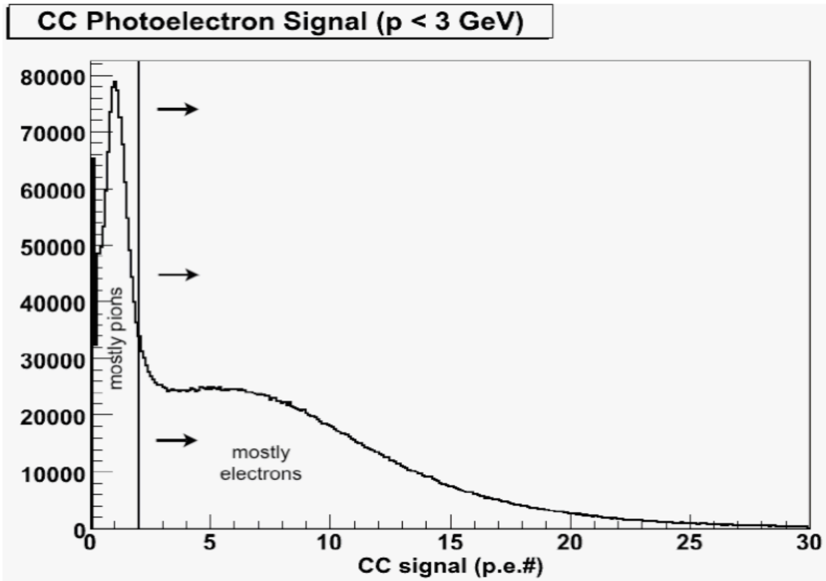


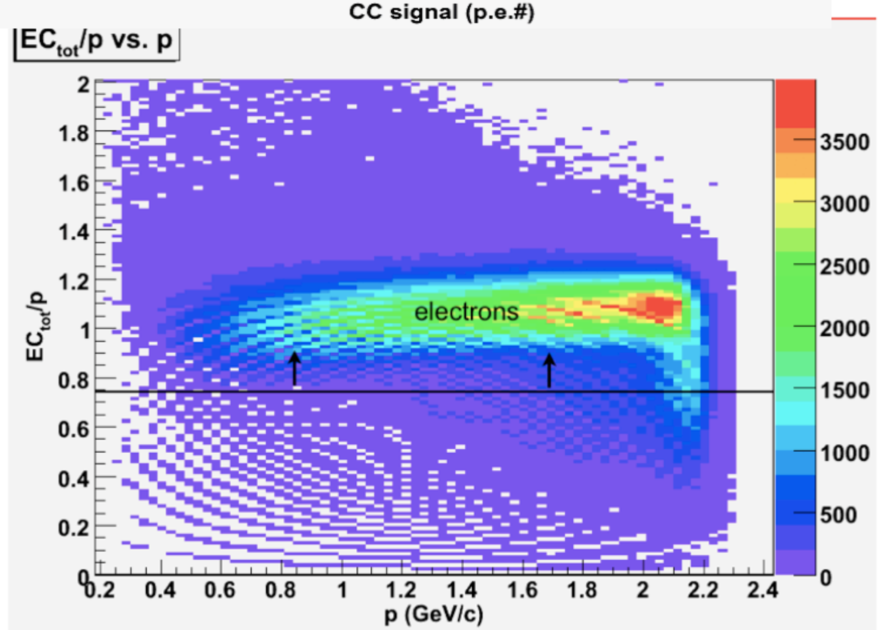
CLAS Detectors

Electromagnetic Calorimeters (EC)

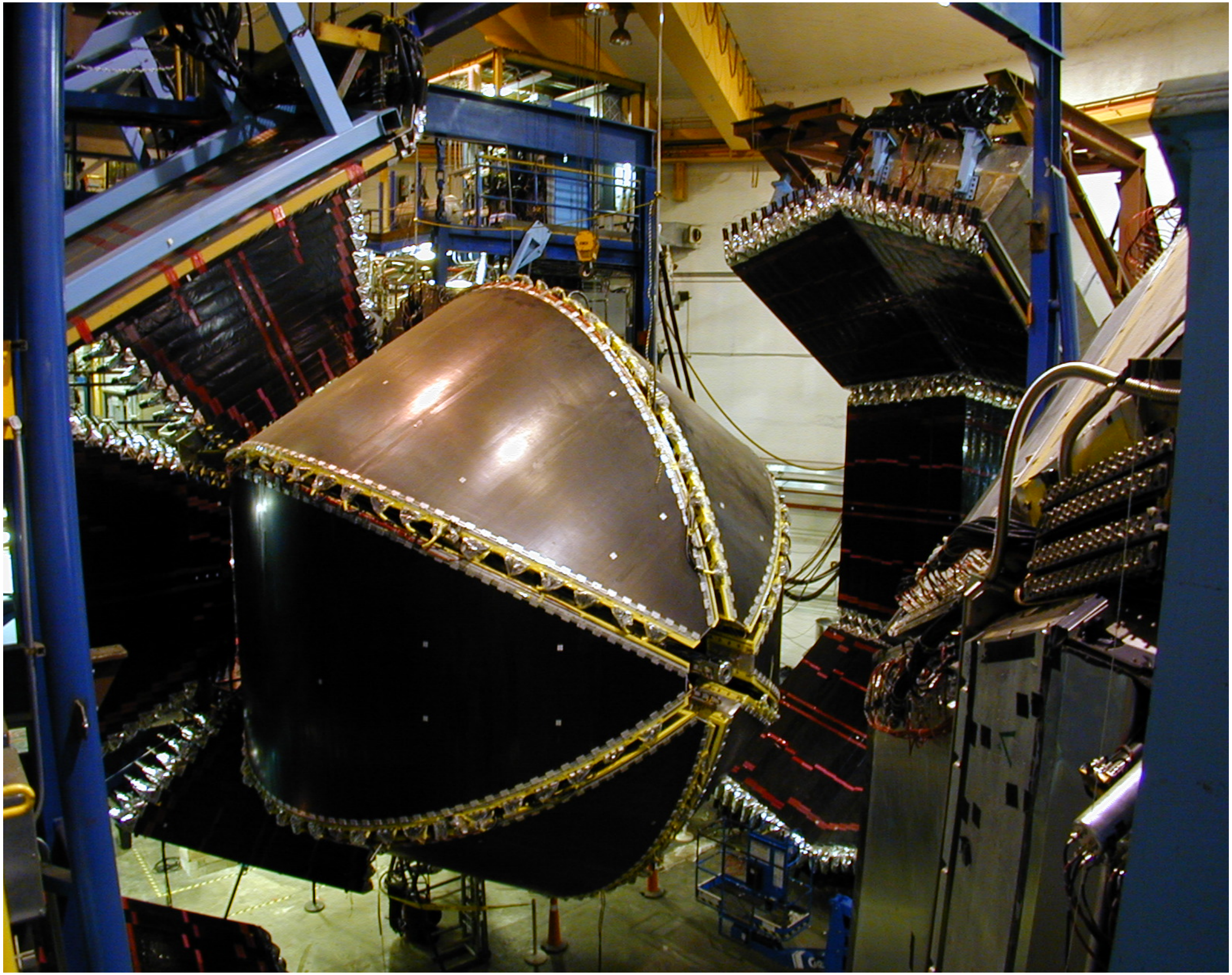




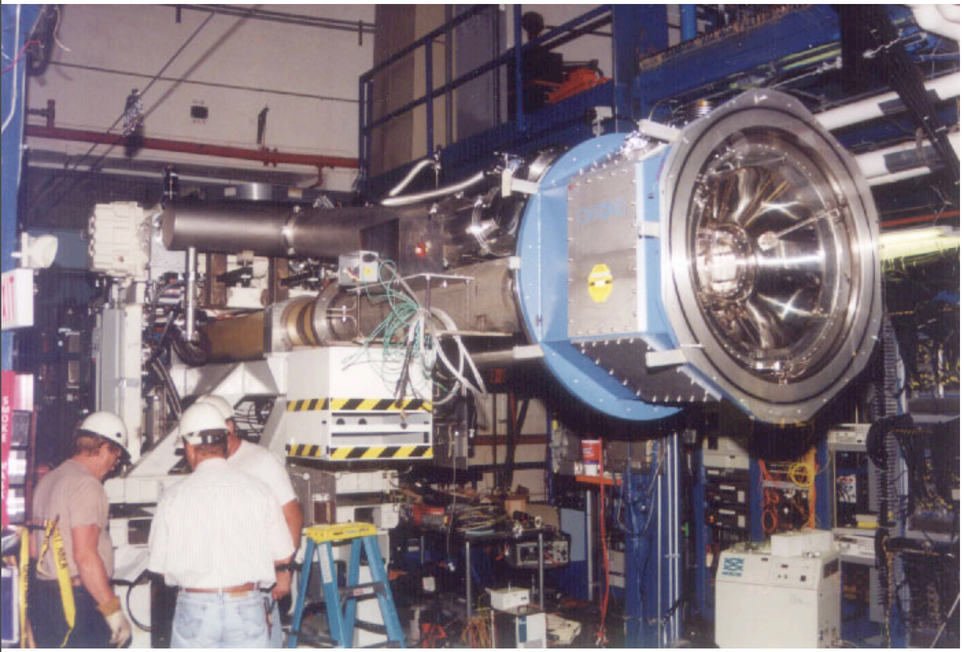


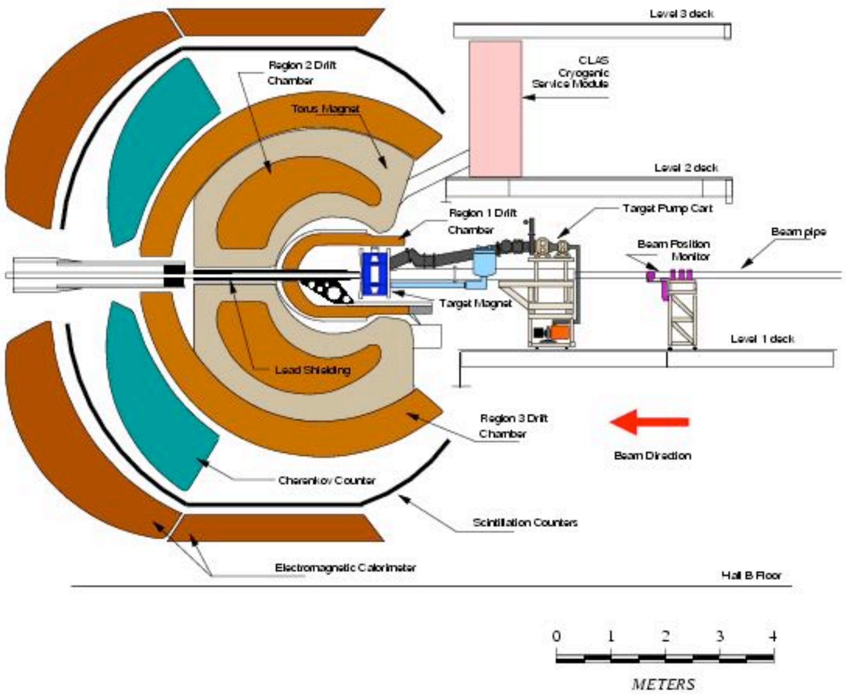


CLAS Photo



Polarized Target



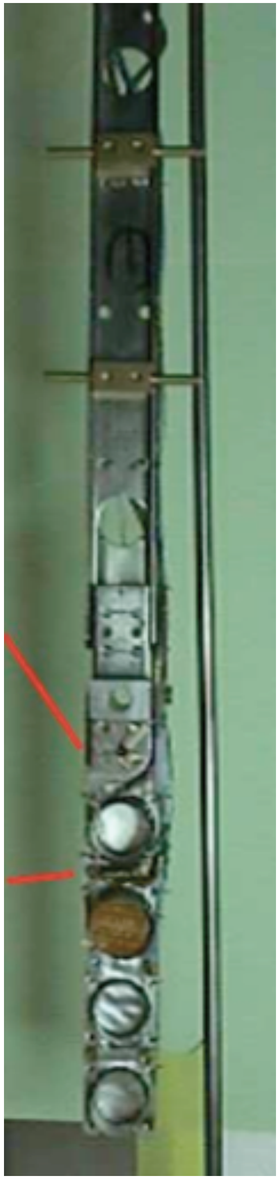


- Dynamic nuclear polarization of NH_3 and ND_3
- Polarizations of 70-80% for p and 20-30% for d
- Luminosity 10^{35} $cm^{-2}s^{-1}$

Polarized Target

Dynamic Nuclear Polarization:

- Freeze ammonia
- Make it paramagnetic through irradiation
- Put it into a 5 T magnetic field
- Drive transitions with microwaves
- Protons will accumulate into a single hyperfine state with spins aligned



Asymmetries and g_1/F_1

Beam polarization

Target polarization

$$A_{\parallel} = \frac{1}{P_b P_t f} \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}}$$

Dilution factor: ratio of scattering from protons to everything else in the target

Depolarization factor (photons are not in the beam direction)

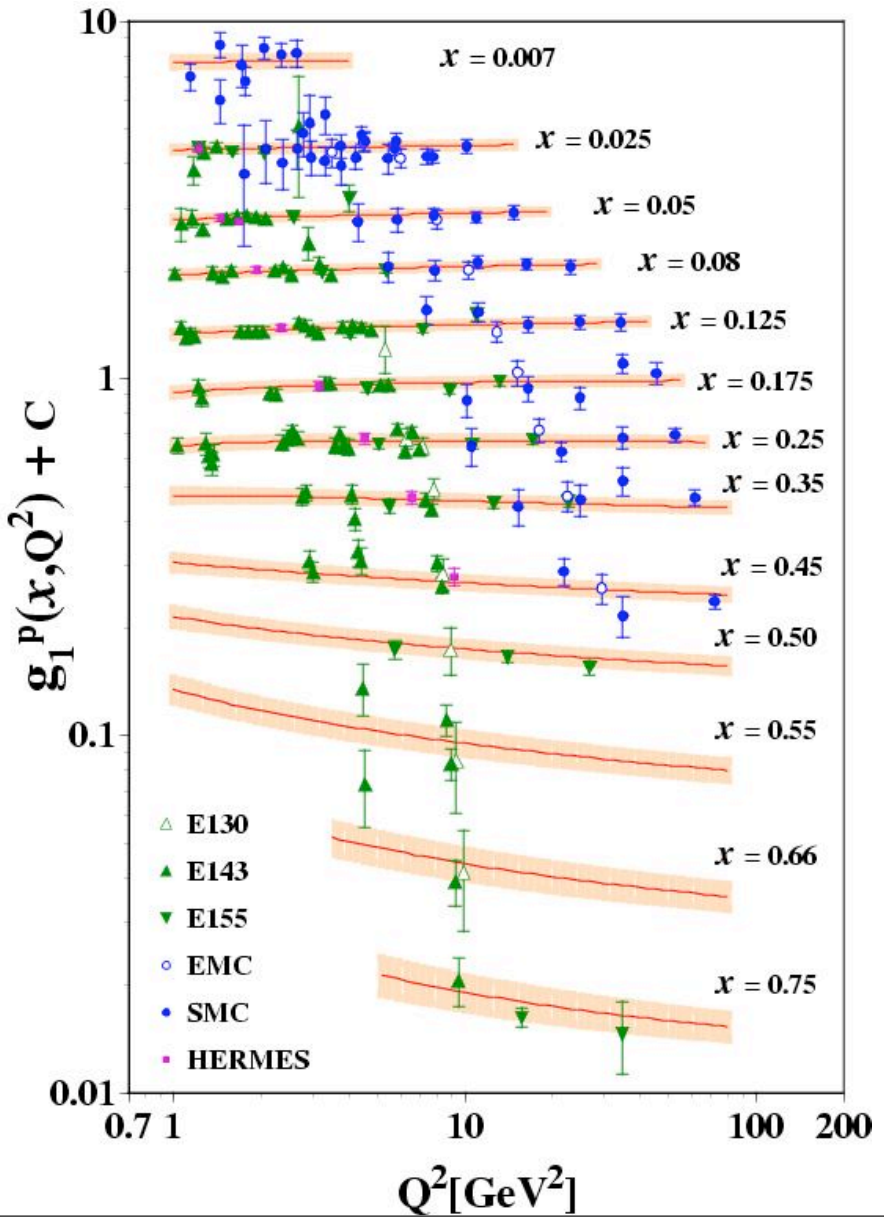
small

tiny

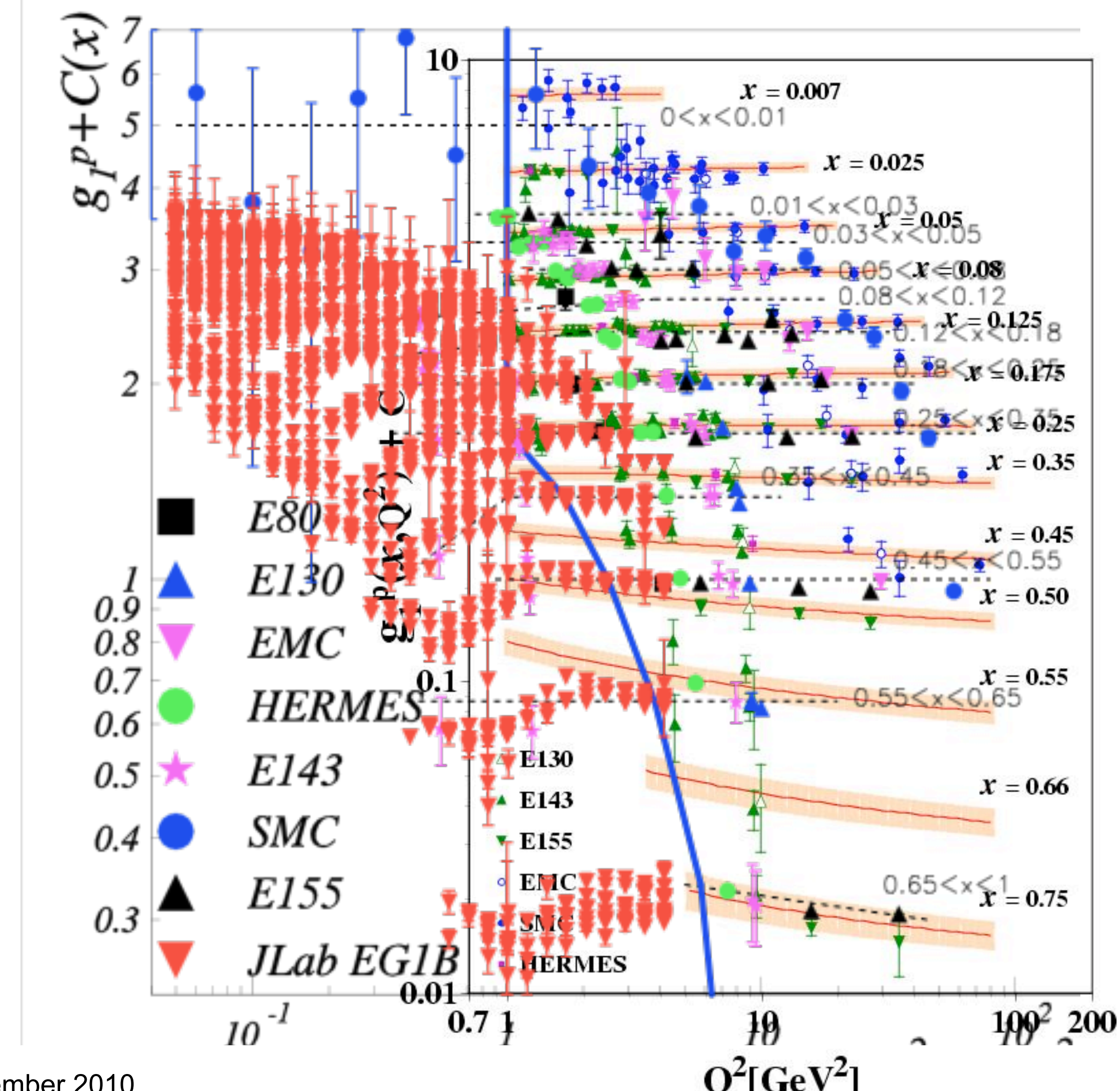
small

$$\frac{A_{\parallel}}{D} = \frac{(1 - \eta\gamma) g_1}{F_1} + \frac{\gamma(\eta - \gamma) g_2}{F_1}$$

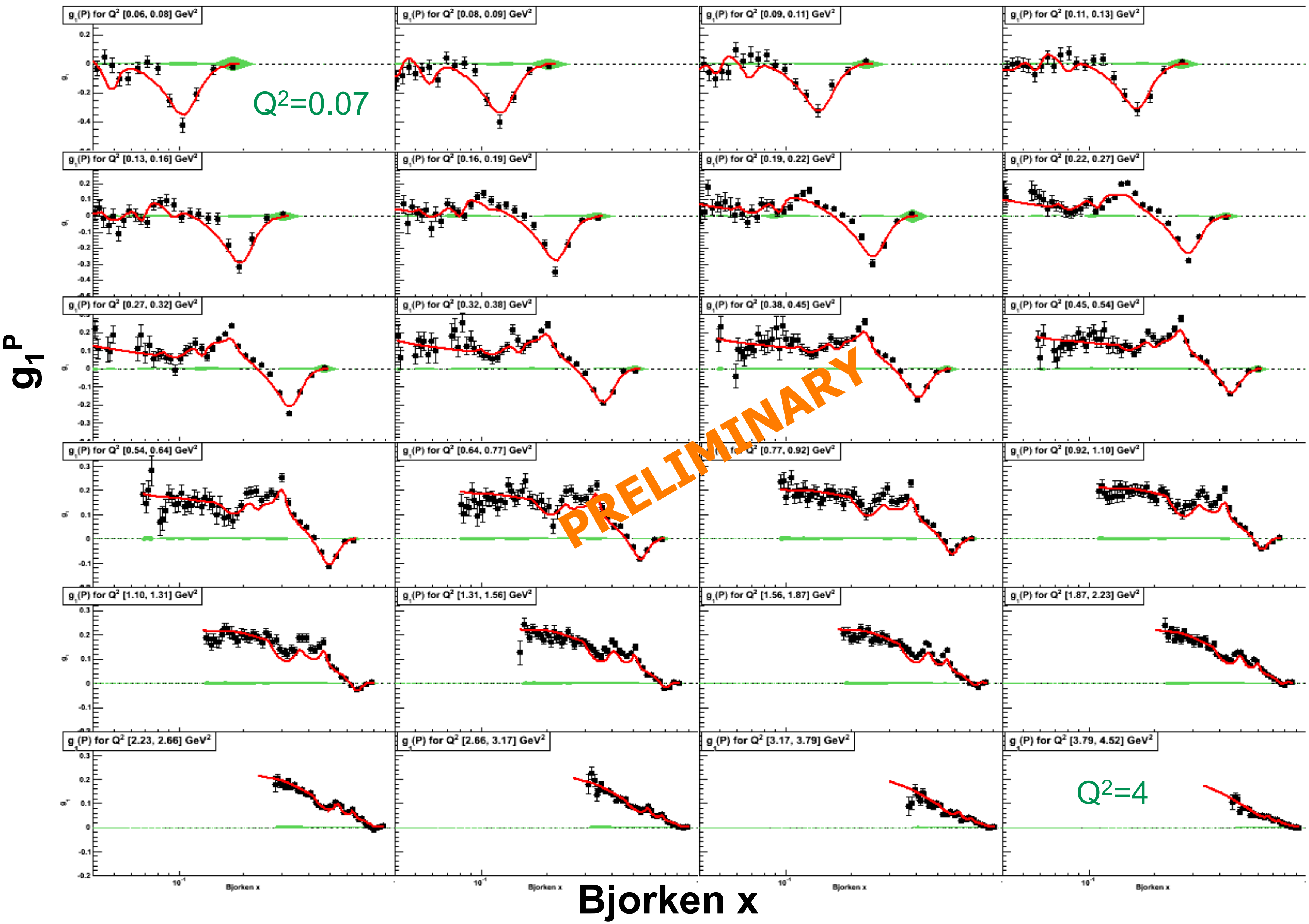
$g_1^p(x,Q^2)$


x = 0.007
x = 0.025
x = 0.05
x = 0.08
x = 0.125
x = 0.175
x = 0.25
x = 0.35
x = 0.45
x = 0.50
x = 0.55
x = 0.66
x = 0.75
E130
E143
E155
EMC
SMC
HERMES
10
1
0.1
0.01
0.7
1
10
100
200
g1p(x,Q2) + C
Q2[GeV2]

$g_1^p(x,Q^2)$ with JLab CLAS



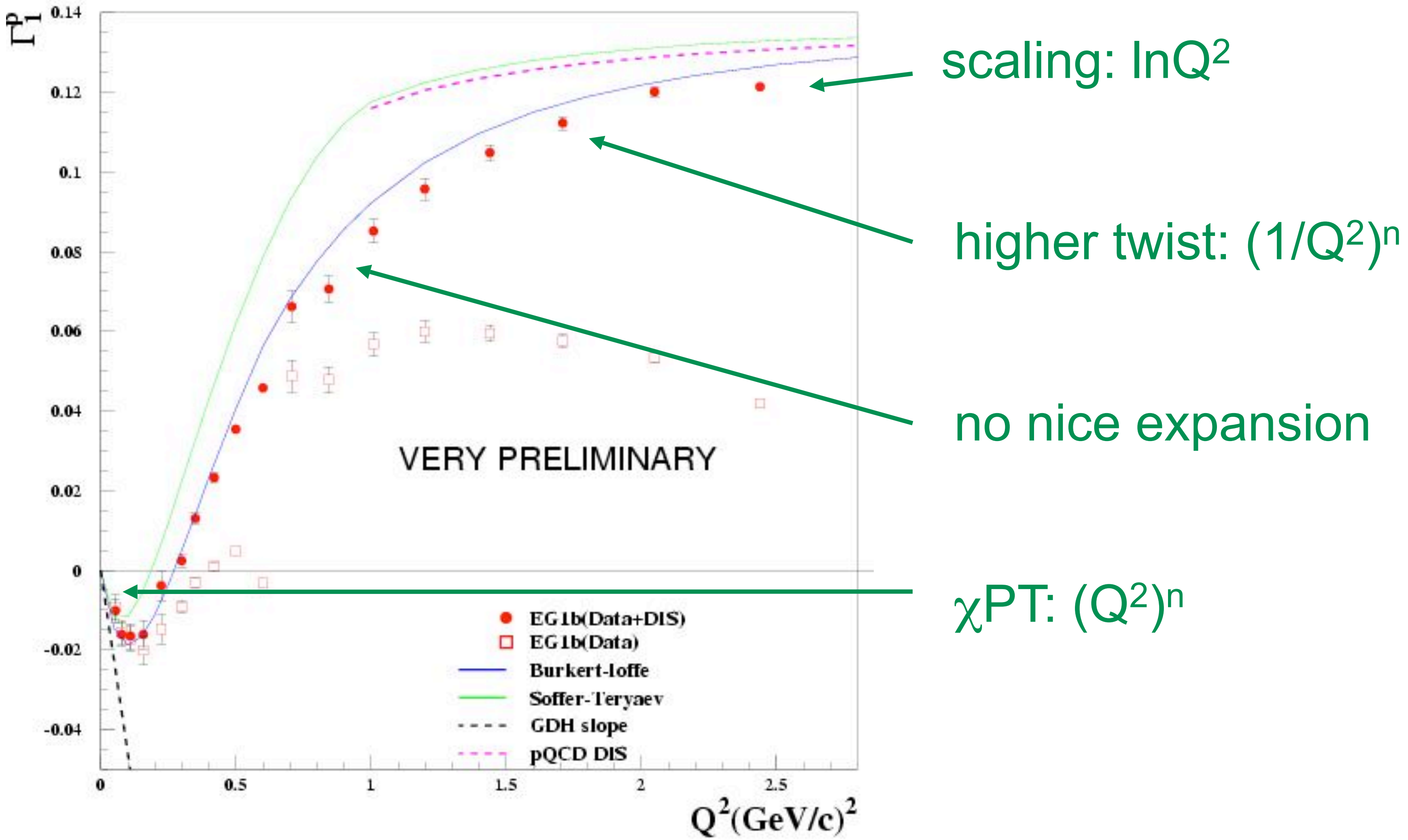
EG1b g_1^p



g1(P) for Q² [0.06, 0.08] GeV²
g1(P) for Q² [0.08, 0.09] GeV²
g1(P) for Q² [0.09, 0.11] GeV²
g1(P) for Q² [0.11, 0.13] GeV²
g1(P) for Q² [0.13, 0.16] GeV²
g1(P) for Q² [0.16, 0.19] GeV²
g1(P) for Q² [0.19, 0.22] GeV²
g1(P) for Q² [0.22, 0.27] GeV²
g1(P) for Q² [0.27, 0.32] GeV²
g1(P) for Q² [0.32, 0.38] GeV²
g1(P) for Q² [0.38, 0.45] GeV²
g1(P) for Q² [0.45, 0.54] GeV²
g1(P) for Q² [0.54, 0.64] GeV²
g1(P) for Q² [0.64, 0.77] GeV²
g1(P) for Q² [0.77, 0.92] GeV²
g1(P) for Q² [0.92, 1.10] GeV²
g1(P) for Q² [1.10, 1.31] GeV²
g1(P) for Q² [1.31, 1.56] GeV²
g1(P) for Q² [1.56, 1.87] GeV²
g1(P) for Q² [1.87, 2.23] GeV²
g1(P) for Q² [2.23, 2.66] GeV²
g1(P) for Q² [2.66, 3.17] GeV²
g1(P) for Q² [3.17, 3.79] GeV²
g1(P) for Q² [3.79, 4.52] GeV²
Q²=0.07
Q²=4
PRELIMINARY
g1P
Bjorken x

Regions of Q^2

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)dx$$



Sum Rules

Energy-Weighted Sum Rule

S(F) = Σ$_a$(E$_a$-E$_0$)|<a|F|0>|^2 = <0|[F,[H,F]|0>

GDH Sum Rule

$$\int_{k_\pi}^{\infty} \frac{dk}{k} \Delta\sigma^{\gamma N}(k) = \frac{2\pi^2\alpha\kappa^2}{M^2}$$

anomalous magnetic moment

$$\Delta\sigma^{\gamma N} = \sigma_{3/2}^{\gamma N} - \sigma_{1/2}^{\gamma N}$$

$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^2}{8M^2} Q^2$$

at low Q^2

Sum over excited states is tied to properties of the ground state

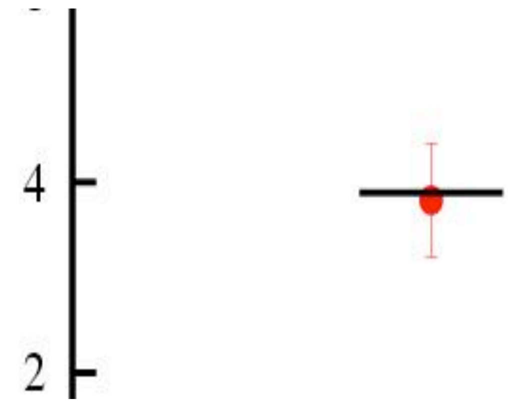
CLAS Moments $\Gamma_1^{p,d}$

$$\Gamma_1^{p,d}(Q^2) = \int_0^1 g_1^{p,d}(x, Q^2)dx \qquad \Gamma_1(Q^2) = aQ^2 + bQ^4 + cQ^6 + dQ^8$$

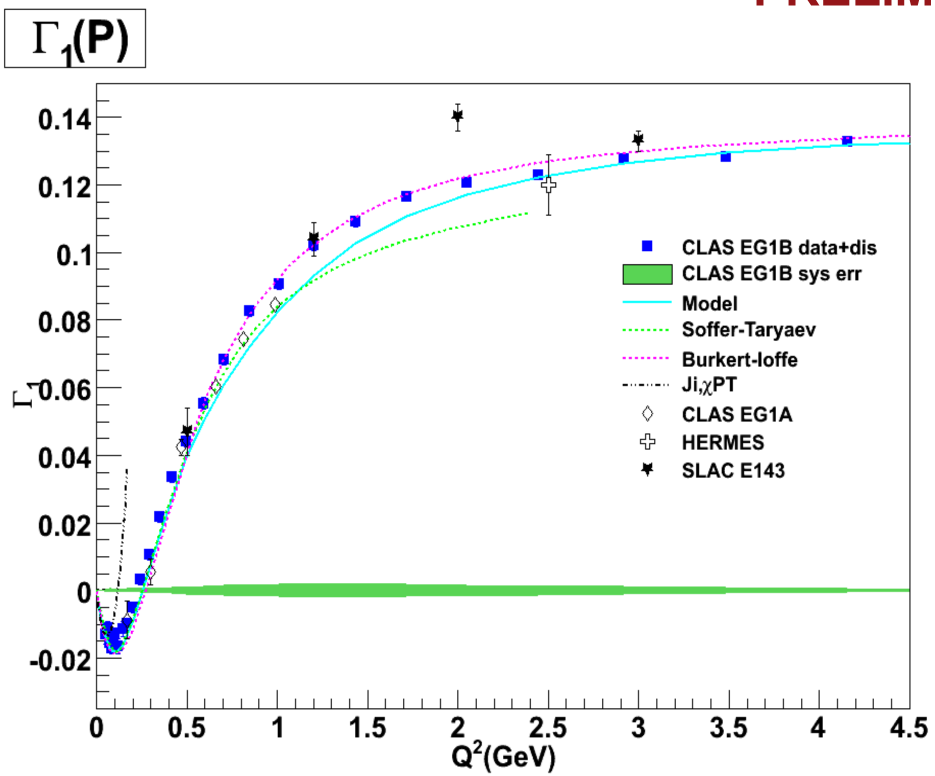
low Q^2 fit

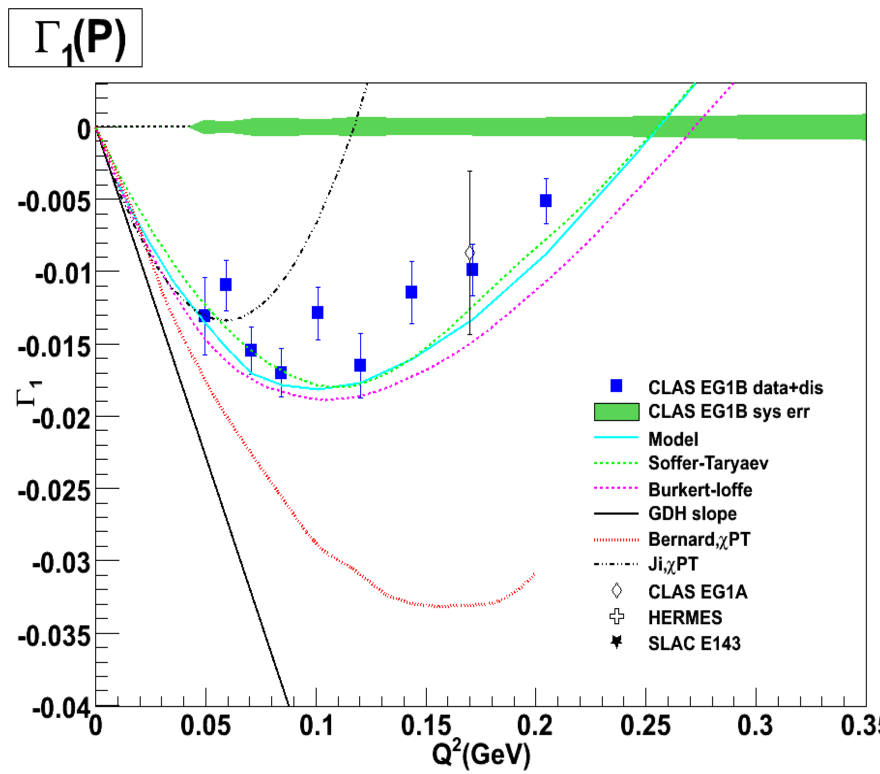
$$\Gamma_1^p(Q^2) = -\frac{\kappa_p^2}{8M^2}Q^2 + 3.89Q^4 + \ldots$$

GDH + χpT

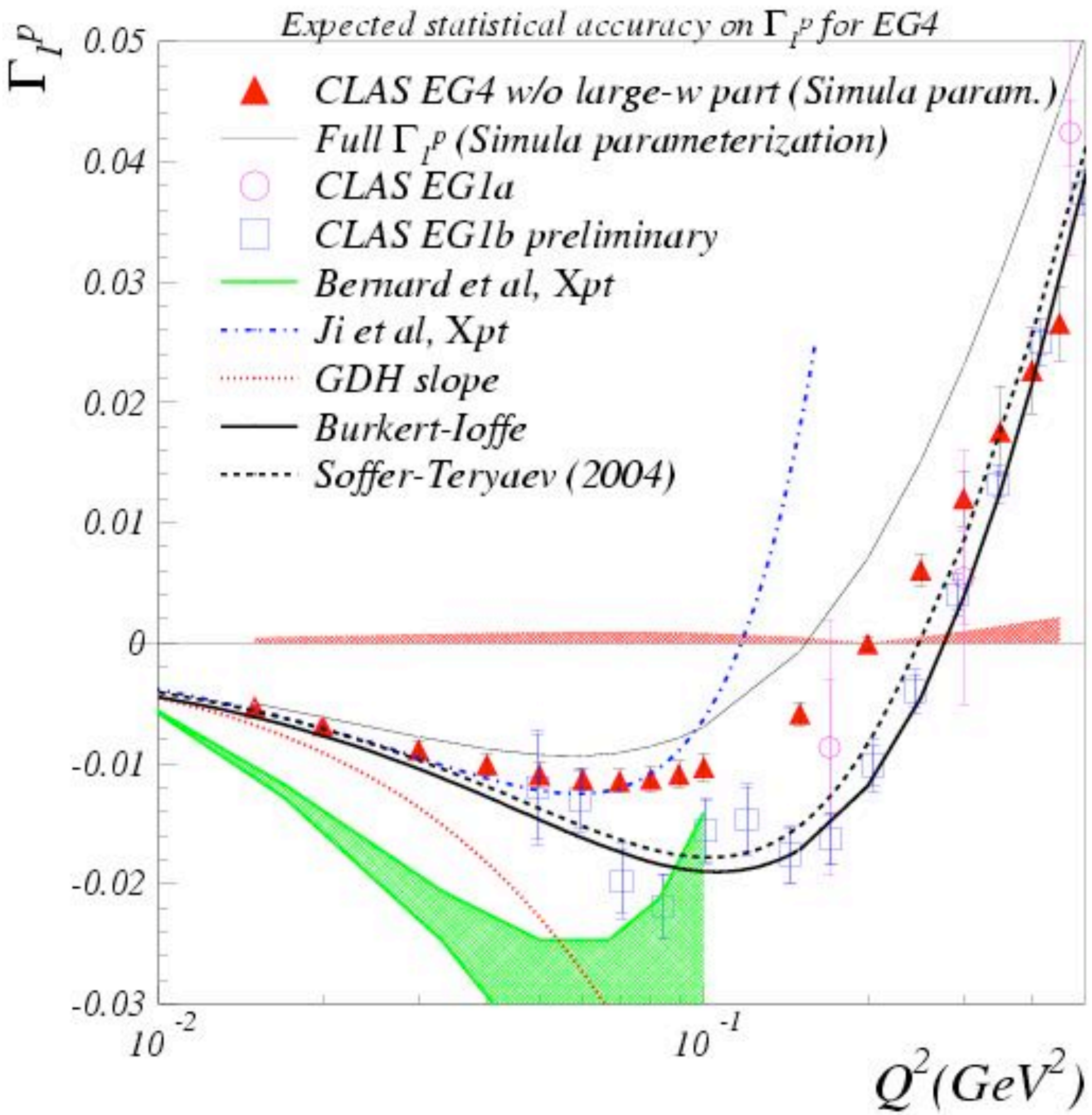


PRELIMINARY



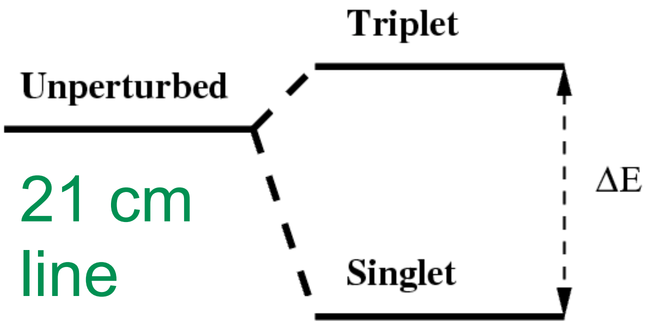


EG4 Γ_1^p Expected

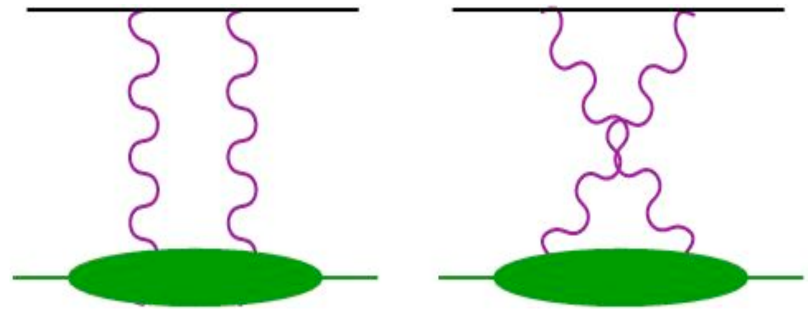


Hydrogen Hyperfine Splitting

Carlson, Nazaryan, Griffioen, PRA**78**(08)022517



21 cm line



$$E_{\mathrm{HFS}}(e^- p) = 1.42040575177667(9)\,\mathrm{GHz} = (1+\Delta_{QED}+\Delta_R^p+\Delta_S)E_F^p$$

$$\Delta_S = \Delta_Z + \Delta_{\mathrm{pol}}$$

Zemach: $\Delta_Z = -2\alpha m_e \langle r\rangle_Z (1+\delta_Z^{\mathrm{rad}})$

$$\delta_Z^{\mathrm{rad}} = \frac{\alpha}{3\pi}\left[2\ln\frac{\Lambda^2}{m^2} - \frac{4111}{420}\right]$$

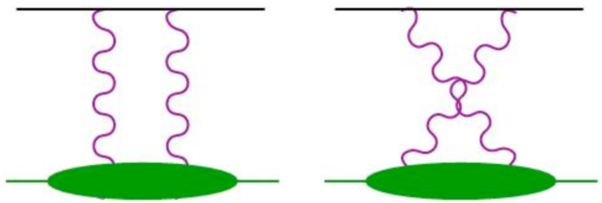
$$\langle r\rangle_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa} - 1\right]$$

$\Delta_Z = -41.0(5)$ ppm

$\Delta_S = -38.62(16)$ ppm

$\Delta_{\mathrm{pol}} = 2.38(58)$ ppm

Hydrogen Hyperfine Splitting



$$\Delta_{\rm pol} = 2.38(58)\ \text{ppm}$$

$$\Delta_{\rm pol} = \frac{\alpha m_e}{2\pi(1+\kappa)M}(\Delta_1 + \Delta_2) = (0.2264798\ \text{ppm})(\Delta_1 + \Delta_2)$$

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\}$$

$$\Delta_2 = -24m_p^2\int_0^\infty \frac{dQ^2}{Q^4}B_2(Q^2).$$

$$B_1 = \int_0^{x_{\rm th}} dx\,\beta(\tau)g_1(x,Q^2)\,,$$

$$B_2 = \int_0^{x_{\rm th}} dx\,\beta_2(\tau)g_2(x,Q^2)\,,$$

kinematic factors

$\Delta_{\rm pol}$ = 1.88(64) ppm from CLAS

Consistent within a half ppm

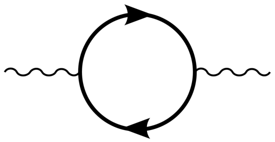
Muonic Hydrogen Lamb Shift

R. Pohl, Max Planck Inst. Quantenoptik, Garching, Germany (2010)

Muonic hydrogen (μp) contains a muon instead of an electron.

A muon is 207 times heavier than an electron. Therefore it spends more time inside the proton.

Finite size correction to the Lamb shift $\Delta E \sim r_{rms}^2 \, |\psi(0)|^2$



$2S_{1/2}^{F=1}$ to $2P_{3/2}^{F=2}$ transition: 49881.88(76) GHz

r_{rms} =0.84184(67) fm

10 times more accurate than all other measurements
4% smaller than accepted radius!

Where We Stand

$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \Delta G + L_z$$

known quite well: 0.13

poorly known but likely small: -0.1

completely unmeasured but likely large: 0.47

$$r_{\rm rms} = \sqrt{\langle r^2 \rangle}$$

0.84184(67) fm
R. Pohl, μp Lamb shift

0.8768(69) fm
CODATA, mostly H spectroscopy

0.895(18) fm
I. Sick, from form factors

Conclusions

After 50 years of studying the proton's internal structure, we still have a long way to go.