# The Proton ~~Radius~~ Radii

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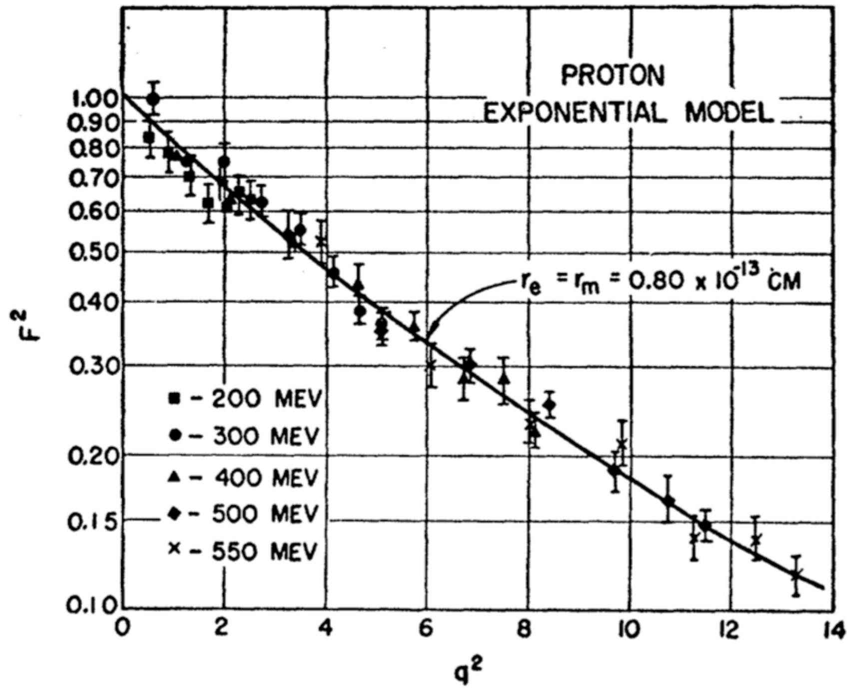
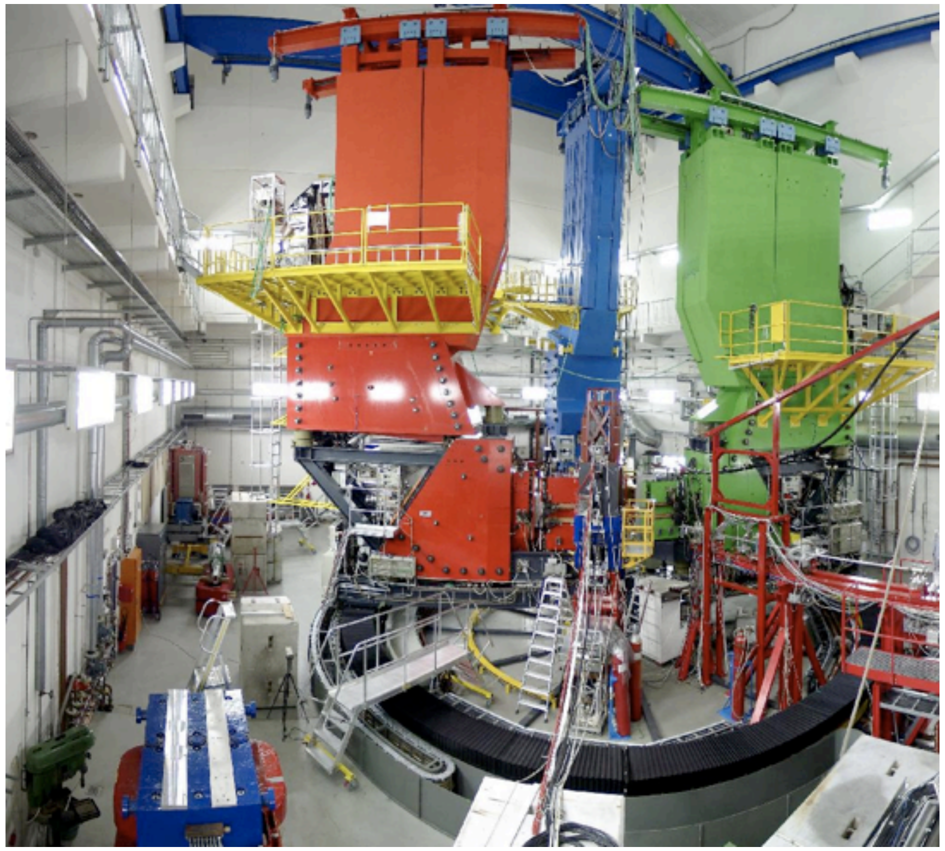
# How to Measure Proton Size



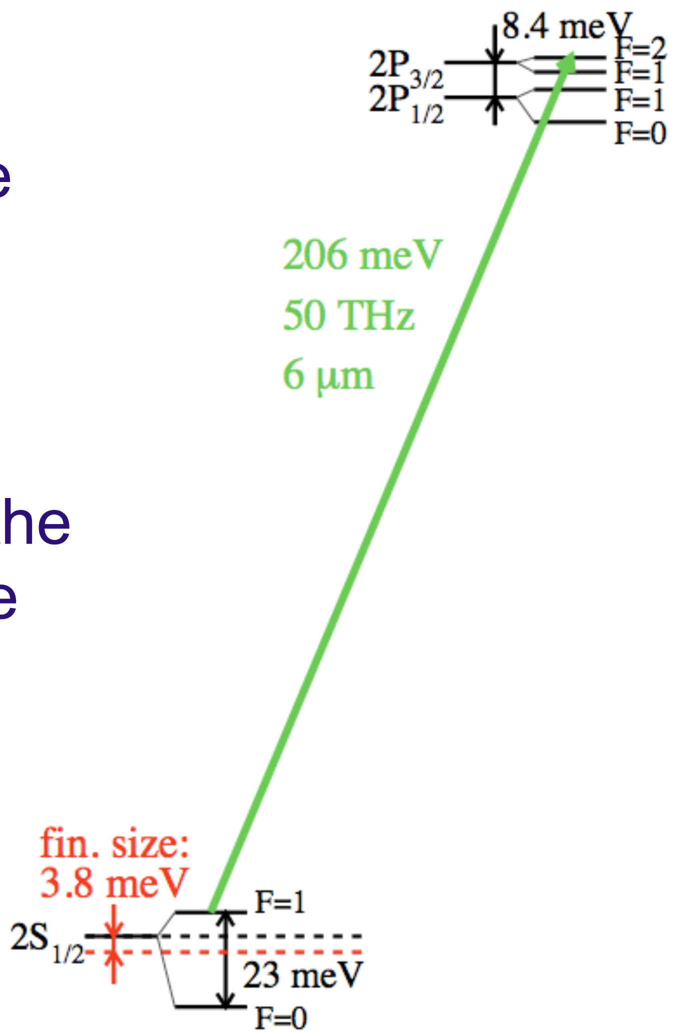

Chambers and Hofstadter, PR**103**(56)14

- Hofstadter at Stanford in the 1950s: electron scattering

- Hadronic physicists around the world in the 1960s-2010s: form factors



Bernauer, PRL**105**(10)242001




- Atomic physicists in the 1990s-2010s: precise atomic transitions in hydrogen

Pohl, Nature**466**(10)213

Question: Why should hadronic physicists pay attention to what atomic physicists are measuring?

Answer: Because atomic physicists can measure some things in nuclear physics more precisely than we can!

# Elastic ep Scattering

Born

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon(1+\tau)}$$

$$\tau = Q^2/(4M_p^2)$$

$$\epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \left(\frac{E'}{E}\right) \frac{\alpha^2\left[1 - \beta^2 \sin^2\frac{\theta}{2}\right]}{4k^2 \sin^4\frac{\theta}{2}}$$

Corrections

- Rosenfelder, PL**B479**(00)381, Sick, NP**A637**(98)559: Coulomb corrections increase the proton radius by 0.008-0.013 fm.
- Guichon&VdH, PRL**91**(03)142303: Two-photon corrections are not well known but are small at low $Q^2$

# Moments of Form Factors

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\vec{k}\cdot\vec{r}}\rho(\vec{r})dxdydz \propto \int_0^{\infty} r^2\rho(r)j_0(kr)dr$$

3-d Fourier Transform for isotropic ρ(**r**)

$$M_n \equiv \int_0^{\infty} drr^{2+n}\rho(r)$$

Radial moments of ρ(**r**)

$$\tilde{\rho}(k) = \int_0^{\infty} r^2\rho(r)j_0(kr)dr = M_0 - \frac{1}{6}M_2k^2 + \frac{1}{120}M_4k^4 - \frac{1}{5040}M_6k^6 + ...$$

Fourier transform of ρ(**r**) in terms of the moments $M_n$

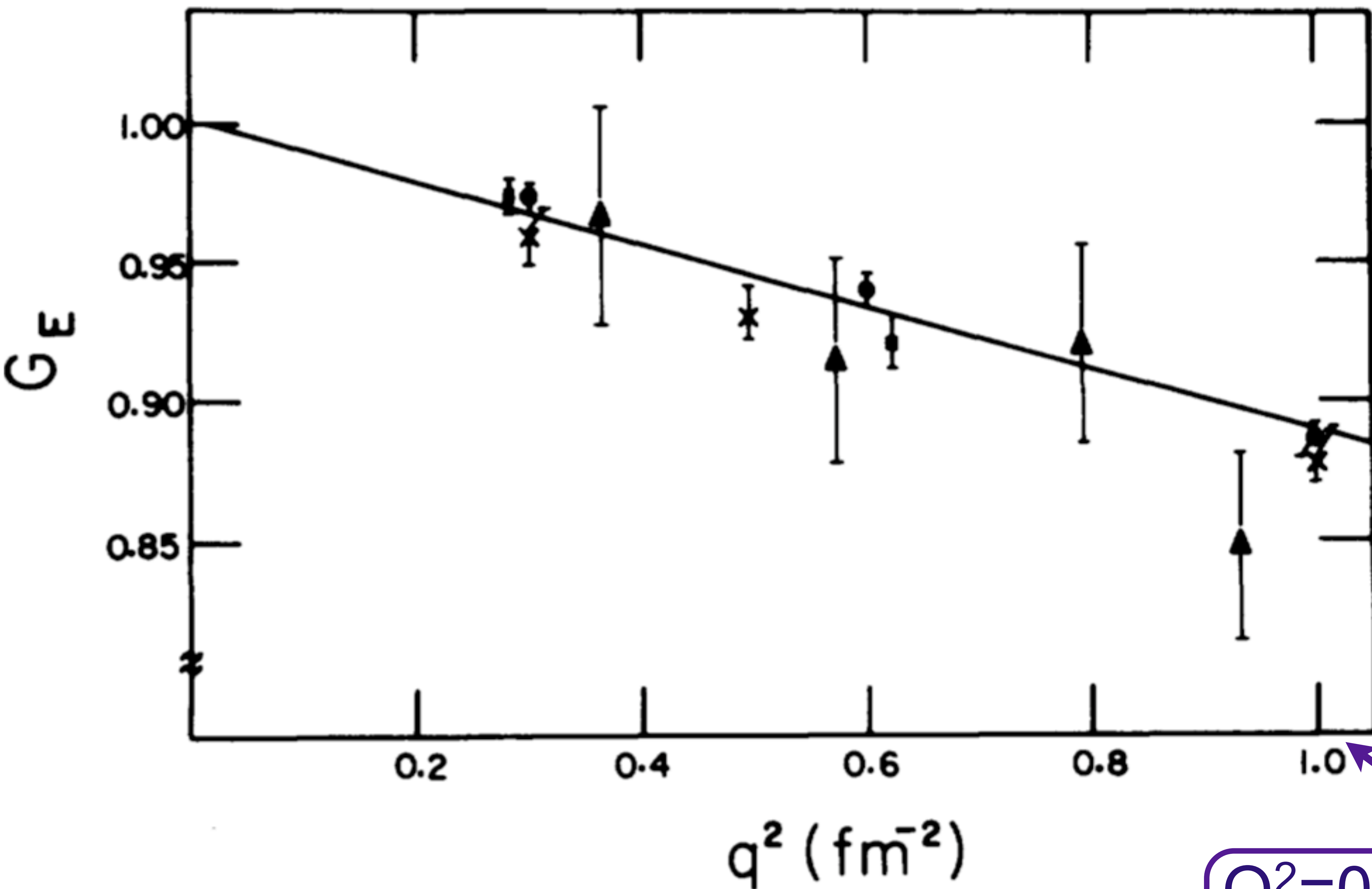
$$G_{E,M}(Q^2) = 1 - \frac{1}{6}\langle r^2_{E,M}\rangle Q^2 + \frac{1}{120}\langle r^4_{E,M}\rangle Q^4 - \frac{1}{5040}\langle r^6_{E,M}\rangle Q^6 + ...$$

Non-relativistic assumption: k = Q; G is F.T. of ρ(**r**)

$$\left(\frac{dG_{E,M}}{dQ^2}\right)_{Q^2=0} = -\frac{1}{6}\langle r^2_{E,M}\rangle \equiv r^2_{E,M}$$

The slope of $G_{E,M}$ at $Q^2$=0 defines the radii $r_{E,M}$ for atomic calculations, and this is what FF experiments quote

# Low $Q^2$ $G_E$ in 1974




$r_E$ = 0.810(40) fm

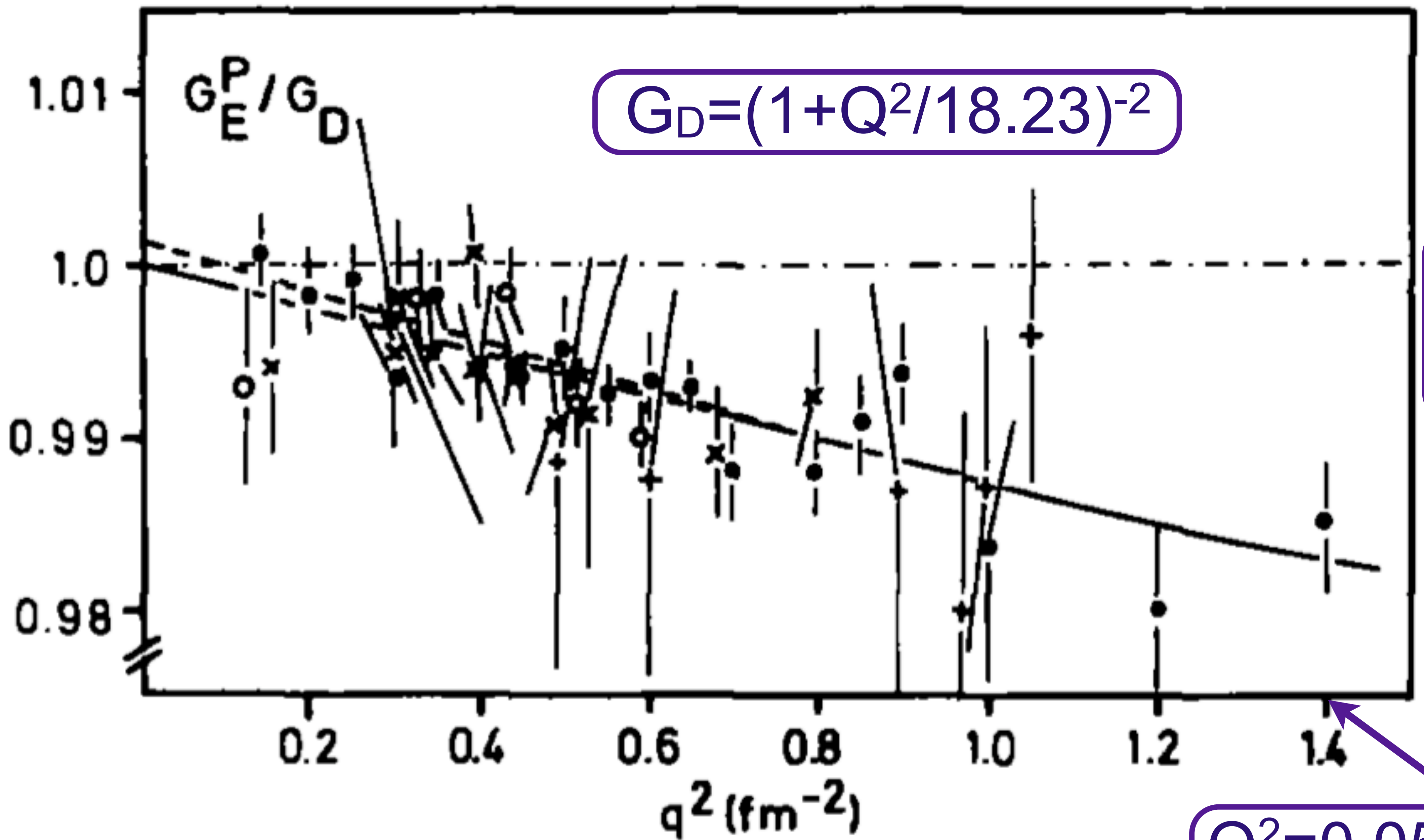
$Q^2$=0.0389 $GeV^2$

Fit to $G_E(Q^2)=a_0+a_1Q^2+a_2Q^4$

Saskatoon 1974

Murphy PRC**9**(74)2125

# Low $Q^2$ $G_E$ in 1980




$G_D=(1+Q^2/18.23)^{-2}$

$r_E$ = 0.862(12) fm

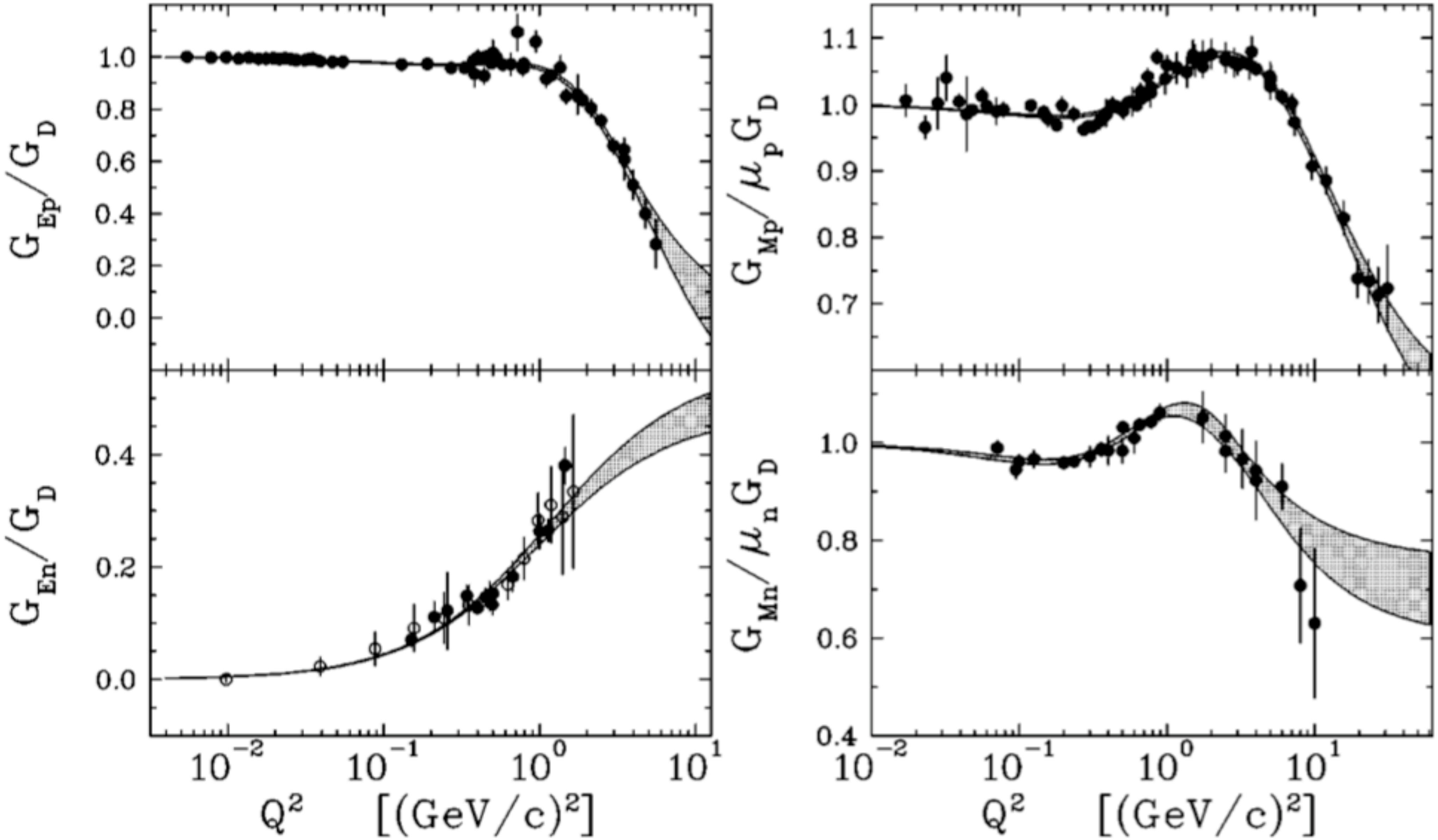
$Q^2$=0.0545 GeV$^2$

Fit to $G_E(Q^2)=a_0+a_1Q^2+a_2Q^4$

Orsay (+) 1965; Saskatoon (x) 1974; Mainz (o) (1980)

Simon NPA**33**(80)381

# The Fitting Industry




- As measurements improved over a wide range of $Q^2$, global fits were made.
- Representative of state of the art in 2004 is this fit by Jim Kelly.
- Fits to mathematical forms with only several parameters

Kelly, PRC**70**(04)068202

# Form Factor Fits

$$G^{E,M}_{\text{dipole}}(Q^2) = \left(1 + \frac{Q^2}{a^{E,M}}\right)^{-2}$$

G'(0)=-2/a (0.71) → $r_D$=0.811 fm

$r^2$=-6G'(0)

$$G^{E,M}_{\text{double dipole}}(Q^2) = a_0^{E/M}\left(1 + \frac{Q^2}{a_1^{E/M}}\right)^{-2} + \left(1 - a_0^{E,M}\right)\left(1 + \frac{Q^2}{a_2^{E/M}}\right)^{-2}$$

G'(0)=-2$a_0$/$a_1$-2(1-$a_0$)/$a_2$

$$G^{E,M}_{\text{polynomial,n}}(Q^2) = 1 + \sum_{i=1}^{n} a_i^{E,M} \cdot Q^{2\cdot i}$$

G'(0)= $a_1$

$$G^{E,M}_{\text{polynomial+dipole,n}}(Q^2) = G_{\text{standard dipole}}(Q^2) + \left(\sum_{i=1}^{n} a_i^{E,M} \cdot Q^{2\cdot i}\right)$$

G'(0)= -2/a+$a_1$

$$G^{E,M}_{\text{polynomial}\times\text{dipole,n}}(Q^2) = G_{\text{standard dipole}}(Q^2) \times \left(1 + \sum_{i=1}^{n} a_i^{E,M} \cdot Q^{2\cdot i}\right)$$

G'(0)= -2/a+$a_1$

$$G^{E,M}_{\text{inv. poly.,n}}(Q^2) = \frac{1}{1 + \sum_{i=1}^{n} a_i^{E,M} \cdot Q^{2\cdot i}}$$

G'(0)= -$a_1$

$r_E$ = 0.883(8) fm
$r_M$= 0.775(16) fm
Bernauer
PRL**105**(10)242001

$$G_e(q) = \frac{1}{1 + \frac{q^2 b_1}{1 + \frac{q^2 b_2}{1 + \cdots}}}$$

G'(0)= -$b_1$ (3.478,3.224) → ($r_E$ = 0.901, $r_M$= 0.868 fm)
Arrington&Sick PRC**76**(07)035201

$$G(Q^2) \propto \frac{\sum_{k=0}^{n} a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k} \qquad \tau = Q^2/4m_p^2$$

G'(0)= $a_1$-$b_1$ (-0.24,10.98,0.12,10.97) → ($r_E$ = 0.863, $r_M$= 0.848 fm)
Kelly PRC**70**(04)068202

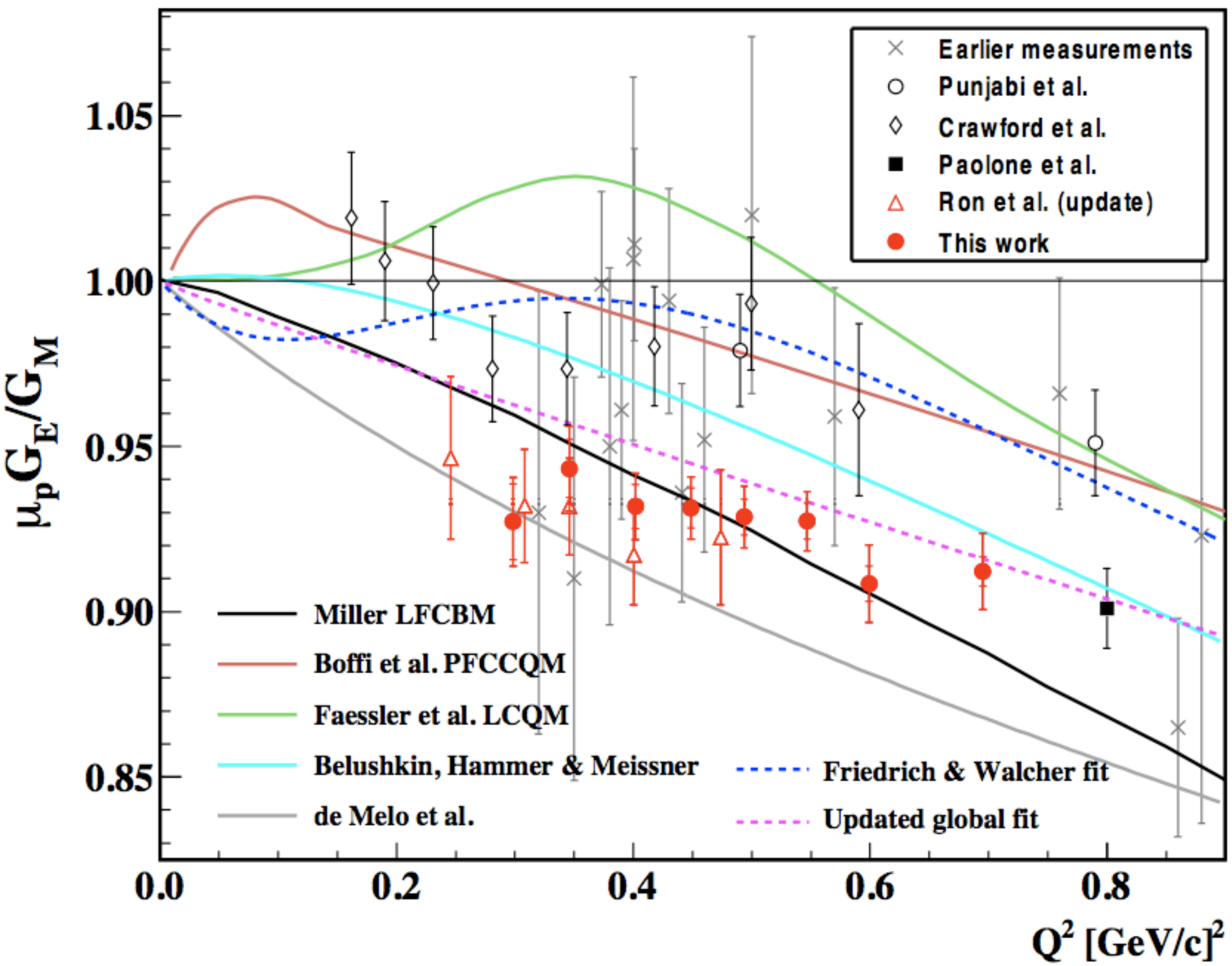
# Recoil Polarization

$$\frac{G_E^P}{G_M^P} = -\mu_p \frac{E_e + E_e'}{2M} \tan\left(\frac{\theta_e}{2}\right) \frac{P_T}{P_L}$$

- $E_e$ is the electron beam energy
- $E'_e$ is the scattered electron energy
- $\theta_e$ is the electron scattering angle
- $P_T$ is the recoil polarization transverse to the proton momentum
- $P_L$ is the recoil polarization longitudinal to the proton momentum
- $\mu_p$ is the proton magnetic moment

- Not affected by two photon processes
- Only the ratio is measured, so to extract form factors, some cross section information is required.

# JLab Hall A E08-007




$$\langle r_E^2 \rangle^{1/2} = 0.875 \pm 0.008_{\rm exp} \pm 0.006_{\rm fit} \text{ fm}$$
$$\langle r_M^2 \rangle^{1/2} = 0.867 \pm 0.009_{\rm exp} \pm 0.018_{\rm fit} \text{ fm}$$

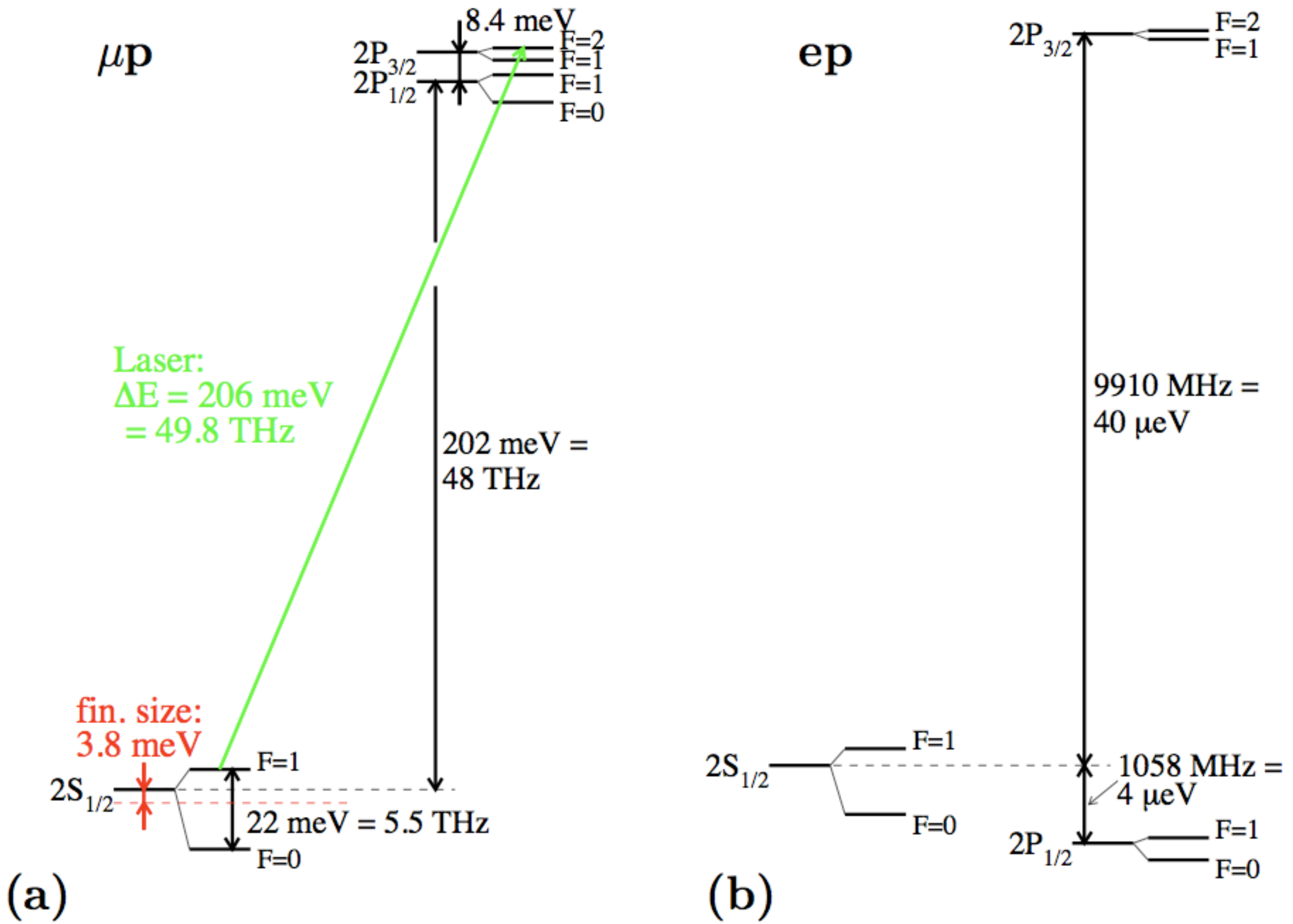
Zhan
arXiv:1102.0318

Recoil Polarization method

Proposal to extend down to $Q^2$=0.02 $GeV^2$

Continued Fractions fit for $Q^2 < 0.5$ $GeV^2$

# Lamb Shift in ep and μp




(a) Vacuum polarization dominates and shifts the $2S_{1/2}$ state downwards
(b) Electron self-energy dominates and shifts the $2S_{1/2}$ state upwards

T. Nebel, PhD Thesis, MPQ

# Lamb Shift Feynman Diagrams

T. Beier et al./Physics Letters A 236 (1997) 329–338 335

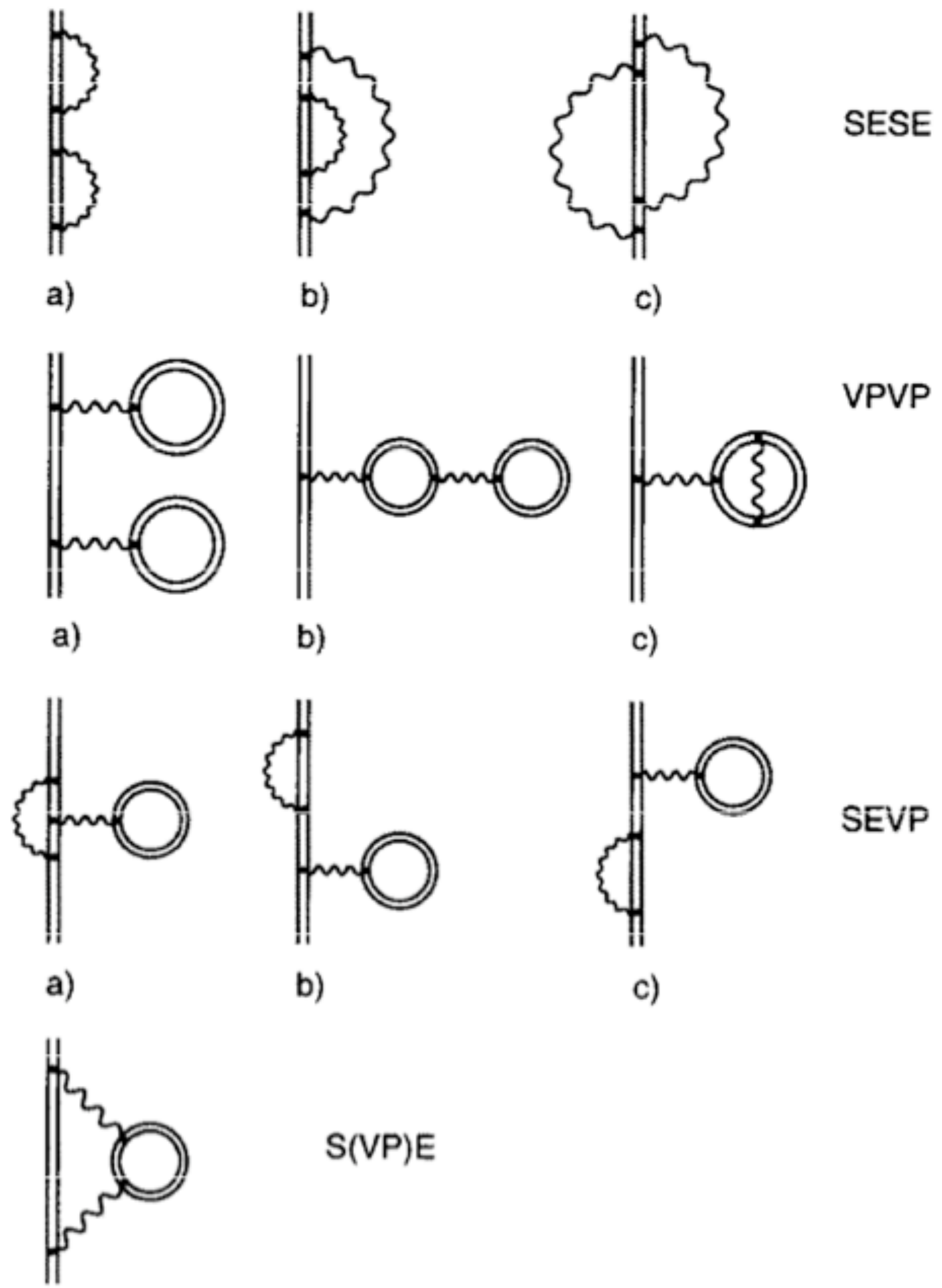



Fig. 3. Feynman diagrams of all QED contributions which contribute to the Lamb shift of order $\alpha^2$, which means two photon lines in the diagram (the wavy lines). The double lines indicate electrons or positrons propagating in the field of the nucleus. We use a naming scheme which indicates whether a diagram consists of two diagrams which are self-energy-like (SESE) or vacuum polarization-like (VPVP) or a mixture of both. The letters (a) to (c) are used to distinguish between diagrams only.

# μp versus ep

"The Lamb shift in electronic hydrogen is dominated by the (repulsive) radiative corrections on the electron line, which are much larger than the (attractive) vacuum polarization corrections on the photon line. The electron spends most of its time outside the polarization cloud induced in the electron Fermi sea. In the muonic-atom case the much smaller Bohr radius is within a significant portion of that cloud and the (electron) vacuum polarization dominates the QED corrections. The smaller radius also means that the hadronic size corrections are significantly more important, as well."

Friar and Sick, nucl-th/0508025

# H-Like Lamb Shift Calculations

Lamb shift is deviation from the unperturbed energy level

$$E^{(0)}(nl_j) = m_R\Big[f(nj) - 1\Big] - \frac{m_R^2}{2(M+m)}\Big[f(nj) - 1\Big]^2$$

Solution to Dirac Eq. for a Coulomb Potential:

$$f(nj) = \left(1 + \frac{(Z\alpha)^2}{\left(n - j - \frac{1}{2} + \sqrt{(j+\frac{1}{2})^2 - (Z\alpha)^2}\,\right)^2}\right)^{-\frac{1}{2}}$$

Lamb Shift ΔE is mostly QED with nuclear size corrections

$$\Delta E = \Delta E_{\mathrm{QED}} + \Delta E_{\mathrm{Nucl}}$$

M = nuclear mass; m = electron mass; $m_R$ is reduced mass; Z is nuclear charge; n = principal Q#; j = ang. mom.

Ivanov & Karshenboim, arXiv:physics/0009069v1

# H-Like Lamb Shift Calculations

$\Delta E_{QED}$ includes 1,2,3 loops for M=∞; $\Delta E_M$ is recoil correction

$$\Delta E_{\mathrm{QED}} = \Delta E_{\infty} + \Delta E_M$$

Rec = pure recoil; RRC = radiative recoil

$$\Delta E_M = \Delta E_{\mathrm{Rec}} + \Delta E_{\mathrm{RRC}}$$

e.g. pure recoil corrections for S-state

$$R_{ns}(Z) = (Z\alpha)\left[\frac{2}{3}\ln\frac{1}{(Z\alpha)} - \frac{8}{3}\ln\left(k_0(ns)\right) + \frac{187}{18} + (Z\alpha)^2\pi\left(4\ln(2) - \frac{7}{2}\right)\right]$$

$$\Delta E_{\mathrm{Rec}} = \frac{1}{\pi}\frac{1}{n^3}\frac{m^2}{M}(Z\alpha)^4 R(Z)$$

Radiative recoil term only know to first order:

$$\Delta E_{\mathrm{RRC}}(nl) = \frac{\alpha(Z\alpha)^5}{\pi}\frac{m}{n^3}\frac{m}{M}\left(-1.36449\right)\delta_{l0}$$

Ivanov & Karshenboim, arXiv:physics/0009069v1

# H-Like Lamb Shift Calculations

## e.g. 1-loop S-state vacuum polarization looks like:

$$F_{1s}^{VP}(Z) = -\frac{4}{15} + \frac{5\pi}{48}(Z\alpha) + (Z\alpha)^2\left(-\frac{2}{15}\ln\frac{1}{(Z\alpha)^2} + \frac{4}{15}\log(2) - \frac{1289}{1575}\right) + (Z\alpha)^3\left(\frac{5\pi}{96}\ln\frac{1}{(Z\alpha)^2} + \frac{5\pi}{48}\ln(2) + \frac{23\pi}{288}\right),$$

## e.g. 1-loop S-state self-energy looks like:

$$F_{ns}^{SE}(Z) = \frac{4}{3}\ln\frac{m}{(Z\alpha)^2 m_R} - \frac{4}{3}\ln\left(k_0(ns)\right) + \frac{10}{9} + (Z\alpha)\,4\pi\left(\frac{139}{128} - \frac{1}{2}\ln(2)\right) + (Z\alpha)^2\left(-\ln^2\frac{1}{(Z\alpha)^2} + A_{61}(ns)\ln\frac{1}{(Z\alpha)^2} + G_{ns}(Z)\right),$$

$$A_{61}(1s) = \frac{28}{3}\ln(2) - \frac{21}{20}$$

$\ln(k_0)$ Bethe log.; $G_{ns}$ higher order

Ivanov & Karshenboim, arXiv:physics/0009069v1

# H Lamb Nuclear Dependence

$$\Delta E_{\mathrm{Nucl}}(nl) = \frac{2}{3}\frac{(Z\alpha)^4 m}{n^3}\left(m R_{\mathrm{N}}\right)^2 \delta_{l0}\left(1 + (Z\alpha)^2 \ln\frac{1}{Z\alpha m R_{\mathrm{N}}}\right)$$

$$\Delta E_{\mathrm{Nucl}}(2p_{1/2}) = \frac{1}{16}(Z\alpha)^6 m \left(m R_{\mathrm{N}}\right)^2$$

$$\Delta E_{\mathrm{Nucl}}(2p_{3/2}) = 0 \,.$$

$\Delta E_{\mathrm{Lamb}}(1S)$ = 8172.582(40) MHz

$\Delta E_{\mathrm{Nucl}}(1S)$ = 1.269 MHz for $r_p$ = 0.9 fm

$\Delta E_{\mathrm{Nucl}}(1S)$ = 1.003 MHz for $r_p$ = 0.8 fm

$\Delta E_{\mathrm{Lamb}}(2S)$ = 1057.8450(29) MHz

$\Delta E_{\mathrm{Nucl}}(2S)$ = 0.1586 MHz for $r_p$ = 0.9 fm

$\Delta E_{\mathrm{Nucl}}(2S)$ = 0.1254 MHz for $r_p$ = 0.8 fm

Ivanov & Karshenboim, arXiv:physics/0009069v1

# Paris Group 2000 Summary

## $1S_{1/2}$ Lamb Shift in Hydrogen

| | | $L_H(1S_{1/2})$ (MHz) |
|---|---|---|
| general least squares adjustment in hydrogen and deuterium | | |
| 2S–2P, 2S–8S/D, 2S–12D, 1S–2S and $1/n^3$ law | (36–44) | 8 172.840(22) |
| theory $r_p = 0.862(12)$ fm [56] | | 8 172.731(40) |
| theory $r_p = 0.805(11)$ fm [56] | | 8 172.582(40) |

## $2S_{1/2}$ Lamb Shift in Hydrogen

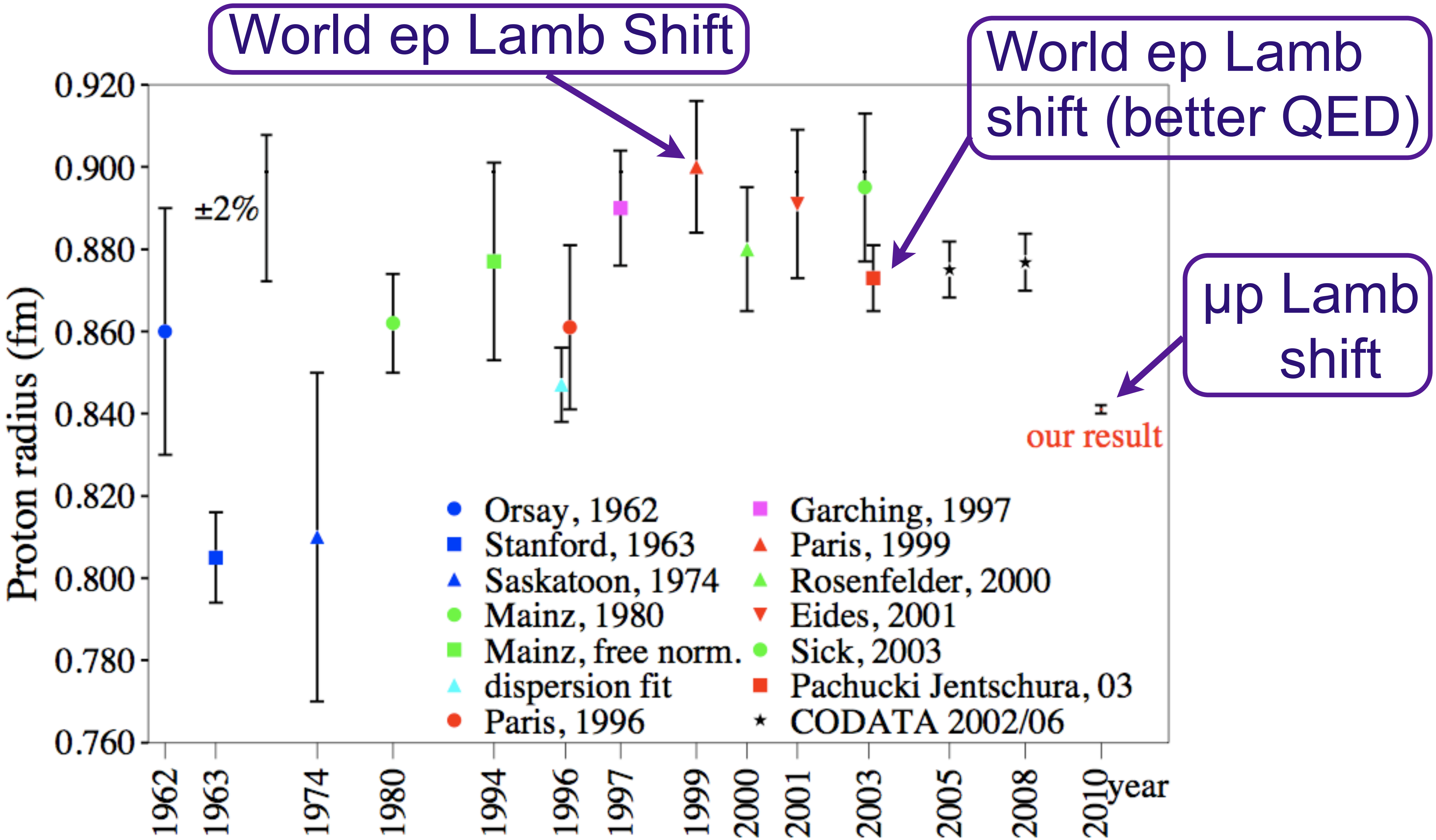
| | | $\nu_H(2S_{1/2} - 2P_{1/2})$ (MHz) |
|---|---|---|
| general least squares adjustment in hydrogen and deuterium | | |
| 2S–2P, 2S–8S/D, 2S–12D, 1S–2S and $1/n^3$ law | (36–44) | 1 057.8450(29) |
| theory $r_p = 0.862(12)$ fm [56] | | 1 057.836(6) |
| theory $r_p = 0.805(11)$ fm [56] | | 1 057.812(6) |

$\Delta E_{Nucl}(1S) = 1.2720$ MHz for $r_p = 0.901(16)$ [best fit] (0.02%)

$\Delta E_{Nucl}(2S) = 0.1590$ MHz for $r_p = 0.901(16)$ [best fit] (0.02%)

de Beauvoir, et al., EPJ**D12**(00)61

# Evolution of Proton Radius




Blues: form factors; Reds: Lamb shifts

Points are misleading; often new analyses of the same data

T. Nebel, PhD Thesis, MPQ

# The size of the proton

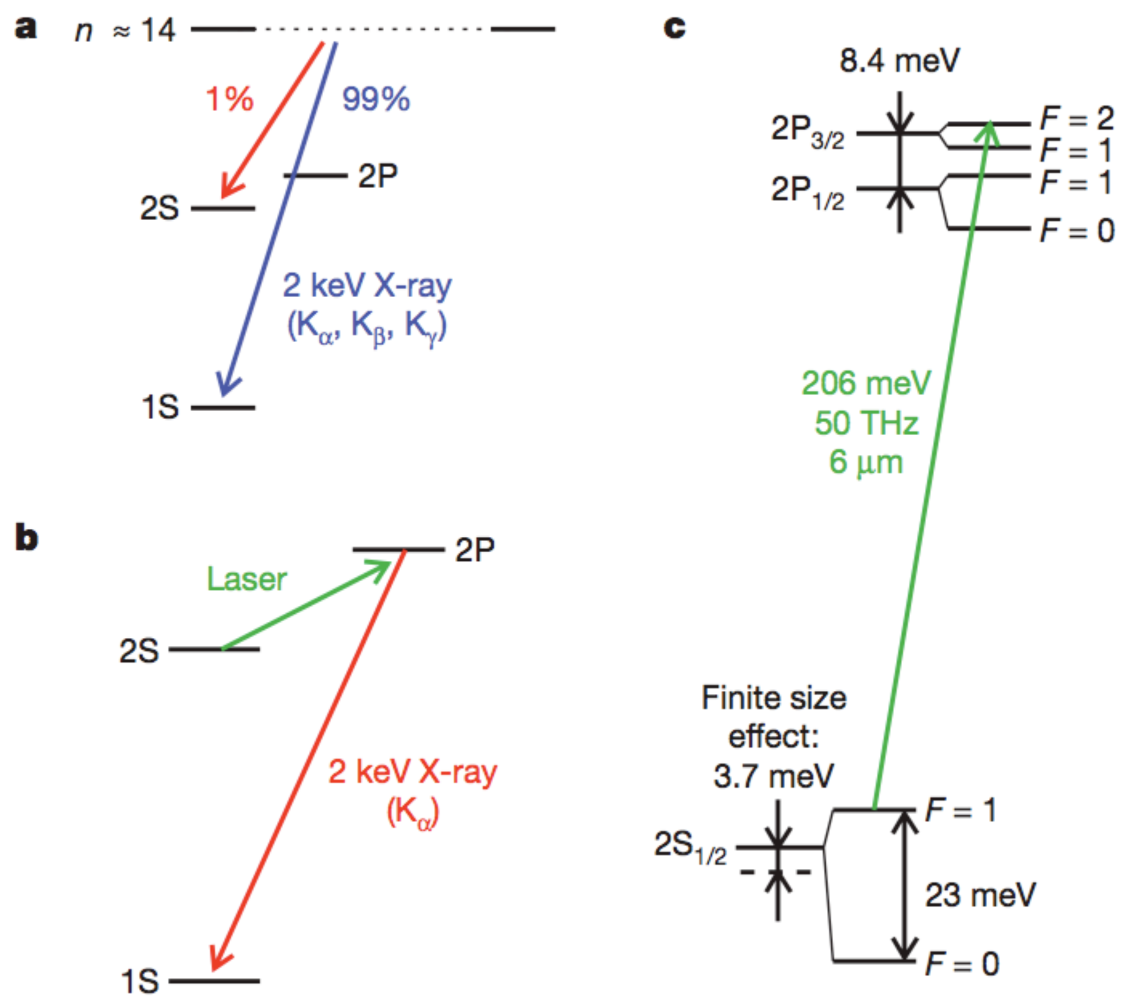
Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

- Nature **466** (10) 213
- Lamb shift at 49.88188(76) THz
- Atomic energy levels depend on nuclear size!
- Extracted $r_p$ = 0.84184(67) fm
- $r_p$(CODATA) = 0.8768(69) fm
- This is the nucleon size crisis

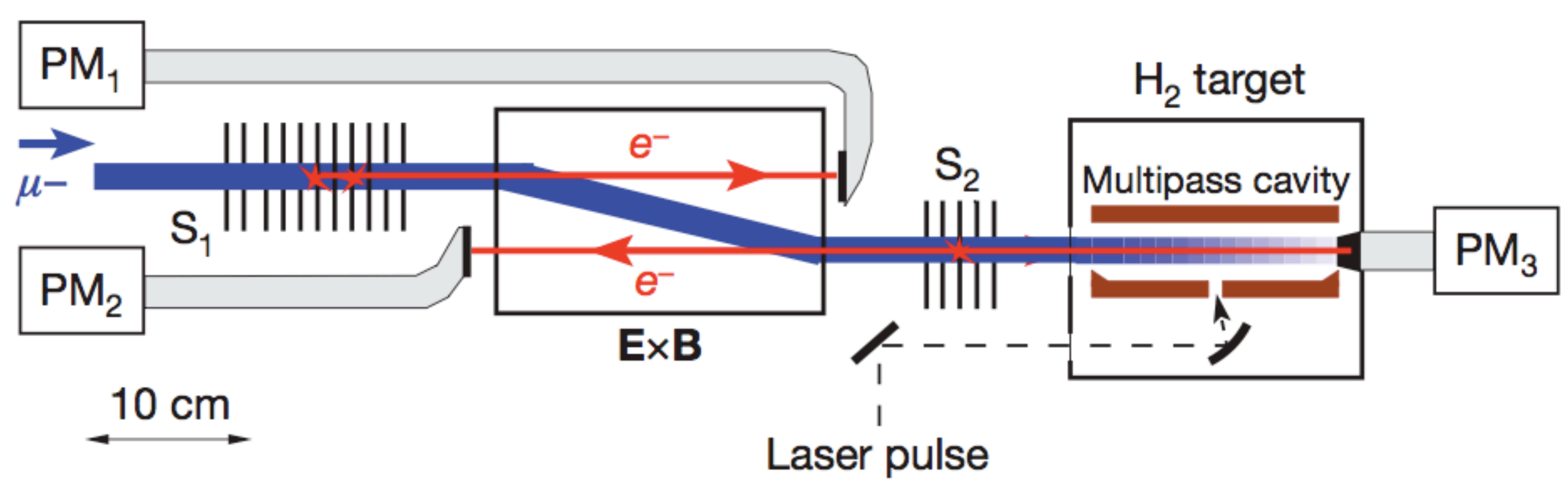
$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + 0.0347\ r_p^3\ \text{meV}$$

The proton is the primary building block of the visible Universe, but many of its properties—such as its charge radius and its anomalous magnetic moment—are not well understood. The root-mean-square charge radius, $r_p$, has been determined with an accuracy of 2 per cent (at best) by electron–proton scattering experiments[1,2]. The present most accurate value of $r_p$ (with an uncertainty of 1 per cent) is given by the CODATA compilation of physical constants[3]. This value is based mainly on precision spectroscopy of atomic hydrogen[4–7] and calculations of bound-state quantum electrodynamics (QED; refs 8, 9). The accuracy of $r_p$ as deduced from electron–proton scattering limits the testing of bound-state QED in atomic hydrogen as well as the determination of the Rydberg constant (currently the most accurately measured fundamental physical constant[3]). An attractive means to improve the accuracy in the measurement of $r_p$ is provided by muonic hydrogen (a proton orbited by a negative muon); its much smaller Bohr radius compared to ordinary atomic hydrogen causes enhancement of effects related to the finite size of the proton. In particular, the Lamb shift[10] (the energy difference between the $2S_{1/2}$ and $2P_{1/2}$ states) is affected by as much as 2 per cent. Here we use pulsed laser spectroscopy to measure a muonic Lamb shift of 49,881.88(76) GHz. On the basis of present calculations[11–15] of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67)$ fm, which differs by 5.0 standard deviations from the CODATA value[3] of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by $-110\,\text{kHz}/c$ (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient.

# μp Lamb Shift Measurement

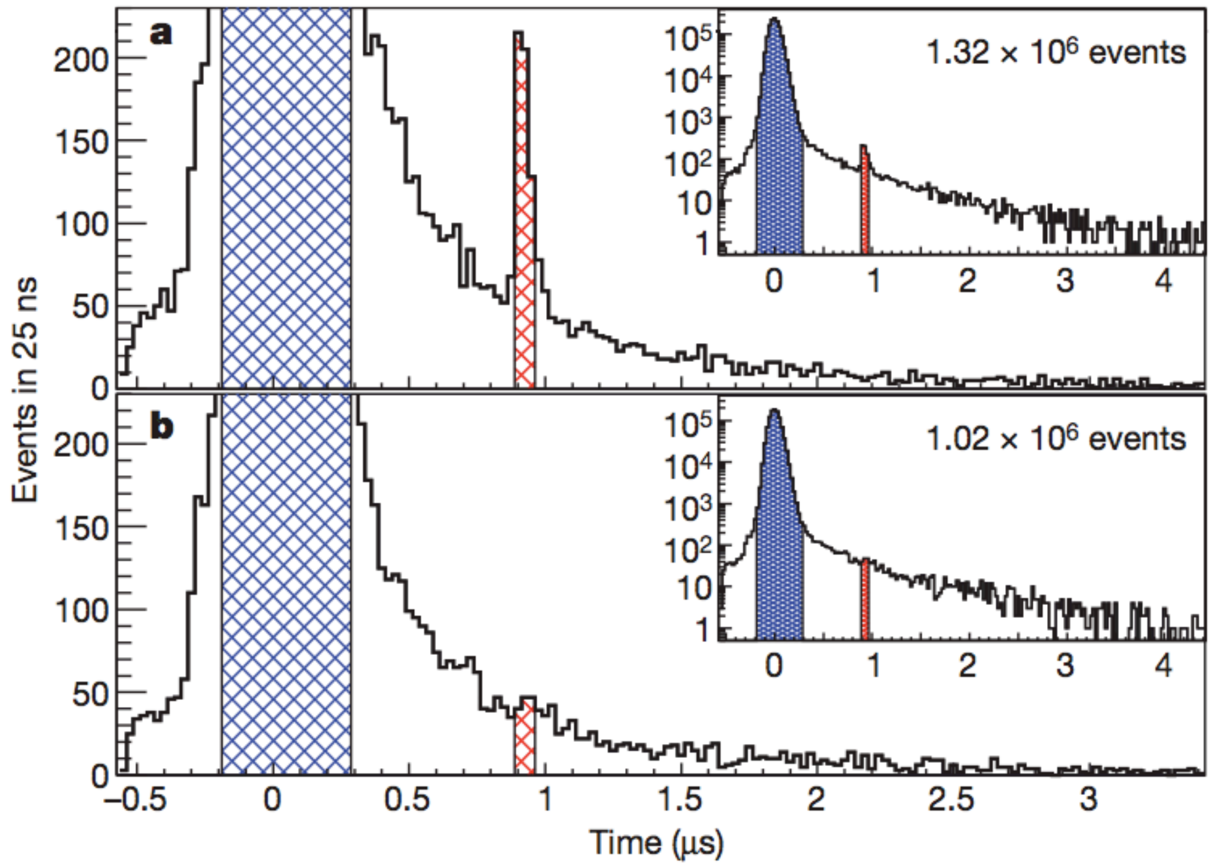



- μ from πE5 beamline at PSI (20 keV)
- μ's with 5 keV kinetic energy after carbon foils $S_{1-2}$
- Arrival of the pulsed beam is timed by secondary electrons in $PM_{1-3}$
- μ's are absorbed in the $H_2$ target at high excitation followed by decay to the 2S metastable level (which has a 1 μs lifetime)
- A laser pulse timed by the PMs excites the $2S_{1/2}^{F=1}$ to $2P_{3/2}^{F=2}$ transition
- The 2 keV X-rays from 2P to 1S are detected.

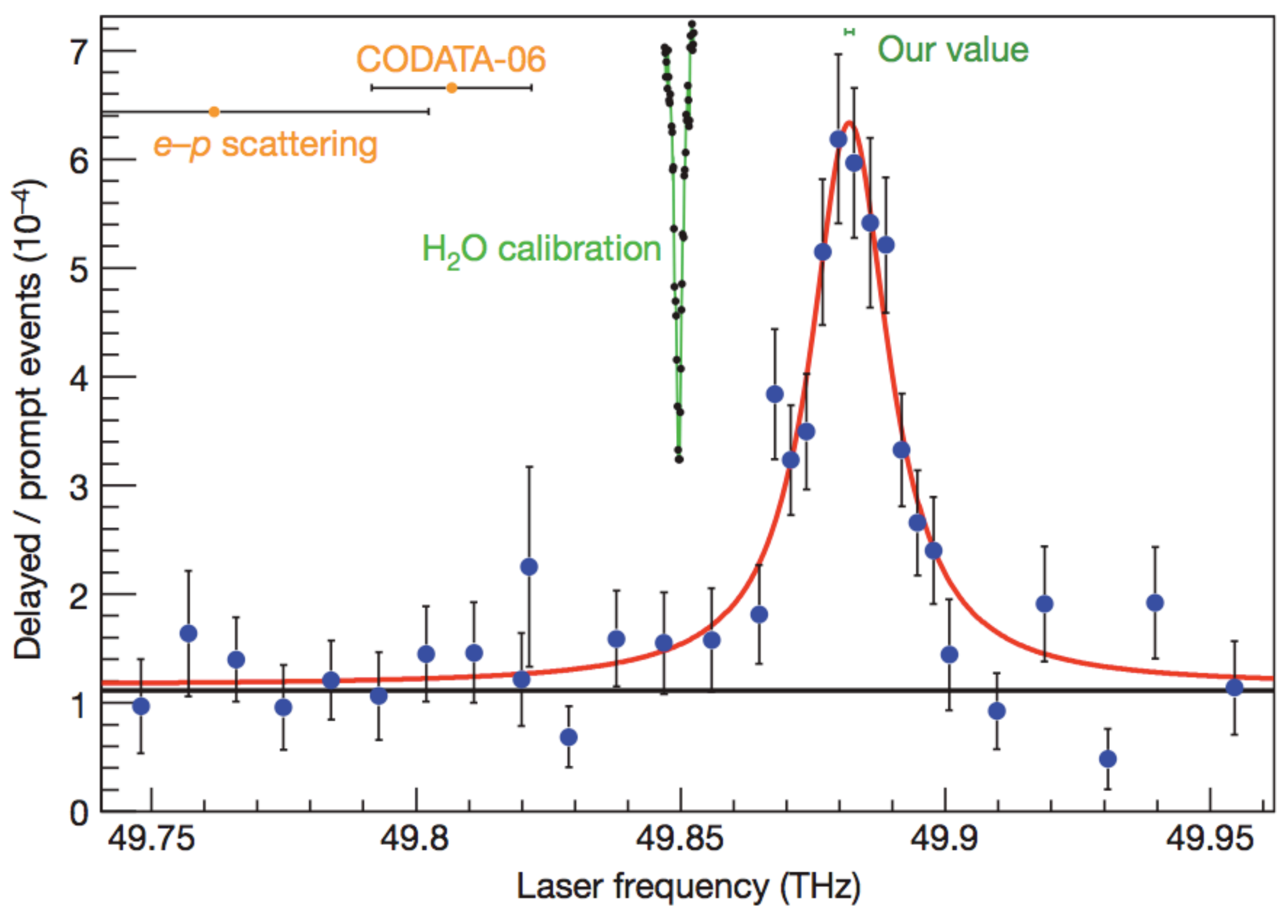



Pohl, Nature **466**(10)213

# μp Lamb Shift Measurement




- (a) on resonance
- (b) off resonance
- Blue peak: prompt X-Rays
- Red peak: Signal from 2P to 1S decay




- 2S-1P resonance shape
- Ratio of red counts (signal X-rays) to blue counts (prompt X-rays)
- Shape is a Lorentzian with flat background
- Central frequency comes from a least-squares fit

Pohl, Nature **466**(10)213

# Intermission

- Will the true $r_E$ please stand up?
- First low $Q^2$ form factor results had $r_E$~0.80-0.87 fm
- Hydrogen Lamb shift results put $r_E$~0.87-0.90 fm
- Form factor fits also gave $r_E$~0.88 fm
- Then the muonic hydrogen weighed in at $r_E$ = 0.8418(7) fm (very accurate and very low)

- Could the muonic hydrogen measurements be wrong?
  - New experiments on muonic deuterium will help resolve this issue since we think we understand the deuteron radius very well.
- Could the electronic hydrogen measurements be wrong?
  - These should be repeated with modern equipment and much better accuracy
- Could the form factor slopes at low $Q^2$ be wrong?
  - New very low $Q^2$ measurements should be done.
- Could the QED in higher-orders still need work?
- Could there be new physics such as dark photons that shift the states?

# State of the Art

PRL **105,** 242001 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

## High-Precision Determination of the Electric and Magnetic Form Factors of the Proton

J. C. Bernauer,[1,*] P. Achenbach,[1] C. Ayerbe Gayoso,[1] R. Böhm,[1] D. Bosnar,[2] L. Debenjak,[3] M. O. Distler,[1,†] L. Doria,[1] A. Esser,[1] H. Fonvieille,[4] J. M. Friedrich,[5] J. Friedrich,[1] M. Gómez Rodríguez de la Paz,[1] M. Makek,[2] H. Merkel,[1] D. G. Middleton,[1] U. Müller,[1] L. Nungesser,[1] J. Pochodzalla,[1] M. Potokar,[3] S. Sánchez Majos,[1] B. S. Schlimme,[1] S. Širca,[6,3] Th. Walcher,[1] and M. Weinriefer[1]

(A1 Collaboration)

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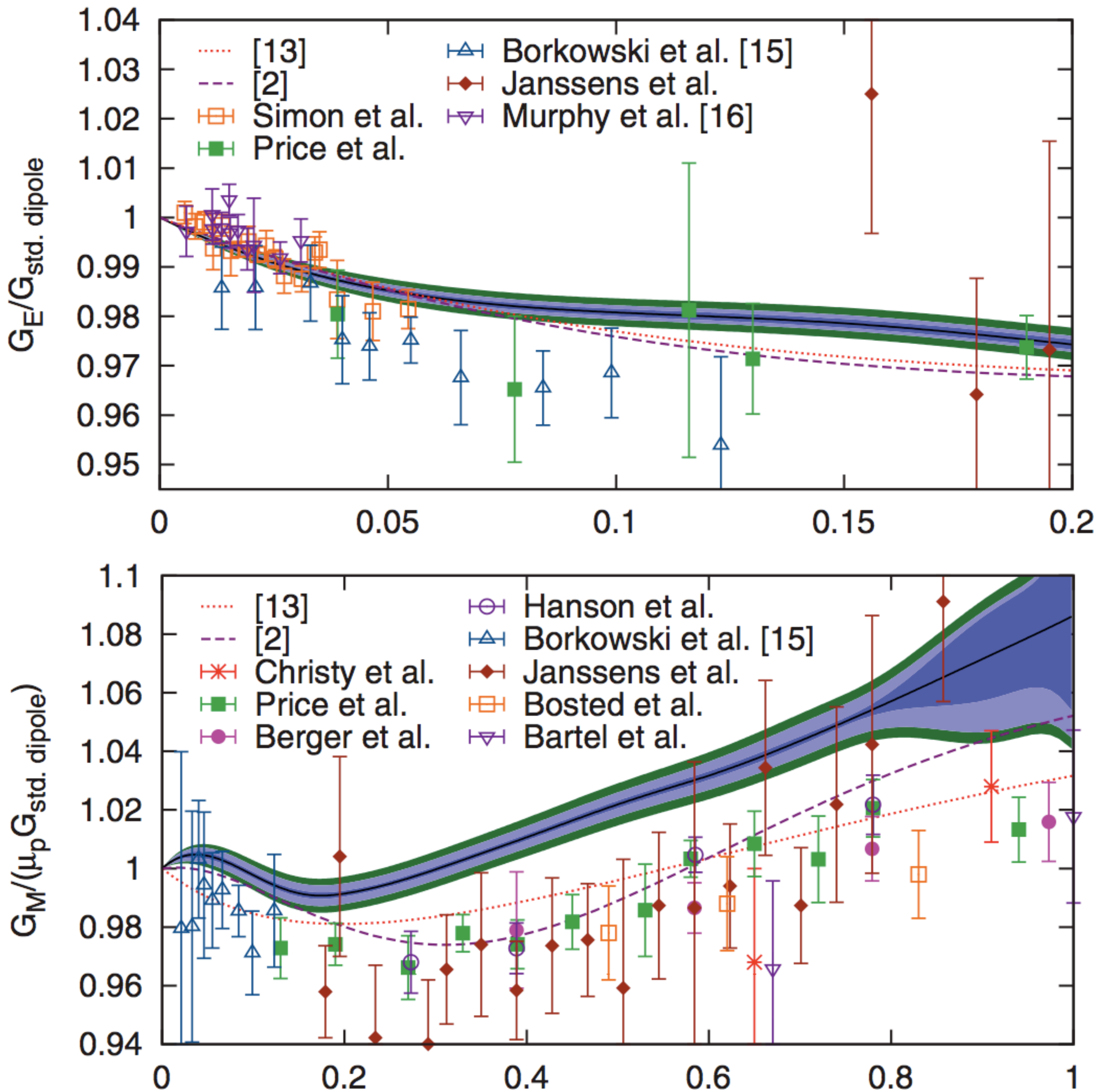
(Received 29 July 2010; published 10 December 2010)

New precise results of a measurement of the elastic electron-proton scattering cross section performed at the Mainz Microtron MAMI are presented. About 1400 cross sections were measured with negative four-momentum transfers squared up to $Q^2 = 1\ (\mathrm{GeV}/c)^2$ with statistical errors below 0.2%. The electric and magnetic form factors of the proton were extracted by fits of a large variety of form factor models directly to the cross sections. The form factors show some features at the scale of the pion cloud. The charge and magnetic radii are determined to be $\langle r_E^2 \rangle^{1/2} = 0.879(5)_{\mathrm{stat}}(4)_{\mathrm{syst}}(2)_{\mathrm{model}}(4)_{\mathrm{group}}$ fm and $\langle r_M^2 \rangle^{1/2} = 0.777(13)_{\mathrm{stat}}(9)_{\mathrm{syst}}(5)_{\mathrm{model}}(2)_{\mathrm{group}}$ fm.

Small, but first good low $Q^2$ data

Similar to others

# Mainz 2010




$r_E$ = 0.883(8) fm
$r_M$= 0.775(16) fm
Bernauer
PRL**105**(10)242001

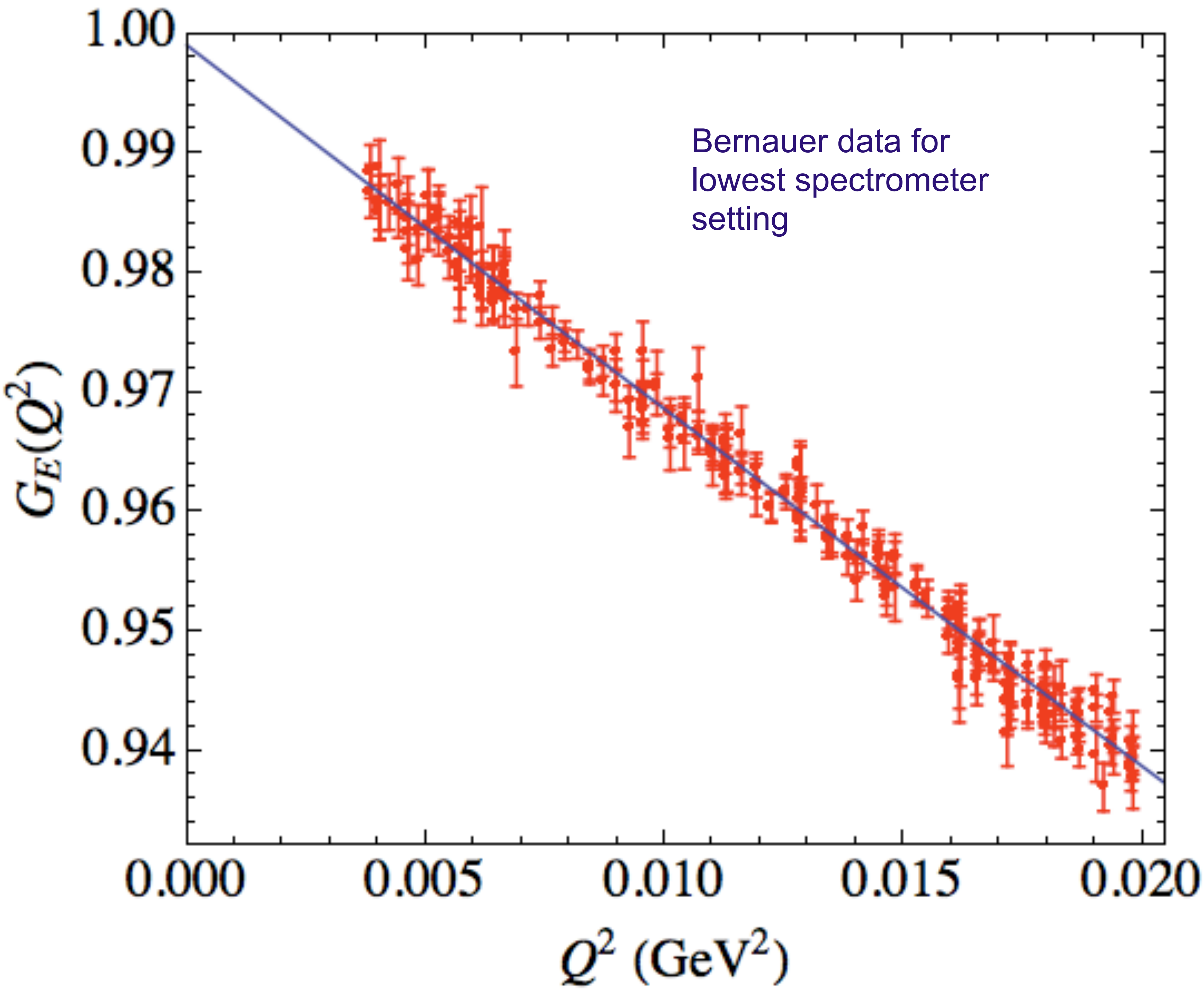
Black line:
best fit
Blue shade:
stat. 68% confidence
Purple:
exp. systematic errors
Green:
±50% variation of
Coulomb correction

This data set supersedes all others for $Q^2$<1 $GeV^2$

But notice data are several percent above the average

Bernauer, PRL**105**(10)242001

# Low $Q^2$ $G_E$ in 2010




$r_E$ = 0.848(19) fm

Fit to $G_E(Q^2)=a_0+a_1Q^2+a_2Q^4$ by C. Carlson
Mainz 2010 low-$Q^2$ data

# Hyperfine Splittings & $r_Z$

$$E_{\rm hfs}(e^- p) = 1420.405\,751\,766\,7(9)\ \text{MHz}$$

Ground State Hydrogen Hyperfine splitting measured to 13-digit accuracy.

$$E_{\rm hfs}(e^- p) = (1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_{\rm hvp}^p + \Delta_{\mu\rm vp}^p + \Delta_{\rm weak}^p + \Delta_S)E_F^p$$

The splitting depends on QED, recoil, vacuum polarization, weak forces and the proton structure.

$$E_F^p = \frac{8}{3\pi}\alpha^3 \mu_B \mu_p \frac{m_e^3 m_p^3}{(m_p + m_e)^3}$$

$$\Delta_S = \Delta_Z + \Delta_{\rm pol} = -38.58(16)\ \text{ppm}$$

Nuclear contribution of about 39 ppm

$$\Delta_Z = -2\alpha m_e r_Z (1 + \delta_Z^{\rm rad})$$

Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa_p} - 1\right]$$

$$\Delta_{\rm pol} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2)$$

Polarizability:
$\Delta_{\rm pol}$=1.32±0.24 ppm
Zemach radius dominates!

$$\tau = \nu^2/Q^2$$

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\}$$

$$B_1 = \int_0^{x_{\rm th}} dx\,\beta(\tau) g_1(x, Q^2),$$

$$\Delta_2 = -24 m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2).$$

$$B_2 = \int_0^{x_{\rm th}} dx\,\beta_2(\tau) g_2(x, Q^2)$$

$$\beta(\tau) = \frac{4}{9}[-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}]$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)},$$

Nazaryan, Carlson, Griffioen, PRL**96**(06)163001

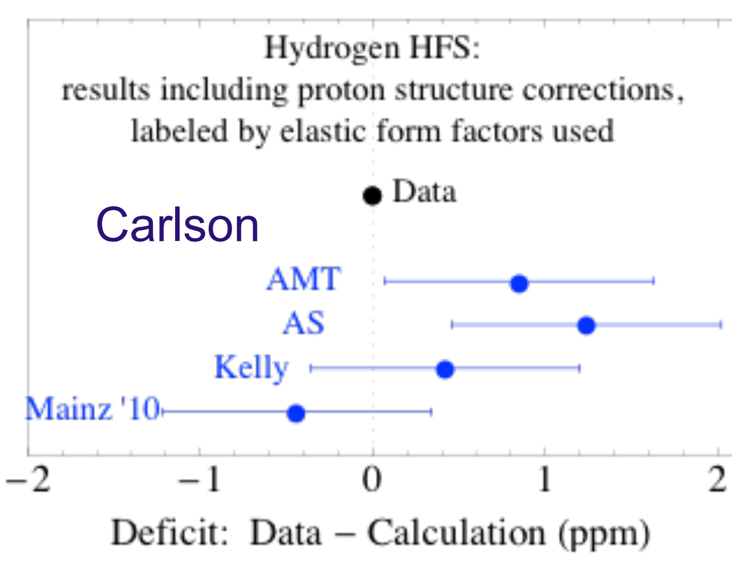
# Hyperfine in μp Lamb Shift

TABLE I. Hyperfine splitting for the 2$S$ state of muonic hydrogen, using different modern analytic fits in the terms that involve elastic form factors.

| Form factor fit | $E^{2S}_{\mathrm{HFS}}$ (meV) | $r_Z$ (fm) |
|---|---|---|
| AMT [18] | 22.8123 | 1.080 |
| Kelly [19] | 22.8141 | 1.069 |
| AS [20] | 22.8105 | 1.091 |
| Mainz 2010 [21–23] | 22.8187 | 1.045 |

Smaller because of smaller $r_M$

The dependence of the muon Lamb shift measurements on the Zemach radius are too small to change the extracted proton charge radius.
However, the smaller Mainz Zemach radius brings theory and experiment closer for the hydrogen hyperfine splitting.




Nazaryan, Carlson, Griffioen, PRA**83**(11)042509

# Proposal for Very Low $Q^2$

PR12-11-106 Hall B with PrimEx-II Detectors

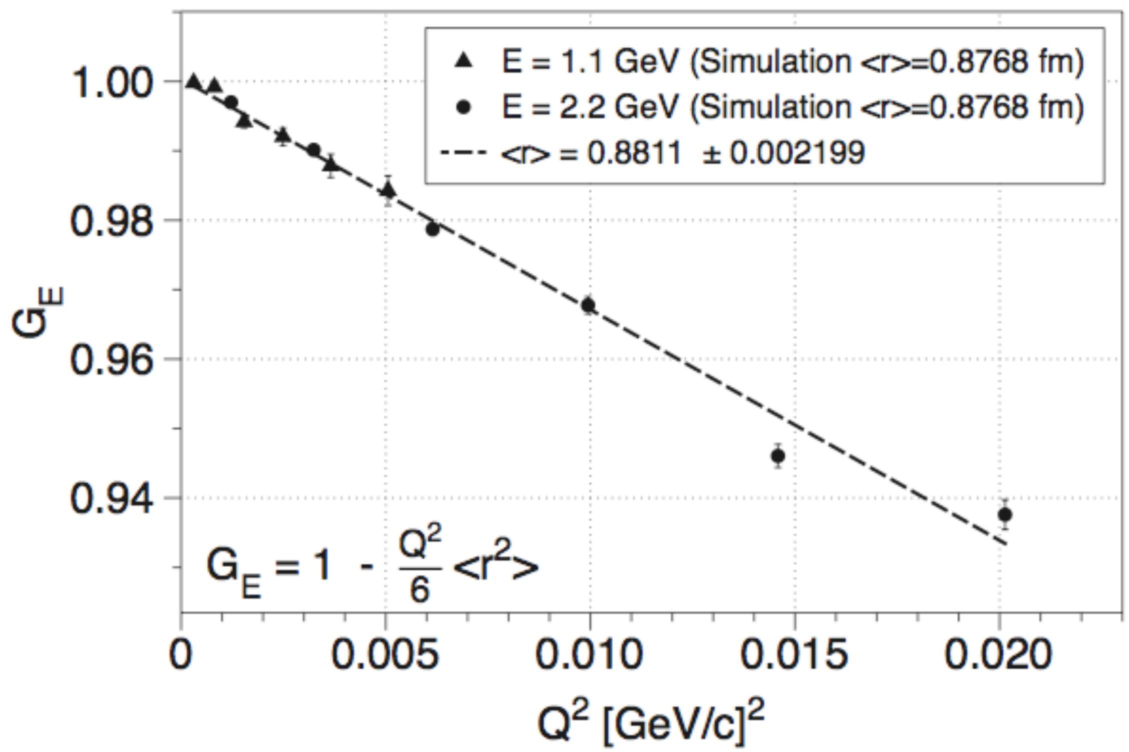
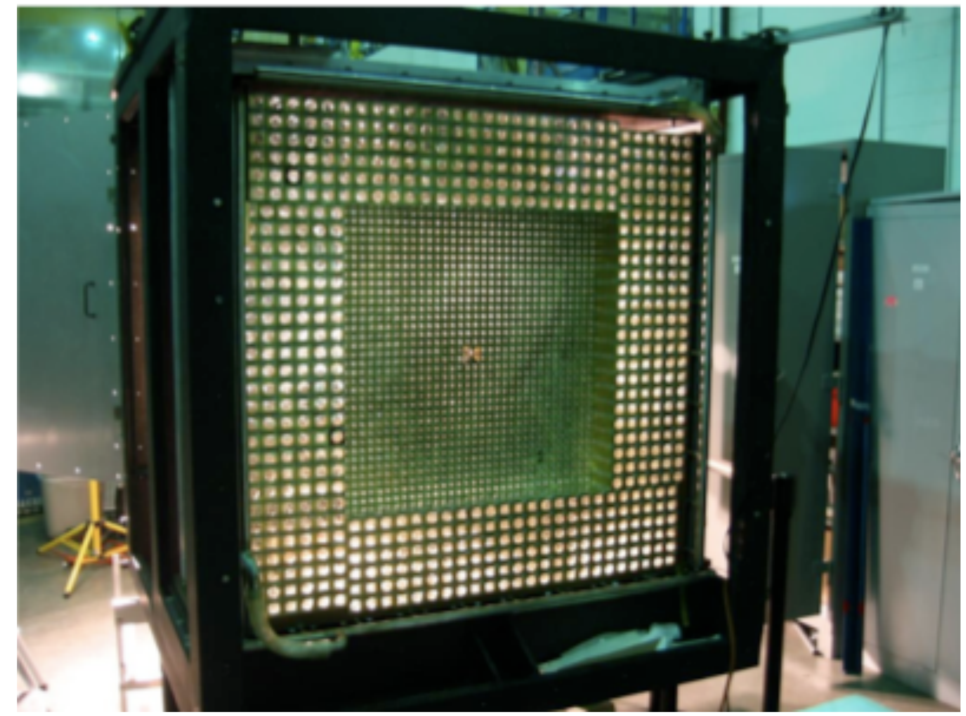
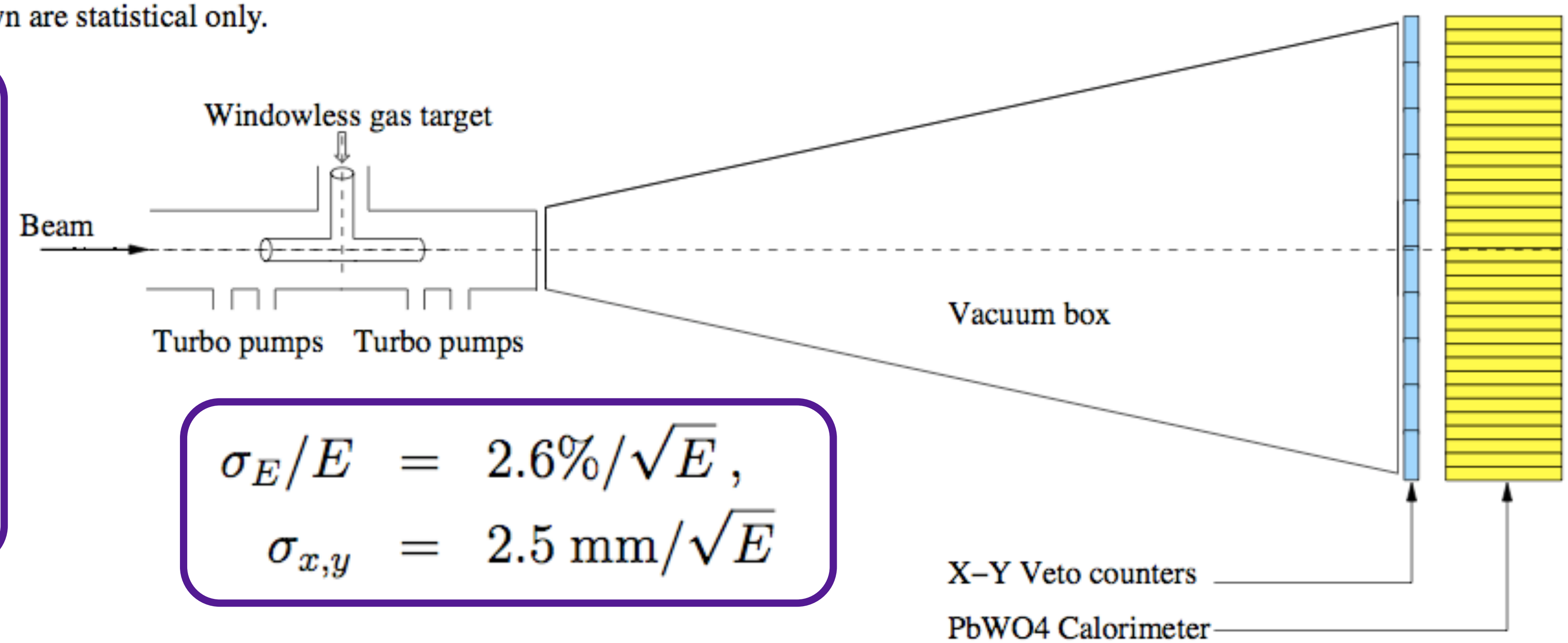



Figure 30: Extraction of $G_E^p$ from Monte Carlo generated data set for $E_0 = 1.1$ GeV and $E_0 = 2.2$ GeV runs for the value of $r_p = 0.8768$ fm. The error bars shown are statistical only.



HYCAL

- 70 cm x 70 cm calorimeter
- 2.05 cm x 2.05 cm $PbWO_4$
- Measure ep and ee (Moller) simultaneously
- $E_{beam}$ = 1.1 and 2.2 GeV
- Conditionally approved




$$\sigma_E/E = 2.6\%/\sqrt{E}\,,$$
$$\sigma_{x,y} = 2.5\ \mathrm{mm}/\sqrt{E}$$

# The Nucleon Size Crisis

- First there was the nucleon spin crisis, now there is the nucleon size crisis.
- There is good reason to believe the small Mainz 2010 magnetic radius because this is the first set of data that extends to low enough $Q^2$ so that this radius isn't just an artifact of the global fit. Moreover, the smaller magnetic radius drives the Zemach radius to a smaller value, which makes the hydrogen hyperfine theory and experiment agree.
- There is no good reason to doubt the muonic Lamb shift measurements. However, the electronic Lamb shifts and the form factor analyses have fairly consistently given much bigger radii.
- If the muonic Lamb shift is correct, then both the electronic Lamb shift and the form factor results are wrong, unless there is some unique physics related to the muon.
- As usual, new, precise data are needed to answer these questions. In the mean time, we have plenty of room for speculation.